

# Symbolic Math Toolbox™ 5

## User's Guide

MATLAB®

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*Symbolic Math Toolbox™ User's Guide*

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# Introduction

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- “Accessing Symbolic Math Toolbox Functionality” on page 1-3

## Product Overview

Symbolic Math Toolbox™ software lets you to perform symbolic computations within the MATLAB® numeric environment. It provides tools for solving and manipulating symbolic math expressions and performing variable-precision arithmetic. The toolbox contains hundreds of symbolic functions that leverage the MuPAD® engine for a broad range of mathematical tasks such as:

- Differentiation
- Integration
- Linear algebraic operations
- Simplification
- Transforms
- Variable-precision arithmetic
- Equation solving

Symbolic Math Toolbox software also includes the MuPAD language, which is optimized for handling and operating on symbolic math expressions. In addition to covering common mathematical tasks, the libraries of MuPAD functions cover specialized areas such as number theory and combinatorics. You can extend the built-in functionality by writing custom symbolic functions and libraries in the MuPAD language.

# Accessing Symbolic Math Toolbox Functionality

## In this section...

- “Key Features” on page 1-3
- “Working from MATLAB” on page 1-3
- “Working from MuPAD” on page 1-3

## Key Features

Symbolic Math Toolbox software provides a complete set of tools for symbolic computing that augments the numeric capabilities of MATLAB. The toolbox includes extensive symbolic functionality that you can access directly from the MATLAB command line or from the MuPAD Notebook Interface. You can extend the functionality available in the toolbox by writing custom symbolic functions or libraries in the MuPAD language.

## Working from MATLAB

You can access the Symbolic Math Toolbox functionality directly from the MATLAB Command Window. This environment lets you call functions using familiar MATLAB syntax.

The MATLAB Help browser presents the documentation that covers working from the MATLAB Command Window. To access the MATLAB Help browser:

- Select **Help > Product Help**, and then select **Symbolic Math Toolbox** in the left pane.
- Enter `doc` at the MATLAB command line.

If you are a new user, begin with Chapter 2, “Getting Started”.

## Working from MuPAD

Also you can access the Symbolic Math Toolbox functionality from the MuPAD Notebook Interface using the MuPAD language. The MuPAD Notebook Interface includes a symbol palette for accessing common MuPAD functions. All results are displayed in typeset math. You also can convert the results

into MathML and TeX. You can embed graphics, animations, and descriptive text within your notebook.

An editor, debugger, and other programming utilities provide tools for authoring custom symbolic functions and libraries in the MuPAD language. The MuPAD language supports multiple programming styles including imperative, functional, and object-oriented programming. The language treats variables as symbolic by default and is optimized for handling and operating on symbolic math expressions. You can call functions written in the MuPAD language from the MATLAB Command Window. For more information, see “Calling Built-In MuPAD Functions from the MATLAB Command Window” on page 4-40

The MuPAD Help browser presents documentation covering the MuPAD Notebook Interface. To access the MuPAD Help browser:

- From the MuPAD Notebook Interface, select **Help > Open Help**.
- From the MATLAB Command Window, enter `doc(symengine)`.

If you are a new user of the MuPAD Notebook Interface, read the Getting Started chapter of the MuPAD documentation.

There is also a MuPAD Tutorial PDF file available at <http://www.mathworks.com/help/toolbox/mupad/pdflinks/tutorium.pdf>.

# Getting Started

---

- “Symbolic Objects” on page 2-2
- “Creating Symbolic Variables and Expressions” on page 2-6
- “Performing Symbolic Computations” on page 2-12
- “Assumptions for Symbolic Objects” on page 2-31

## Symbolic Objects

In this section...
“Overview of Symbolic Objects” on page 2-2
“Symbolic Variables” on page 2-2
“Symbolic Numbers” on page 2-3

### Overview of Symbolic Objects

Symbolic objects are a special MATLAB data type introduced by the Symbolic Math Toolbox software. They allow you to perform mathematical operations in the MATLAB workspace analytically, without calculating numeric values. You can use symbolic objects to perform a wide variety of analytical computations:

- Differentiation, including partial differentiation
- Definite and indefinite integration
- Taking limits, including one-sided limits
- Summation, including Taylor series
- Matrix operations
- Solving algebraic and differential equations
- Variable-precision arithmetic
- Integral transforms

Symbolic objects present symbolic variables, symbolic numbers, symbolic expressions, and symbolic matrices.

### Symbolic Variables

To declare variables  $x$  and  $y$  as symbolic objects use the `syms` command:

```
syms x y
```

You can manipulate the symbolic objects according to the usual rules of mathematics. For example:



```
x + x + y
```

```
ans =  
2*x + y
```

You also can create formal symbolic mathematical expressions and symbolic matrices. See “Creating Symbolic Variables and Expressions” on page 2-6 for more information.

## Symbolic Numbers

Symbolic Math Toolbox software also enables you to convert numbers to symbolic objects. To create a symbolic number, use the `sym` command:

```
a = sym('2')
```

If you create a symbolic number with 10 or fewer decimal digits, you can skip the quotes:

```
a = sym(2)
```

The following example illustrates the difference between a standard double-precision MATLAB data and the corresponding symbolic number. The MATLAB command

```
sqrt(2)
```

returns a double-precision floating-point number:

```
ans =  
1.4142
```

On the other hand, if you calculate a square root of a symbolic number 2:

```
a = sqrt(sym(2))
```

you get the precise symbolic result:

```
a =  
2^(1/2)
```

Symbolic results are not indented. Standard MATLAB double-precision results are indented. The difference in output form shows what type of data is presented as a result.

To evaluate a symbolic number numerically, use the `double` command:

```
double(a)

ans =
    1.4142
```

You also can create a rational fraction involving symbolic numbers:

```
sym(2)/sym(5)

ans =
    2/5
```

or more efficiently:

```
sym(2/5)

ans =
    2/5
```

MATLAB performs arithmetic on symbolic fractions differently than it does on standard numeric fractions. By default, MATLAB stores all numeric values as double-precision floating-point data. For example:

```
2/5 + 1/3

ans =
    0.7333
```

If you add the same fractions as symbolic objects, MATLAB finds their common denominator and combines them in the usual procedure for adding rational numbers:

```
sym(2/5) + sym(1/3)

ans =
    11/15
```

To learn more about symbolic representation of rational and decimal fractions, see “Estimating the Precision of Numeric to Symbolic Conversions” on page 2-17.

## Creating Symbolic Variables and Expressions

### In this section...

“Creating Symbolic Variables” on page 2-6

“Creating Symbolic Expressions” on page 2-7

“Creating Symbolic Objects with Identical Names” on page 2-8

“Creating a Matrix of Symbolic Variables” on page 2-9

“Creating a Matrix of Symbolic Numbers” on page 2-10

“Finding Symbolic Variables in Expressions and Matrices” on page 2-11

### Creating Symbolic Variables

The `sym` command creates symbolic variables and expressions. For example, the commands

```
x = sym('x');  
a = sym('alpha');
```

create a symbolic variable `x` with the value `x` assigned to it in the MATLAB workspace and a symbolic variable `a` with the value `alpha` assigned to it. An alternate way to create a symbolic object is to use the `syms` command:

```
syms x;  
a = sym('alpha');
```

You can use `sym` or `syms` to create symbolic variables. The `syms` command:

- Does not use parentheses and quotation marks: `syms x`
- Can create multiple objects with one call
- Serves best for creating individual single and multiple symbolic variables

The `sym` command:

- Requires parentheses and quotation marks: `x = sym('x')`. When creating a symbolic number with 10 or fewer decimal digits, you can skip the quotation marks: `f = sym(5)`.

- Creates one symbolic object with each call.
- Serves best for creating symbolic numbers and symbolic expressions.
- Serves best for creating symbolic objects in functions and scripts.

---

**Note** In Symbolic Math Toolbox, `pi` is a reserved word.

---

## Creating Symbolic Expressions

Suppose you want to use a symbolic variable to represent the golden ratio

$$\rho = \frac{1 + \sqrt{5}}{2}$$

The command

```
phi = sym('(1 + sqrt(5))/2');
```

achieves this goal. Now you can perform various mathematical operations on `phi`. For example,

```
f = phi^2 - phi - 1
```

returns

```
f =
(5^(1/2)/2 + 1/2)^2 - 5^(1/2)/2 - 3/2
```

Now suppose you want to study the quadratic function  $f = ax^2 + bx + c$ . One approach is to enter the command

```
f = sym('a*x^2 + b*x + c');
```

which assigns the symbolic expression  $ax^2 + bx + c$  to the variable `f`. However, in this case, Symbolic Math Toolbox software does not create variables corresponding to the terms of the expression: `a`, `b`, `c`, and `x`. To perform symbolic math operations on `f`, you need to create the variables explicitly. A better alternative is to enter the commands

```
a = sym('a');
```

```
b = sym('b');  
c = sym('c');  
x = sym('x');
```

or simply

```
syms a b c x
```

Then, enter

```
f = a*x^2 + b*x + c;
```

---

**Tip** To create a symbolic expression that is a constant, you must use the `sym` command. Do not use `syms` command to create a symbolic expression that is a constant. For example, to create the expression whose value is 5, enter `f = sym(5)`. The command `f = 5` does *not* define `f` as a symbolic expression.

---

## Creating Symbolic Objects with Identical Names

If you set a variable equal to a symbolic expression, and then apply the `syms` command to the variable, MATLAB software removes the previously defined expression from the variable. For example,

```
syms a b;  
f = a + b
```

returns

```
f =  
a + b
```

If later you enter

```
syms f;  
f
```

then MATLAB removes the value `a + b` from the expression `f`:

```
f =  
f
```

You can use the `syms` command to clear variables of definitions that you previously assigned to them in your MATLAB session. However, `syms` does not clear the following assumptions of the variables: complex, real, and positive. These assumptions are stored separately from the symbolic object. See “Deleting Symbolic Objects and Their Assumptions” on page 2-32 for more information.

## Creating a Matrix of Symbolic Variables

### Using Existing Symbolic Objects as Elements

A circulant matrix has the property that each row is obtained from the previous one by cyclically permuting the entries one step forward. For example, create the symbolic circulant matrix whose elements are `a`, `b`, and `c`, using the commands:

```
syms a b c;
A = [a b c; c a b; b c a]
```

```
A =
[ a, b, c]
[ c, a, b]
[ b, c, a]
```

Since matrix `A` is circulant, the sum of elements over each row and each column is the same. Find the sum of all the elements of the first row:

```
sum(A(1,:))

ans =
a + b + c
```

Check if the sum of the elements of the first row equals the sum of the elements of the second column:

```
sum(A(1,:)) == sum(A(:,2))
```

The sums are equal:

```
ans =
1
```

From this example, you can see that using symbolic objects is very similar to using regular MATLAB numeric objects.

### Generating Elements While Creating a Matrix

The `sym` function also lets you define a symbolic matrix or vector without having to define its elements in advance. In this case, the `sym` function generates the elements of a symbolic matrix at the same time when it creates a matrix. The function presents all generated elements using the same form: the base (which must be a valid variable name), a row index, and a column index. Use the first argument of `sym` to specify the base for the names of generated elements. You can use any valid variable name as a base. To check whether the name is a valid variable name, use the `isvarname` function. By default, `sym` separates a row index and a column index by underscore. For example, create the 2-by-4 matrix `A` with the elements `A1_1`, ..., `A2_4`:

```
A = sym('A', [2 4])

A =
 [ A1_1, A1_2, A1_3, A1_4]
 [ A2_1, A2_2, A2_3, A2_4]
```

To control the format of the generated names of matrix elements, use `%d` in the first argument:

```
A = sym('A%d%d', [2 4])

A =
 [ A11, A12, A13, A14]
 [ A21, A22, A23, A24]
```

### Creating a Matrix of Symbolic Numbers

A particularly effective use of `sym` is to convert a matrix from numeric to symbolic form. The command

```
A = hilb(3)
```

generates the 3-by-3 Hilbert matrix:

```
A =
    1.0000    0.5000    0.3333
```



```

0.5000    0.3333    0.2500
0.3333    0.2500    0.2000

```

By applying `sym` to `A`

```
A = sym(A)
```

you can obtain the precise symbolic form of the 3-by-3 Hilbert matrix:

```

A =
[ 1, 1/2, 1/3]
[ 1/2, 1/3, 1/4]
[ 1/3, 1/4, 1/5]

```

For more information on numeric to symbolic conversions, see “Estimating the Precision of Numeric to Symbolic Conversions” on page 2-17.

## Finding Symbolic Variables in Expressions and Matrices

To determine what symbolic variables are present in an expression, use the `symvar` command. For example, given the symbolic expressions `f` and `g` defined by

```

syms a b n t x z;
f = x^n;
g = sin(a*t + b);

```

you can find the symbolic variables in `f` by entering:

```

symvar(f)

ans =
[ n, x]

```

Similarly, you can find the symbolic variables in `g` by entering:

```

symvar(g)

ans =
[ a, b, t]

```

## Performing Symbolic Computations

### In this section...

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### Simplifying Symbolic Expressions

Symbolic Math Toolbox provides a set of simplification functions allowing you to manipulate an output of a symbolic expression. For example, the following polynomial of the golden ratio phi

```
phi = sym('(1 + sqrt(5))/2');
f = phi^2 - phi - 1
```

returns

```
f =
(5^(1/2)/2 + 1/2)^2 - 5^(1/2)/2 - 3/2
```

You can simplify this answer by entering

```
simplify(f)
```

and get a very short answer:

```
ans =
0
```

Symbolic simplification is not always so straightforward. There is no universal simplification function, because the meaning of a simplest representation of

a symbolic expression cannot be defined clearly. Different problems require different forms of the same mathematical expression. Knowing what form is more effective for solving your particular problem, you can choose the appropriate simplification function.

For example, to show the order of a polynomial or symbolically differentiate or integrate a polynomial, use the standard polynomial form with all the parenthesis multiplied out and all the similar terms summed up. To rewrite a polynomial in the standard form, use the `expand` function:

```
syms x;
f = (x ^2- 1)*(x^4 + x^3 + x^2 + x + 1)*(x^4 - x^3 + x^2 - x + 1);
expand(f)

ans =
x^10 - 1
```

The `factor` simplification function shows the polynomial roots. If a polynomial cannot be factored over the rational numbers, the output of the `factor` function is the standard polynomial form. For example, to factor the third-order polynomial, enter:

```
syms x;
g = x^3 + 6*x^2 + 11*x + 6;
factor(g)

ans =
(x + 3)*(x + 2)*(x + 1)
```

The nested (Horner) representation of a polynomial is the most efficient for numerical evaluations:

```
syms x;
h = x^5 + x^4 + x^3 + x^2 + x;
horner(h)

ans =
x*(x*(x*(x*(x + 1) + 1) + 1) + 1)
```

For a list of Symbolic Math Toolbox simplification functions, see “Simplifications” on page 3-30.

## Substituting in Symbolic Expressions

### Substituting Symbolic Variables with Numbers

You can substitute a symbolic variable with a numeric value by using the `subs` function. For example, evaluate the symbolic expression `f` at the point  $x = 2/3$ :

```
syms x;  
f = 2*x^2 - 3*x + 1;  
subs(f, 1/3)  
  
ans =  
    0.2222
```

The `subs` function does not change the original expression `f`:

```
f  
  
f =  
2*x^2 - 3*x + 1
```

### Substituting in Multivariate Expressions

When your expression contains more than one variable, you can specify the variable for which you want to make the substitution. For example, to substitute the value  $x = 3$  in the symbolic expression

```
syms x y;  
f = x^2*y + 5*x*sqrt(y);
```

enter the command

```
subs(f, x, 3)  
  
ans =  
9*y + 15*y^(1/2)
```

### Substituting One Symbolic Variable for Another

You also can substitute one symbolic variable for another symbolic variable. For example to replace the variable `y` with the variable `x`, enter

```
subs(f, y, x)
```

```
ans =
x^3 + 5*x^(3/2)
```

## Substituting a Matrix into a Polynomial

You can also substitute a matrix into a symbolic polynomial with numeric coefficients. There are two ways to substitute a matrix into a polynomial: element by element and according to matrix multiplication rules.

**Element-by-Element Substitution.** To substitute a matrix at each element, use the `subs` command:

```
A = [1 2 3;4 5 6];
syms x; f = x^3 - 15*x^2 - 24*x + 350;
subs(f,A)

ans =
    312    250    170
     78    -20   -118
```

You can do element-by-element substitution for rectangular or square matrices.

**Substitution in a Matrix Sense.** If you want to substitute a matrix into a polynomial using standard matrix multiplication rules, a matrix must be square. For example, you can substitute the magic square A into a polynomial f:

**1** Create the polynomial:

```
syms x;
f = x^3 - 15*x^2 - 24*x + 350;
```

**2** Create the magic square matrix:

```
A = magic(3)

A =
     8     1     6
     3     5     7
     4     9     2
```

- 3** Get a row vector containing the numeric coefficients of the polynomial  $f$ :

```
b = sym2poly(f)
```

```
b =  
    1   -15   -24   350
```

- 4** Substitute the magic square matrix  $A$  into the polynomial  $f$ . Matrix  $A$  replaces all occurrences of  $x$  in the polynomial. The constant times the identity matrix  $\text{eye}(3)$  replaces the constant term of  $f$ :

```
A^3 - 15*A^2 - 24*A + 350*eye(3)
```

```
ans =  
   -10     0     0  
     0   -10     0  
     0     0   -10
```

The `polyvalm` command provides an easy way to obtain the same result:

```
polyvalm(sym2poly(f),A)
```

```
ans =  
   -10     0     0  
     0   -10     0  
     0     0   -10
```

### **Substituting the Elements of a Symbolic Matrix**

To substitute a set of elements in a symbolic matrix, also use the `subs` command. Suppose you want to replace some of the elements of a symbolic circulant matrix  $A$

```
syms a b c;  
A = [a b c; c a b; b c a]
```

```
A =  
[ a, b, c]  
[ c, a, b]  
[ b, c, a]
```

To replace the (2, 1) element of A with beta and the variable b throughout the matrix with variable alpha, enter

```
alpha = sym('alpha');
beta = sym('beta');
A(2,1) = beta;
A = subs(A,b,alpha)
```

The result is the matrix:

```
A =
[      a, alpha,      c]
[ beta,      a, alpha]
[ alpha,      c,      a]
```

For more information on the subs command see “Substitutions” on page 3-41.

## Estimating the Precision of Numeric to Symbolic Conversions

The sym command converts a numeric scalar or matrix to symbolic form. By default, the sym command returns a rational approximation of a numeric expression. For example, you can convert the standard double-precision variable into a symbolic object:

```
t = 0.1;
sym(t)

ans =
1/10
```

The technique for converting floating-point numbers is specified by the optional second argument, which can be 'f', 'r', 'e' or 'd'. The default option is 'r' that stands for rational approximation “Converting to Rational Symbolic Form” on page 2-18.

## Converting to Floating-Point Symbolic Form

The 'f' option to sym converts a double-precision floating-point number to a sum of two binary numbers. All values are represented as rational numbers  $N \cdot 2^e$ , where e and N are integers, and N is nonnegative. For example,

```
sym(t, 'f')
```

returns the symbolic floating-point representation:

```
ans =  
3602879701896397/36028797018963968
```

### **Converting to Rational Symbolic Form**

If you call `sym` command with the 'r' option

```
sym(t, 'r')
```

you get the results in the rational form:

```
ans =  
1/10
```

This is the default setting for the `sym` command. If you call this command without any option, you get the result in the same rational form:

```
sym(t)  
  
ans =  
1/10
```

### **Converting to Rational Symbolic Form with Machine Precision**

If you call the `sym` command with the option 'e', it returns the rational form of `t` plus the difference between the theoretical rational expression for `t` and its actual (machine) floating-point value in terms of `eps` (the floating-point relative precision):

```
sym(t, 'e')  
  
ans =  
eps/40 + 1/10
```

### **Converting to Decimal Symbolic Form**

If you call the `sym` command with the option 'd', it returns the decimal expansion of `t` up to the number of significant digits:



```
sym(t, 'd')
```

```
ans =
0.100000000000000000555111512312578
```

By default, the `sym(t, 'd')` command returns a number with 32 significant digits. To change the number of significant digits, use the `digits` command:

```
digits(7);
sym(t, 'd')
```

```
ans =
0.1
```

## Differentiating Symbolic Expressions

With the Symbolic Math Toolbox software, you can find

- Derivatives of single-variable expressions
- Partial derivatives
- Second and higher order derivatives
- Mixed derivatives

For in-depth information on taking symbolic derivatives see “Differentiation” on page 3-2.

## Expressions with One Variable

To differentiate a symbolic expression, use the `diff` command. The following example illustrates how to take a first derivative of a symbolic expression:

```
syms x;
f = sin(x)^2;
diff(f)

ans =
2*cos(x)*sin(x)
```

## Partial Derivatives

For multivariable expressions, you can specify the differentiation variable. If you do not specify any variable, MATLAB chooses a default variable by the proximity to the letter *x*:

```
syms x y;  
f = sin(x)^2 + cos(y)^2;  
diff(f)  
  
ans =  
2*cos(x)*sin(x)
```

For the complete set of rules MATLAB applies for choosing a default variable, see “Finding a Default Symbolic Variable” on page 2-25.

To differentiate the symbolic expression *f* with respect to a variable *y*, enter:

```
syms x y;  
f = sin(x)^2 + cos(y)^2;  
diff(f, y)  
  
ans =  
-2*cos(y)*sin(y)
```

## Second Partial and Mixed Derivatives

To take a second derivative of the symbolic expression *f* with respect to a variable *y*, enter:

```
syms x y;  
f = sin(x)^2 + cos(y)^2;  
diff(f, y, 2)  
  
ans =  
2*sin(y)^2 - 2*cos(y)^2
```

You get the same result by taking derivative twice: `diff(diff(f, y))`. To take mixed derivatives, use two differentiation commands. For example:

```
syms x y;  
f = sin(x)^2 + cos(y)^2;  
diff(diff(f, y), x)
```

```
ans =
0
```

## Integrating Symbolic Expressions

You can perform symbolic integration including:

- Indefinite and definite integration
- Integration of multivariable expressions

For in-depth information on the `int` command including integration with real and complex parameters, see “Integration” on page 3-11.

### Indefinite Integrals of One-Variable Expressions

Suppose you want to integrate a symbolic expression. The first step is to create the symbolic expression:

```
syms x;
f = sin(x)^2;
```

To find the indefinite integral, enter

```
int(f)

ans =
x/2 - sin(2*x)/4
```

### Indefinite Integrals of Multivariable Expressions

If the expression depends on multiple symbolic variables, you can designate a variable of integration. If you do not specify any variable, MATLAB chooses a default variable by the proximity to the letter `x`:

```
syms x y n;
f = x^n + y^n;
int(f)

ans =
x*y^n + (x*x^n)/(n + 1)
```

For the complete set of rules MATLAB applies for choosing a default variable, see “Finding a Default Symbolic Variable” on page 2-25.

You also can integrate the expression  $f = x^n + y^n$  with respect to  $y$

```
syms x y n;  
f = x^n + y^n;  
int(f, y)  
  
ans =  
x^n*y + (y*y^n)/(n + 1)
```

If the integration variable is  $n$ , enter

```
syms x y n;  
f = x^n + y^n;  
int(f, n)  
  
ans =  
x^n/log(x) + y^n/log(y)
```

## Definite Integrals

To find a definite integral, pass the limits of integration as the final two arguments of the `int` function:

```
syms x y n;  
f = x^n + y^n;  
int(f, 1, 10)  
  
ans =  
piecewise([n = -1, log(10) + 9/y],...  
[n <> -1, (10*10^n - 1)/(n + 1) + 9*y^n])
```

## If MATLAB Cannot Find a Closed Form of an Integral

If the `int` function cannot compute an integral, MATLAB issues a warning and returns an unresolved integral:

```
syms x y n;  
f = sin(x)^(1/sqrt(n));
```

```
int(f, n, 1, 10)
```

Warning: Explicit integral could not be found.

```
ans =
int(sin(x)^(1/n^(1/2)), n = 1..10)
```

## Solving Equations

You can solve different types of symbolic equations including:

- Algebraic equations with one symbolic variable
- Algebraic equations with several symbolic variables
- Systems of algebraic equations

For in-depth information on solving symbolic equations including differential equations, see “Solving Equations” on page 3-81.

### Algebraic Equations with One Symbolic Variable

You can find the values of variable  $x$  for which the following expression is equal to zero:

```
syms x;
solve(x^3 - 6*x^2 + 11*x - 6)
```

```
ans =
1
2
3
```

By default, the `solve` command assumes that the right-side of the equation is equal to zero. If you want to solve an equation with a nonzero right part, use quotation marks around the equation:

```
syms x;
solve('x^3 - 6*x^2 + 11*x - 5 = 1')
```

```
ans =  
1  
2  
3
```

### **Algebraic Equations with Several Symbolic Variables**

If an equation contains several symbolic variables, you can designate a variable for which this equation should be solved. For example, you can solve the multivariable equation:

```
syms x y;  
f = 6*x^2 - 6*x^2*y + x*y^2 - x*y + y^3 - y^2;
```

with respect to a symbolic variable  $y$ :

```
solve(f, y)  
  
ans =  
1  
2*x  
-3*x
```

If you do not specify any variable, you get the solution of an equation for the alphabetically closest to  $x$  variable. For the complete set of rules MATLAB applies for choosing a default variable see “Finding a Default Symbolic Variable” on page 2-25.

### **Systems of Algebraic Equations**

You also can solve systems of equations. For example:

```
syms x y z;  
[x, y, z] = solve('z = 4*x', 'x = y', 'z = x^2 + y^2')
```

```
x =  
0  
2  
  
y =  
0  
2  
  
z =  
0  
8
```

## Finding a Default Symbolic Variable

When performing substitution, differentiation, or integration, if you do not specify a variable to use, MATLAB uses a *default* variable. The default variable is basically the one closest alphabetically to  $x$ . To find which variable is chosen as a default variable, use the `symvar(expression, 1)` command.

For example:

```
syms s t;  
g = s + t;  
symvar(g, 1)
```

```
ans =  
t
```

```
syms sx tx;  
g = sx + tx;  
symvar(g, 1)
```

```
ans =  
tx
```

For more information on choosing the default symbolic variable, see the `symvar` command.

## Creating Plots of Symbolic Functions

You can create different types of graphs including:

- Plots of explicit functions
- Plots of implicit functions
- 3-D parametric plots
- Surface plots

See “Pedagogical and Graphical Applications” on page 5-6 for in-depth coverage of Symbolic Math Toolbox graphics and visualization tools.



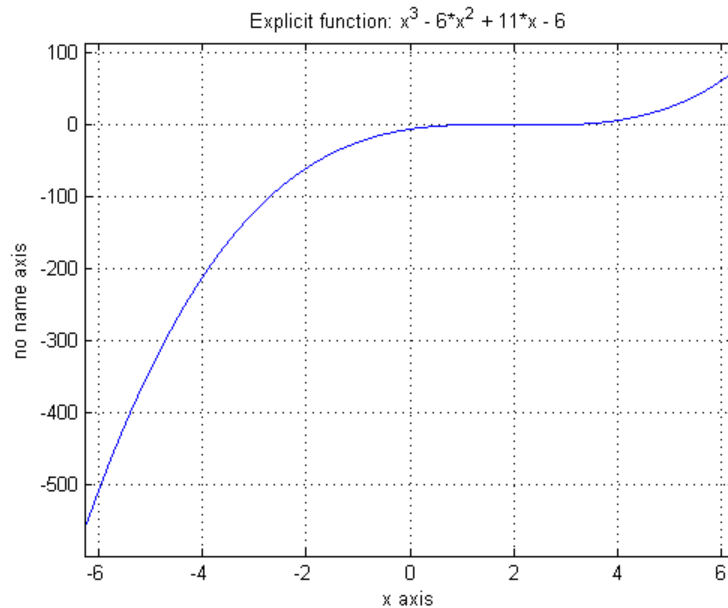
## Explicit Function Plot

The simplest way to create a plot is to use the `ezplot` command:

```
syms x;  
ezplot(x^3 - 6*x^2 + 11*x - 6);  
hold on;
```

The `hold on` command retains the existing plot allowing you to add new elements and change the appearance of the plot. For example, now you can change the names of the axes and add a new title and grid lines. When you finish working with the current plot, enter the `hold off` command:

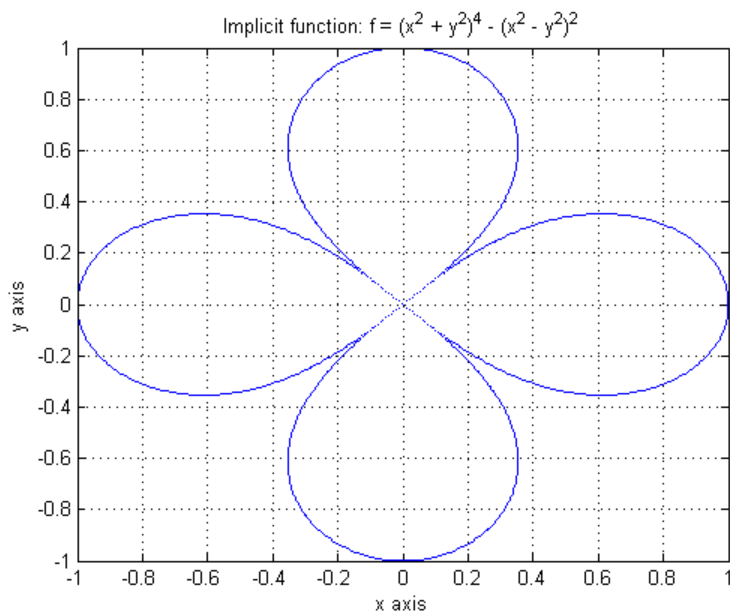
```
xlabel('x axis');  
ylabel('no name axis');  
title('Explicit function: x^3 - 6*x^2 + 11*x - 6');  
grid on;  
hold off
```



## Implicit Function Plot

You can plot implicitly defined functions. For example, create a plot for the following implicit function over the domain  $-1 < x < 1$ :

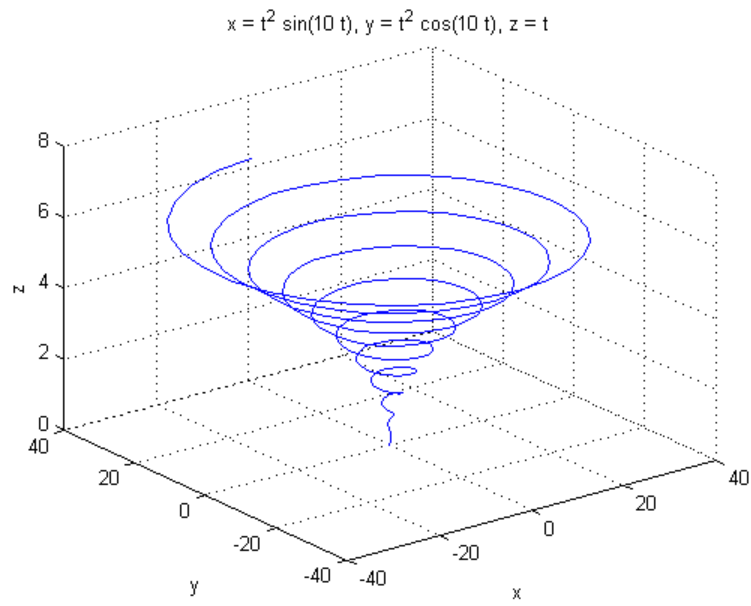
```
syms x y;  
f = (x^2 + y^2)^4 - (x^2 - y^2)^2;  
ezplot(f, [-1 1]);  
hold on;  
xlabel('x axis');  
ylabel('y axis');  
title('Implicit function: f = (x^2 + y^2)^4 - (x^2 - y^2)^2');  
grid on;  
hold off
```



### 3-D Plot

3-D graphics is also available in Symbolic Math Toolbox. To create a 3-D plot, use the `ezplot3` command. For example:

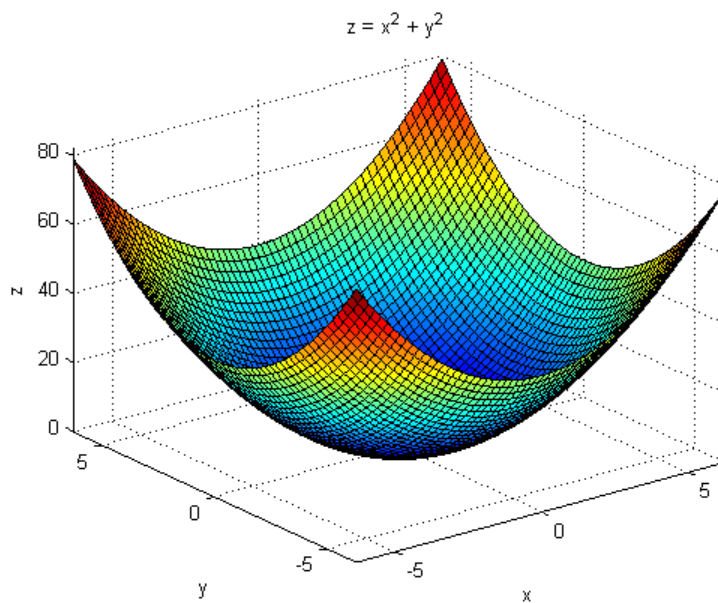
```
syms t;  
ezplot3(t^2*sin(10*t), t^2*cos(10*t), t);
```



## Surface Plot

If you want to create a surface plot, use the `ezsurf` command. For example, to plot a paraboloid  $z = x^2 + y^2$ , enter:

```
syms x y;  
ezsurf(x^2 + y^2);  
hold on;  
zlabel('z');  
title('z = x^2 + y^2');  
hold off
```



## Assumptions for Symbolic Objects

### In this section...

“Default Assumption” on page 2-31

“Setting Assumptions for Symbolic Variables” on page 2-31

“Deleting Symbolic Objects and Their Assumptions” on page 2-32

### Default Assumption

In Symbolic Math Toolbox, symbolic variables are single complex variables by default. For example, if you declare  $z$  as a symbolic variable:

```
syms z
```

MATLAB assumes  $z$  is a complex variable. You can always check if a symbolic variable is assumed to be complex or real by entering `conj` command. If `conj(x) == x` returns 1,  $x$  is a real variable:

```
z == conj(z)

ans =
    0
```

### Setting Assumptions for Symbolic Variables

The `sym` and `syms` commands allow you to set up assumptions for symbolic variables. For example, create the real symbolic variables  $x$  and  $y$ , and the positive symbolic variable  $z$ :

```
x = sym('x', 'real');
y = sym('y', 'real');
z = sym('z', 'positive');
```

or more efficiently

```
syms x y real;
syms z positive;
```

There are two assumptions you can assign to a symbolic object within the `sym` command: real and positive. Together with the default complex property of a

symbolic variable, it gives you three choices for an assumption for a symbolic variable: complex, real, and positive.

## Deleting Symbolic Objects and Their Assumptions

When you declare  $x$  to be real with the command

```
syms x real
```

you create a symbolic object  $x$  and the assumption that the object is real. Symbolic objects and their assumptions are stored separately. When you delete a symbolic object from the MATLAB workspace by using

```
clear x
```

the assumption that  $x$  is real stays in the symbolic engine. If you declare a new symbolic variable  $x$  later, it inherits the assumption that  $x$  is real instead of getting a default assumption. If later you solve an equation and simplify an expression with the symbolic variable  $x$ , you could get incomplete results. For example, the assumption that  $x$  is real causes the polynomial  $x^2+1$  to have no roots:

```
syms x real;  
clear x;  
syms x;  
solve(x^2+1)
```

```
Warning: Explicit solution could not be found.  
> In solve at 81
```

```
ans =  
[ empty sym ]
```

The complex roots of this polynomial disappear because the symbolic variable  $x$  still has the assumption that  $x$  is real stored in the symbolic engine. To clear the assumption, enter

```
syms x clear
```

After you clear the assumption, the symbolic object stays in the MATLAB workspace. If you want to remove both the symbolic object and its assumption, use two subsequent commands:

**1** To clear the assumption, enter

```
syms x clear
```

**2** To delete the symbolic object, enter

```
clear x
```

For more information on clearing symbolic variables, see “Clearing Assumptions and Resetting the Symbolic Engine” on page 4-51.





# Using Symbolic Math Toolbox Software

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- “Calculus” on page 3-2
- “Simplifications and Substitutions” on page 3-30
- “Variable-Precision Arithmetic” on page 3-48
- “Linear Algebra” on page 3-54
- “Solving Equations” on page 3-81
- “Integral Transforms and Z-Transforms” on page 3-91
- “Special Functions of Applied Mathematics” on page 3-108
- “Using Graphics” on page 3-118
- “Generating Code from Symbolic Expressions” on page 3-134

## Calculus

### In this section...

“Differentiation” on page 3-2

“Limits” on page 3-8

“Integration” on page 3-11

“Symbolic Summation” on page 3-18

“Taylor Series” on page 3-19

“Calculus Example” on page 3-21

### Differentiation

To illustrate how to take derivatives using Symbolic Math Toolbox software, first create a symbolic expression:

```
syms x
f = sin(5*x)
```

The command

```
diff(f)
```

differentiates  $f$  with respect to  $x$ :

```
ans =
5*cos(5*x)
```

As another example, let

```
g = exp(x)*cos(x)
```

where  $\exp(x)$  denotes  $e^x$ , and differentiate  $g$ :

```
diff(g)
ans =
exp(x)*cos(x) - exp(x)*sin(x)
```

To take the second derivative of  $g$ , enter

```
diff(g,2)
ans =
-2*exp(x)*sin(x)
```

You can get the same result by taking the derivative twice:

```
diff(diff(g))
ans =
-2*exp(x)*sin(x)
```

In this example, MATLAB software automatically simplifies the answer. However, in some cases, MATLAB might not simply an answer, in which case you can use the `simplify` command. For an example of such simplification, see “More Examples” on page 3-5.

Note that to take the derivative of a constant, you must first define the constant as a symbolic expression. For example, entering

```
c = sym('5');
diff(c)
```

returns

```
ans =
0
```

If you just enter

```
diff(5)
```

MATLAB returns

```
ans =
[]
```

because 5 is not a symbolic expression.

### **Derivatives of Expressions with Several Variables**

To differentiate an expression that contains more than one symbolic variable, specify the variable that you want to differentiate with respect to. The `diff`

command then calculates the partial derivative of the expression with respect to that variable. For example, given the symbolic expression

```
syms s t
f = sin(s*t)
```

the command

```
diff(f,t)
```

calculates the partial derivative  $\partial f / \partial t$ . The result is

```
ans =
s*cos(s*t)
```

To differentiate  $f$  with respect to the variable  $s$ , enter

```
diff(f,s)
```

which returns:

```
ans =
t*cos(s*t)
```

If you do not specify a variable to differentiate with respect to, MATLAB chooses a default variable. Basically, the default variable is the letter closest to  $x$  in the alphabet. See the complete set of rules in “Finding a Default Symbolic Variable” on page 2-25. In the preceding example, `diff(f)` takes the derivative of  $f$  with respect to  $t$  because the letter  $t$  is closer to  $x$  in the alphabet than the letter  $s$  is. To determine the default variable that MATLAB differentiates with respect to, use the `symvar` command:

```
symvar(f, 1)

ans =
t
```

To calculate the second derivative of  $f$  with respect to  $t$ , enter

```
diff(f, t, 2)
```

which returns

```
ans =
-s^2*sin(s*t)
```

Note that `diff(f, 2)` returns the same answer because `t` is the default variable.

### More Examples

To further illustrate the `diff` command, define `a`, `b`, `x`, `n`, `t`, and `theta` in the MATLAB workspace by entering

```
syms a b x n t theta
```

This table illustrates the results of entering `diff(f)`.

<b>f</b>	<b>diff(f)</b>
<pre>syms x n; f = x^n;</pre>	<pre>diff(f)  ans = n*x^(n - 1)</pre>
<pre>syms a b t; f = sin(a*t + b);</pre>	<pre>diff(f)  ans = a*cos(b + a*t)</pre>
<pre>syms theta; f = exp(i*theta);</pre>	<pre>diff(f)  ans = exp(theta*i)*i</pre>

To differentiate the Bessel function of the first kind, `besselj(nu, z)`, with respect to `z`, type

```
syms nu z
b = besselj(nu, z);
db = diff(b)
```

which returns

```
db =  
(nu*besselj(nu, z))/z - besselj(nu + 1, z)
```

The `diff` function can also take a symbolic matrix as its input. In this case, the differentiation is done element-by-element. Consider the example

```
syms a x  
A = [cos(a*x), sin(a*x); -sin(a*x), cos(a*x)]
```

which returns

```
A =  
[ cos(a*x), sin(a*x)]  
[ -sin(a*x), cos(a*x)]
```

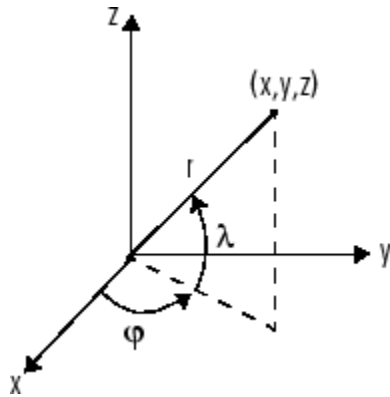
The command

```
diff(A)
```

returns

```
ans =  
[ -a*sin(a*x), a*cos(a*x)]  
[ -a*cos(a*x), -a*sin(a*x)]
```

You can also perform differentiation of a vector function with respect to a vector argument. Consider the transformation from Euclidean  $(x, y, z)$  to spherical  $(r, \lambda, \varphi)$  coordinates as given by  $x = r \cos \lambda \cos \varphi$ ,  $y = r \cos \lambda \sin \varphi$ , and  $z = r \sin \lambda$ . Note that  $\lambda$  corresponds to elevation or latitude while  $\varphi$  denotes azimuth or longitude.



To calculate the Jacobian matrix,  $J$ , of this transformation, use the `jacobian` function. The mathematical notation for  $J$  is

$$J = \frac{\partial(x, y, z)}{\partial(r, \lambda, \varphi)}.$$

For the purposes of toolbox syntax, use `l` for  $\lambda$  and `f` for  $\varphi$ . The commands

```
syms r l f
x = r*cos(l)*cos(f); y = r*cos(l)*sin(f); z = r*sin(l);
J = jacobian([x; y; z], [r l f])
```

return the Jacobian

```
J =
[ cos(f)*cos(l), -r*cos(f)*sin(l), -r*cos(l)*sin(f)]
[ cos(l)*sin(f), -r*sin(f)*sin(l),  r*cos(f)*cos(l)]
[          sin(l),          r*cos(l),          0]
```

and the command

```
detJ = simple(det(J))
```

returns

```
detJ =
-r^2*cos(l)
```

The arguments of the `jacobian` function can be column or row vectors. Moreover, since the determinant of the Jacobian is a rather complicated trigonometric expression, you can use the `simple` command to make trigonometric substitutions and reductions (simplifications). “Simplifications and Substitutions” on page 3-30 discusses simplification in more detail.

A table summarizing `diff` and `jacobian` follows.

Mathematical Operator	MATLAB Command
$\frac{df}{dx}$	<code>diff(f)</code> or <code>diff(f, x)</code>
$\frac{df}{da}$	<code>diff(f, a)</code>
$\frac{d^2f}{db^2}$	<code>diff(f, b, 2)</code>
$J = \frac{\partial(r,t)}{\partial(u,v)}$	<code>J = jacobian([r; t],[u; v])</code>

## Limits

The fundamental idea in calculus is to make calculations on functions as a variable “gets close to” or approaches a certain value. Recall that the definition of the derivative is given by a limit

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h},$$

provided this limit exists. Symbolic Math Toolbox software enables you to calculate the limits of functions directly. The commands

```
syms h n x
limit((cos(x+h) - cos(x))/h, h, 0)
```

which return



```
ans =  
-sin(x)
```

and

```
limit((1 + x/n)^n, n, inf)
```

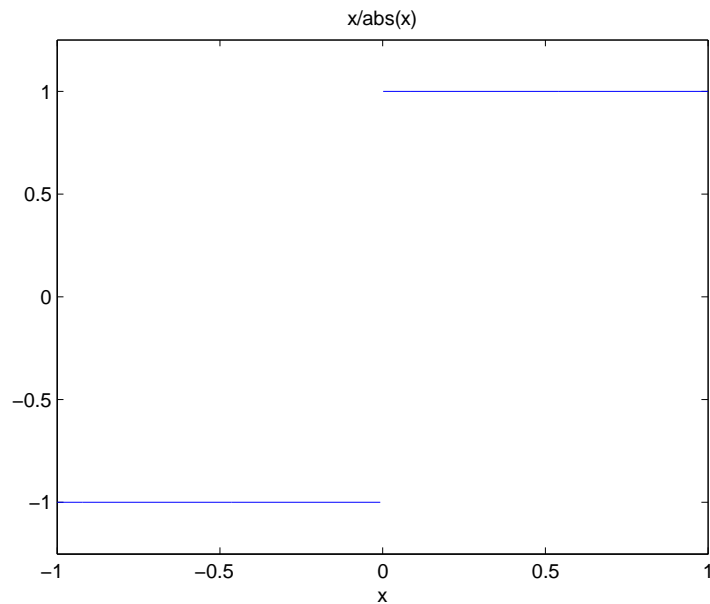
which returns

```
ans =  
exp(x)
```

illustrate two of the most important limits in mathematics: the derivative (in this case of  $\cos(x)$ ) and the exponential function.

### One-Sided Limits

You can also calculate one-sided limits with Symbolic Math Toolbox software. For example, you can calculate the limit of  $x/|x|$ , whose graph is shown in the following figure, as  $x$  approaches 0 from the left or from the right.



To calculate the limit as  $x$  approaches 0 from the left,

$$\lim_{x \rightarrow 0^-} \frac{x}{|x|},$$

enter

```
syms x;  
limit(x/abs(x), x, 0, 'left')
```

This returns

```
ans =  
-1
```

To calculate the limit as  $x$  approaches 0 from the right,

$$\lim_{x \rightarrow 0^+} \frac{x}{|x|} = 1,$$

enter

```
syms x;  
limit(x/abs(x), x, 0, 'right')
```

This returns

```
ans =  
1
```

Since the limit from the left does not equal the limit from the right, the two-sided limit does not exist. In the case of undefined limits, MATLAB returns NaN (not a number). For example,

```
syms x;  
limit(x/abs(x), x, 0)
```

returns

```
ans =  
NaN
```

Observe that the default case, `limit(f)` is the same as `limit(f,x,0)`. Explore the options for the `limit` command in this table, where `f` is a function of the symbolic object `x`.

Mathematical Operation	MATLAB Command
$\lim_{x \rightarrow 0} f(x)$	<code>limit(f)</code>
$\lim_{x \rightarrow a} f(x)$	<code>limit(f, x, a)</code> or <code>limit(f, a)</code>
$\lim_{x \rightarrow a^-} f(x)$	<code>limit(f, x, a, 'left')</code>
$\lim_{x \rightarrow a^+} f(x)$	<code>limit(f, x, a, 'right')</code>

## Integration

If `f` is a symbolic expression, then

```
int(f)
```

attempts to find another symbolic expression, `F`, so that `diff(F) = f`. That is, `int(f)` returns the indefinite integral or antiderivative of `f` (provided one exists in closed form). Similar to differentiation,

```
int(f,v)
```

uses the symbolic object `v` as the variable of integration, rather than the variable determined by `symvar`. See how `int` works by looking at this table.

Mathematical Operation	MATLAB Command
$\int x^n dx = \begin{cases} \log(x) & \text{if } n = -1 \\ \frac{x^{n+1}}{n+1} & \text{otherwise.} \end{cases}$	<code>int(x^n) or int(x^n, x)</code>
$\int_0^{\pi/2} \sin(2x) dx = 1$	<code>int(sin(2*x), 0, pi/2) or int(sin(2*x), x, 0, pi/2)</code>
$g = \cos(at + b)$ $\int g(t) dt = \sin(at + b) / a$	<code>g = cos(a*t + b) int(g) or int(g, t)</code>
$\int J_1(z) dz = -J_0(z)$	<code>int(besselj(1, z)) or int(besselj(1, z), z)</code>

In contrast to differentiation, symbolic integration is a more complicated task. A number of difficulties can arise in computing the integral:

- The antiderivative, F, may not exist in closed form.
- The antiderivative may define an unfamiliar function.
- The antiderivative may exist, but the software can't find it.
- The software could find the antiderivative on a larger computer, but runs out of time or memory on the available machine.

Nevertheless, in many cases, MATLAB can perform symbolic integration successfully. For example, create the symbolic variables

```
syms a b theta x y n u z
```

The following table illustrates integration of expressions containing those variables.

<b>f</b>	<b>int(f)</b>
<pre>syms x n; f = x^n;</pre>	<pre>int(f)  ans = piecewise([n = -1, log(x)], [n &lt;&gt; -1, x^(n + 1)/(n + 1)])</pre>
<pre>syms y; f = y^(-1);</pre>	<pre>int(f)  ans = log(y)</pre>
<pre>syms x n; f = n^x;</pre>	<pre>int(f)  ans = n^x/log(n)</pre>
<pre>syms a b theta; f = sin(a*theta+b);</pre>	<pre>int(f)  ans = -cos(b + a*theta)/a</pre>
<pre>syms u; f = 1/(1+u^2);</pre>	<pre>int(f)  ans = atan(u)</pre>
<pre>syms x; f = exp(-x^2);</pre>	<pre>int(f)  ans = (pi^(1/2)*erf(x))/2</pre>

In the last example,  $\exp(-x^2)$ , there is no formula for the integral involving standard calculus expressions, such as trigonometric and exponential functions. In this case, MATLAB returns an answer in terms of the error function `erf`.

If MATLAB is unable to find an answer to the integral of a function  $f$ , it just returns `int(f)`.

Definite integration is also possible.

Definite Integral	Command
$\int_a^b f(x)dx$	<code>int(f, a, b)</code>
$\int_a^b f(v)dv$	<code>int(f, v, a, b)</code>

Here are some additional examples.

<b>f</b>	<b>a, b</b>	<b>int(f, a, b)</b>
<code>syms x; f = x^7;</code>	<code>a = 0; b = 1;</code>	<code>int(f, a, b)</code>  <code>ans =</code> <code>1/8</code>
<code>syms x; f = 1/x;</code>	<code>a = 1; b = 2;</code>	<code>int(f, a, b)</code>  <code>ans =</code> <code>log(2)</code>
<code>syms x; f = log(x)*sqrt(x);</code>	<code>a = 0; b = 1;</code>	<code>int(f, a, b)</code>  <code>ans =</code> <code>-4/9</code>

<b>f</b>	<b>a, b</b>	<b>int(f, a, b)</b>
<pre>syms x; f = exp(-x^2);</pre>	<pre>a = 0; b = inf;</pre>	<pre>int(f, a, b)  ans = pi^(1/2)/2</pre>
<pre>syms z; f = besselj(1,z)^2;</pre>	<pre>a = 0; b = 1;</pre>	<pre>int(f, a, b)  ans = hypergeom([3/2, 3/2], [2, 5/2, 3], -1)/12</pre>

For the Bessel function (`besselj`) example, it is possible to compute a numerical approximation to the value of the integral, using the `double` function. The commands

```
syms z
a = int(besselj(1,z)^2,0,1)
```

return

```
a =
hypergeom([3/2, 3/2], [2, 5/2, 3], -1)/12
```

and the command

```
a = double(a)
```

returns

```
a =
0.0717
```

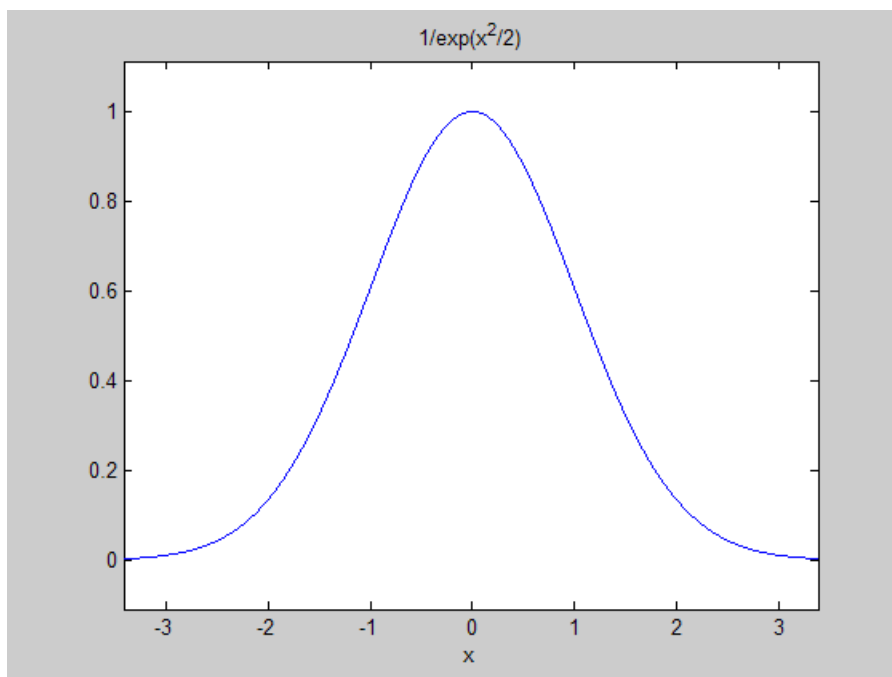
### Integration with Real Parameters

One of the subtleties involved in symbolic integration is the “value” of various parameters. For example, if  $a$  is any positive real number, the expression

$$e^{-ax^2}$$

is the positive, bell shaped curve that tends to 0 as  $x$  tends to  $\pm\infty$ . You can create an example of this curve, for  $a = 1/2$ , using the following commands:

```
syms x
a = sym(1/2);
f = exp(-a*x^2);
ezplot(f)
```



However, if you try to calculate the integral

$$\int_{-\infty}^{\infty} e^{-ax^2} dx$$

without assigning a value to  $a$ , MATLAB assumes that  $a$  represents a complex number, and therefore returns a piecewise answer that depends on the argument of  $a$ . If you are only interested in the case when  $a$  is a positive real number, you can calculate the integral as follows:



```
syms a positive;
```

The argument `positive` in the `syms` command restricts `a` to have positive values. Now you can calculate the preceding integral using the commands

```
syms x;
f = exp(-a*x^2);
int(f, x, -inf, inf)
```

This returns

```
ans =
pi^(1/2)/a^(1/2)
```

### Integration with Complex Parameters

To calculate the integral

$$\int_{-\infty}^{\infty} \frac{1}{a^2 + x^2} dx$$

for complex values of `a`, enter

```
syms a x clear
f = 1/(a^2 + x^2);
F = int(f, x, -inf, inf)
```

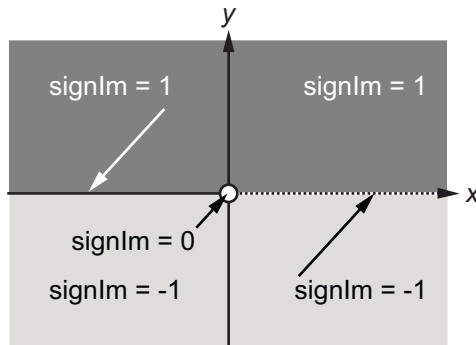
`syms` is used with the `clear` option to clear the real property that was assigned to `a` in the preceding example — see “Deleting Symbolic Objects and Their Assumptions” on page 2-32.

The preceding commands produce the complex output

```
F =
(pi*signIm(i/a))/a
```

The function `signIm` is defined as:

$$\text{signIm}(z) = \begin{cases} 1 & \text{if } \text{Im}(z) > 0, \text{ or } \text{Im}(z) = 0 \text{ and } z < 0 \\ 0 & \text{if } z = 0 \\ -1 & \text{otherwise.} \end{cases}$$



To evaluate F at  $a = 1 + i$ , enter

```
g = subs(F, 1 + i)
```

```
g =  
pi / (2*i)^(1/2)
```

```
double(g)
```

```
ans =  
1.5708 - 1.5708i
```

## Symbolic Summation

You can compute symbolic summations, when they exist, by using the `symsum` command. For example, the p-series

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots$$

sums to  $\pi^2/6$ , while the geometric series

$$1 + x + x^2 + \dots$$

sums to  $1/(1-x)$ , provided  $|x| < 1$ . These summations are demonstrated below:

```
syms x k
s1 = symsum(1/k^2, 1, inf)
s2 = symsum(x^k, k, 0, inf)

s1 =
pi^2/6

s2 =
piecewise([1 <= x, Inf], [abs(x) < 1, -1/(x - 1)])
```

## Taylor Series

The statements

```
syms x
f = 1/(5 + 4*cos(x));
T = taylor(f, 8)

return

T =
(49*x^6)/131220 + (5*x^4)/1458 + (2*x^2)/81 + 1/9
```

which is all the terms up to, but not including, order eight in the Taylor series for  $f(x)$ :

$$\sum_{n=0}^{\infty} (x-a)^n \frac{f^{(n)}(a)}{n!}.$$

Technically,  $T$  is a Maclaurin series, since its base point is  $a = 0$ .

The command

```
pretty(T)
```

prints  $T$  in a format resembling typeset mathematics:

$$\frac{49 x^6}{131220} + \frac{5 x^4}{1458} + \frac{2 x^2}{81} + 1/9$$

These commands

```
syms x
g = exp(x*sin(x))
t = taylor(g, 12, 2);
```

generate the first 12 nonzero terms of the Taylor series for g about x = 2.

t is a large expression; enter

```
size(char(t))

ans =
      1      99791
```

to find that t has about 100,000 characters in its printed form. In order to proceed with using t, first simplify its presentation:

```
t = simplify(t);
size(char(t))

ans =
      1      12137
```

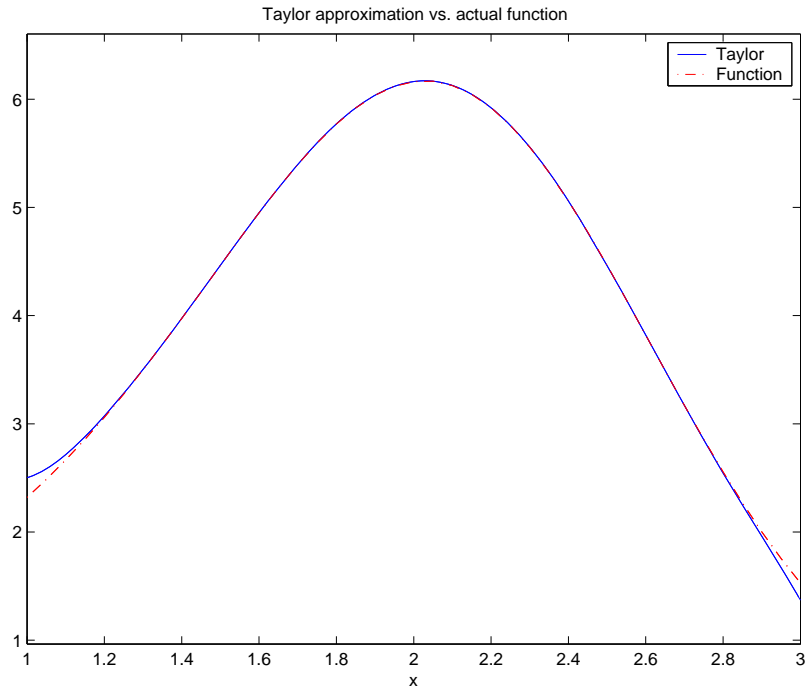
To simplify t even further, use the simple function:

```
t = simple(t);
size(char(t))

ans =
      1      6988
```

Next, plot these functions together to see how well this Taylor approximation compares to the actual function g:

```
xd = 1:0.05:3; yd = subs(g,x,xd);
ezplot(t, [1, 3]); hold on;
plot(xd, yd, 'r-.')
title('Taylor approximation vs. actual function');
legend('Taylor','Function')
```



Special thanks is given to Professor Gunnar Bäckström of UMEA in Sweden for this example.

## Calculus Example

This section describes how to analyze a simple function to find its asymptotes, maximum, minimum, and inflection point. The section covers the following topics:

- “Defining the Function” on page 3-22
- “Finding the Asymptotes” on page 3-23
- “Finding the Maximum and Minimum” on page 3-25
- “Finding the Inflection Point” on page 3-27

### **Defining the Function**

The function in this example is

$$f(x) = \frac{3x^2 + 6x - 1}{x^2 + x - 3}.$$

To create the function, enter the following commands:

```
syms x
num = 3*x^2 + 6*x - 1;
denom = x^2 + x - 3;
f = num/denom
```

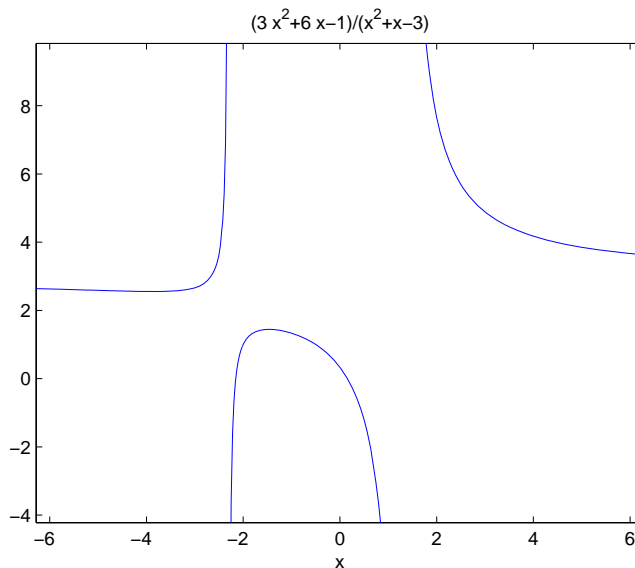
This returns

```
f =
(3*x^2 + 6*x - 1)/(x^2 + x - 3)
```

You can plot the graph of  $f$  by entering

```
ezplot(f)
```

This displays the following plot.



### Finding the Asymptotes

To find the horizontal asymptote of the graph of  $f$ , take the limit of  $f$  as  $x$  approaches positive infinity:

```
limit(f, inf)
ans =
3
```

The limit as  $x$  approaches negative infinity is also 3. This tells you that the line  $y = 3$  is a horizontal asymptote to the graph.

To find the vertical asymptotes of  $f$ , set the denominator equal to 0 and solve by entering the following command:

```
roots = solve(denom)
```

This returns to solutions to  $x^2 + x - 3 = 0$ :

```
roots =
```

$$13^{(1/2)/2} - 1/2$$
$$- 13^{(1/2)/2} - 1/2$$

This tells you that vertical asymptotes are the lines

$$x = \frac{-1 + \sqrt{13}}{2},$$

and

$$x = \frac{-1 - \sqrt{13}}{2}.$$

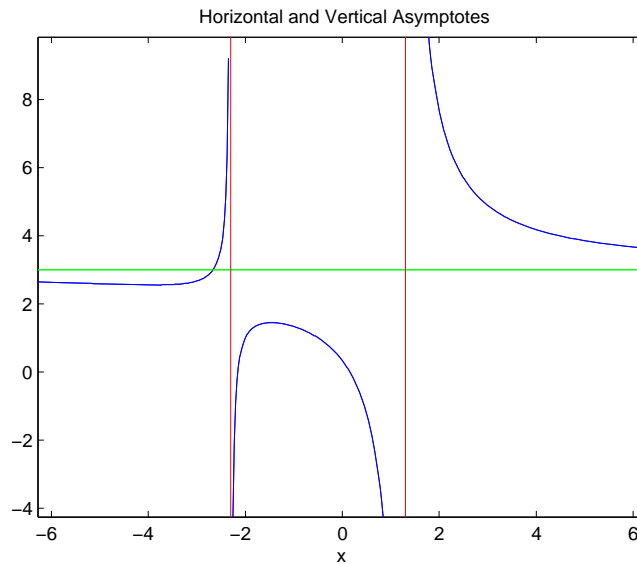
You can plot the horizontal and vertical asymptotes with the following commands:

```
ezplot(f)
hold on % Keep the graph of f in the figure
% Plot horizontal asymptote
plot([-2*pi 2*pi], [3 3], 'g')
% Plot vertical asymptotes
plot(double(roots(1))*[1 1], [-5 10], 'r')
plot(double(roots(2))*[1 1], [-5 10], 'r')
title('Horizontal and Vertical Asymptotes')
hold off
```

Note that `roots` must be converted to `double` to use the `plot` command.

The preceding commands display the following figure.





To recover the graph of  $f$  without the asymptotes, enter

```
ezplot(f)
```

### Finding the Maximum and Minimum

You can see from the graph that  $f$  has a local maximum somewhere between the points  $x = -2$  and  $x = 0$ , and might have a local minimum between  $x = -6$  and  $x = -2$ . To find the  $x$ -coordinates of the maximum and minimum, first take the derivative of  $f$ :

```
f1 = diff(f)
```

This returns

```
f1 =
(6*x + 6)/(x^2 + x - 3) - ((2*x + 1)*(3*x^2 + 6*x - 1))/(x^2 + x - 3)^2
```

To simplify this expression, enter

```
f1 = simplify(f1)
```

which returns

$$f1 = \\ -(3*x^2 + 16*x + 17)/(x^2 + x - 3)^2$$

You can display f1 in a more readable form by entering

```
pretty(f1)
```

which returns

$$\frac{3x^2 + 16x + 17}{(x^2 + x - 3)^2}$$

Next, set the derivative equal to 0 and solve for the critical points:

```
crit_pts = solve(f1)
```

This returns

$$\text{crit\_pts} = \\ \begin{matrix} 13^{(1/2)}/3 - 8/3 \\ - 13^{(1/2)}/3 - 8/3 \end{matrix}$$

It is clear from the graph of f that it has a local minimum at

$$x_1 = \frac{-8 - \sqrt{13}}{3},$$

and a local maximum at

$$x_2 = \frac{-8 + \sqrt{13}}{3}.$$

---

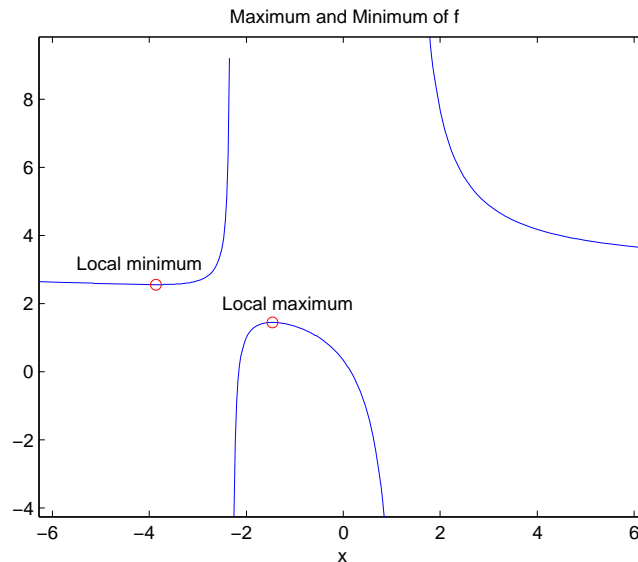
**Note** MATLAB does not always return the roots to an equation in the same order.

---

You can plot the maximum and minimum of  $f$  with the following commands:

```
ezplot(f)
hold on
plot(double(crit_pts), double(subs(f,crit_pts)), 'ro')
title('Maximum and Minimum of f')
text(-5.5,3.2,'Local minimum')
text(-2.5,2,'Local maximum')
hold off
```

This displays the following figure.



### Finding the Inflection Point

To find the inflection point of  $f$ , set the second derivative equal to 0 and solve.

```
f2 = diff(f1);
inflec_pt = solve(f2);
double(inflec_pt)
```

This returns

```
ans =
-5.2635
-1.3682 - 0.8511i
-1.3682 + 0.8511i
```

In this example, only the first entry is a real number, so this is the only inflection point. (Note that in other examples, the real solutions might not be the first entries of the answer.) Since you are only interested in the real solutions, you can discard the last two entries, which are complex numbers.

```
inflec_pt = inflec_pt(1)
```

To see the symbolic expression for the inflection point, enter

```
pretty(simplify(inflec_pt))
```

This returns

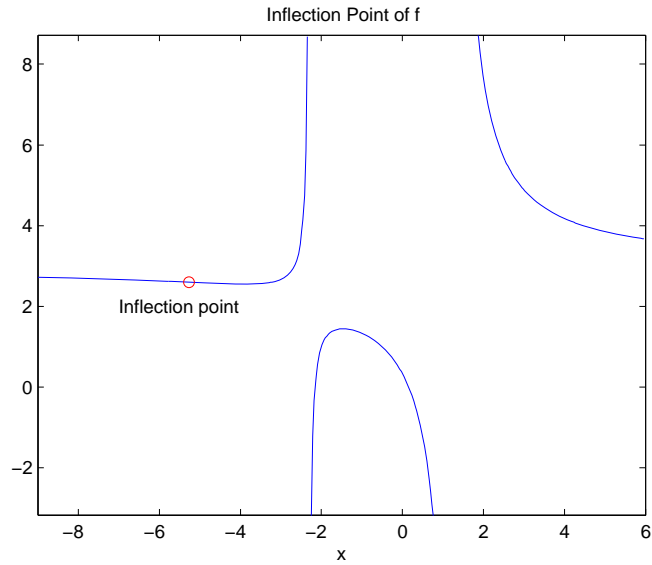
$$-\frac{13}{9\sqrt[3]{169/54} - \sqrt[3]{2197/18}} - \frac{\sqrt[3]{169/54} - \sqrt[3]{2197/18}}{169/54 - \sqrt[3]{2197/18}} - 8/3$$

To plot the inflection point, enter

```
ezplot(f, [-9 6])
hold on
plot(double(inflec_pt), double(subs(f,inflec_pt)), 'ro')
title('Inflection Point of f')
text(-7,2,'Inflection point')
```

hold off

The extra argument, `[-9 6]`, in `ezplot` extends the range of  $x$  values in the plot so that you see the inflection point more clearly, as shown in the following figure.



## Simplifications and Substitutions

### In this section...

“Simplifications” on page 3-30

“Substitutions” on page 3-41

### Simplifications

Here are three different symbolic expressions.

```
syms x
f = x^3 - 6*x^2 + 11*x - 6;
g = (x - 1)*(x - 2)*(x - 3);
h = -6 + (11 + (-6 + x)*x)*x;
```

Here are their prettyprinted forms, generated by

```
pretty(f);
pretty(g);
pretty(h)
```

$$x^3 - 6x^2 + 11x - 6$$

$$(x - 1)(x - 2)(x - 3)$$

$$x(x(x - 6) + 11) - 6$$

These expressions are three different representations of the same mathematical function, a cubic polynomial in  $x$ .

Each of the three forms is preferable to the others in different situations. The first form,  $f$ , is the most commonly used representation of a polynomial. It is simply a linear combination of the powers of  $x$ . The second form,  $g$ , is the factored form. It displays the roots of the polynomial and is the most accurate for numerical evaluation near the roots. But, if a polynomial does not have such simple roots, its factored form may not be so convenient. The third form,  $h$ , is the Horner, or nested, representation. For numerical evaluation, it

involves the fewest arithmetic operations and is the most accurate for some other ranges of  $x$ .

The symbolic simplification problem involves the verification that these three expressions represent the same function. It also involves a less clearly defined objective — which of these representations is “the simplest”?

This toolbox provides several functions that apply various algebraic and trigonometric identities to transform one representation of a function into another, possibly simpler, representation. These functions are `collect`, `expand`, `horner`, `factor`, `simplify`, and `simple`.

### **collect**

The statement `collect(f)` views  $f$  as a polynomial in its symbolic variable, say  $x$ , and collects all the coefficients with the same power of  $x$ . A second argument can specify the variable in which to collect terms if there is more than one candidate. Here are a few examples.

<b>f</b>	<b>collect(f)</b>
<pre>syms x; f = (x-1)*(x-2)*(x-3);</pre>	<pre>collect(f)  ans = x^3 - 6*x^2 + 11*x - 6</pre>
<pre>syms x; f = x*(x*(x - 6) + 11) - 6;</pre>	<pre>collect(f)  ans = x^3 - 6*x^2 + 11*x - 6</pre>
<pre>syms x t; f = (1+x)*t + x*t;</pre>	<pre>collect(f)  ans = (2*t)*x + t</pre>

**expand**

The statement `expand(f)` distributes products over sums and applies other identities involving functions of sums as shown in the examples below.

<b>f</b>	<b>expand(f)</b>
<pre>syms a x y; f = a*(x + y);</pre>	<pre>expand(f)  ans = a*x + a*y</pre>
<pre>syms x; f = (x - 1)*(x - 2)*(x - 3);</pre>	<pre>expand(f)  ans = x^3 - 6*x^2 + 11*x - 6</pre>
<pre>syms x; f = x*(x*(x - 6) + 11) - 6;</pre>	<pre>expand(f)  ans = x^3 - 6*x^2 + 11*x - 6</pre>
<pre>syms a b; f = exp(a + b);</pre>	<pre>expand(f)  ans = exp(a)*exp(b)</pre>
<pre>syms x y; f = cos(x + y);</pre>	<pre>expand(f)  ans = cos(x)*cos(y) - sin(x)*sin(y)</pre>



<b>f</b>	<b>expand(f)</b>
<pre>syms x; f = cos(3*acos(x));</pre>	<pre>expand(f) ans = 4*x^3 - 3*x</pre>
<pre>syms x; f = 3*x*(x^2 - 1) + x^3;</pre>	<pre>expand(f) ans = 4*x^3 - 3*x</pre>

### horner

The statement `horner(f)` transforms a symbolic polynomial `f` into its Horner, or nested, representation as shown in the following examples.

<b>f</b>	<b>horner(f)</b>
<pre>syms x; f = x^3 - 6*x^2 + 11*x - 6;</pre>	<pre>horner(f) ans = x*(x*(x - 6) + 11) - 6</pre>
<pre>syms x; f = 1.1 + 2.2*x + 3.3*x^2;</pre>	<pre>horner(f) ans = x*((33*x)/10 + 11/5) + 11/10</pre>

### factor

If `f` is a polynomial with rational coefficients, the statement

```
factor(f)
```

expresses `f` as a product of polynomials of lower degree with rational coefficients. If `f` cannot be factored over the rational numbers, the result is `f` itself. Here are several examples.

f	factor(f)
<pre>syms x; f = x^3 - 6*x^2 + 11*x - 6;</pre>	<pre>factor(f)  ans = (x - 3)*(x - 1)*(x - 2)</pre>
<pre>syms x; f = x^3 - 6*x^2 + 11*x - 5;</pre>	<pre>factor(f)  ans = x^3 - 6*x^2 + 11*x - 5</pre>
<pre>syms x; f = x^6 + 1;</pre>	<pre>factor(f)  ans = (x^2 + 1)*(x^4 - x^2 + 1)</pre>

Here is another example involving `factor`. It factors polynomials of the form  $x^n + 1$ . This code

```
syms x;
n = (1:9)';
p = x.^n + 1;
f = factor(p);
[p, f]
```

returns a matrix with the polynomials in its first column and their factored forms in its second.

```
ans =
[ x + 1, x + 1]
[ x^2 + 1, x^2 + 1]
[ x^3 + 1, (x + 1)*(x^2 - x + 1)]
[ x^4 + 1, x^4 + 1]
[ x^5 + 1, (x + 1)*(x^4 - x^3 + x^2 - x + 1)]
[ x^6 + 1, (x^2 + 1)*(x^4 - x^2 + 1)]
[ x^7 + 1, (x + 1)*(x^6 - x^5 + x^4 - x^3 + x^2 - x + 1)]
[ x^8 + 1, x^8 + 1]
[ x^9 + 1, (x + 1)*(x^2 - x + 1)*(x^6 - x^3 + 1)]
```

As an aside at this point, `factor` can also factor symbolic objects containing integers. This is an alternative to using the `factor` function in the MATLAB `specfun` folder. For example, the following code segment

```
N = sym(1);
for k = 2:11
    N(k) = 10*N(k-1)+1;
end
[N' factor(N')]
```

displays the factors of symbolic integers consisting of 1s:

```
ans =
[          1,          1]
[          11,         11]
[          111,        3*37]
[          1111,       11*101]
[          11111,      41*271]
[          111111,   3*7*11*13*37]
[          1111111,  239*4649]
[          11111111, 11*73*101*137]
[          111111111, 3^2*37*333667]
[          1111111111, 11*41*271*9091]
[          11111111111, 21649*513239]
```

### **simplify**

The `simplify` function is a powerful, general purpose tool that applies a number of algebraic identities involving sums, integral powers, square roots and other fractional powers, as well as a number of functional identities involving trig functions, exponential and log functions, Bessel functions, hypergeometric functions, and the gamma function. Here are some examples.

<b>f</b>	<b>simplify(f)</b>
<pre>syms x; f = x*(x*(x - 6) + 11) - 6;</pre>	<pre>simplify(f)  ans = (x - 1)*(x - 2)*(x - 3)</pre>
<pre>syms x; f = (1 - x^2)/(1 - x);</pre>	<pre>simplify(f)  ans = x + 1</pre>
<pre>syms a; f = (1/a^3 + 6/a^2 + 12/a + 8)^(1/3);</pre>	<pre>simplify(f)  ans = ((2*a + 1)^3/a^3)^(1/3)</pre>
<pre>syms x y; f = exp(x) * exp(y);</pre>	<pre>simplify(f)  ans = exp(x + y)</pre>
<pre>syms x; f = besselj(2, x) + besselj(0, x);</pre>	<pre>simplify(f)  ans = (2*besselj(1, x))/x</pre>
<pre>syms x; f = gamma(x + 1) - x*gamma(x);</pre>	<pre>simplify(f)  ans = 0</pre>
<pre>syms x; f = cos(x)^2 + sin(x)^2;</pre>	<pre>simplify(f)  ans = 1</pre>

You can also use the syntax `simplify(f, n)` where `n` is a positive integer that controls how many steps `simplify` takes. The default, when you don't provide an argument `n`, is 100 steps. For example,

```
syms x;
z = (cos(3*acos(x))*sin(x)^2*(cos(x)^2 + sin(x)^2)^2)/cos(x)^2

z =
(cos(3*acos(x))*sin(x)^2*(cos(x)^2 + sin(x)^2)^2)/cos(x)^2

simplify(z)

ans =
3*x - (3*x)/cos(x)^2 + (4*x^3)/cos(x)^2 - 4*x^3

simplify(z, 150)

ans =
-(x*(4*x^2 - 3)*(cos(x)^2 - 1))/cos(x)^2
```

## simple

The `simple` function has the unorthodox mathematical goal of finding a simplification of an expression that has the fewest number of characters. Of course, there is little mathematical justification for claiming that one expression is “simpler” than another just because its ASCII representation is shorter, but this often proves satisfactory in practice.

The `simple` function achieves its goal by independently applying `simplify`, `collect`, `factor`, and other simplification functions to an expression and keeping track of the lengths of the results. The `simple` function then returns the shortest result.

The `simple` function has several forms, each returning different output. The form `simple(f)` displays each trial simplification and the simplification function that produced it in the MATLAB command window. The `simple` function then returns the shortest result. For example, the command

```
syms x;
simple(cos(x)^2 + sin(x)^2)
```

displays the following alternative simplifications in the MATLAB command window along with the result:

```
simplify:
1

radsimp:
cos(x)^2 + sin(x)^2

simplify(100):
1

combine(sincos):
1

combine(sinhcosh):
cos(x)^2 + sin(x)^2

combine(ln):
cos(x)^2 + sin(x)^2

factor:
cos(x)^2 + sin(x)^2

expand:
cos(x)^2 + sin(x)^2

combine:
cos(x)^2 + sin(x)^2

rewrite(exp):
(1/(2*exp(x*i)) + exp(x*i)/2)^2 + (i/(2*exp(x*i)) - (exp(x*i)*i)/2)^2

rewrite(sincos):
cos(x)^2 + sin(x)^2

rewrite(sinhcosh):
cosh(x*i)^2 - sinh(x*i)^2

rewrite(tan):
```

```
(tan(x/2)^2 - 1)^2/(tan(x/2)^2 + 1)^2 + (4*tan(x/2)^2)/(tan(x/2)^2 + 1)^2

mwcoss2sin:
1

collect(x):
cos(x)^2 + sin(x)^2

ans =
1
```

This form is useful when you want to check, for example, whether the shortest form is indeed the simplest. If you are not interested in how `simple` achieves its result, use the form `f = simple(f)`. This form simply returns the shortest expression found. For example, the statement

```
f = simple(cos(x)^2 + sin(x)^2)
```

returns

```
f =
1
```

If you want to know which simplification returned the shortest result, use the multiple output form `[f, how] = simple(f)`. This form returns the shortest result in the first variable and the simplification method used to achieve the result in the second variable. For example, the statement

```
[f, how] = simple(cos(x)^2 + sin(x)^2)
```

returns

```
f =
1

how =
simplify
```

The `simple` function sometimes improves on the result returned by `simplify`, one of the simplifications that it tries. For example, when applied to the examples given for `simplify`, `simple` returns a simpler (or at least shorter) result as shown:

<b>f</b>	<b>simplify(f)</b>	<b>simple(f)</b>
<pre>syms a positive; f = (1/a^3 + 6/a^2 + 12/a + 8)^(1/3);</pre>	<pre>simplify(f)  ans = (8*a^3 + 12*a^2 + 6*a + 1)^(1/3)/a</pre>	<pre>g = simple(f)  g = 1/a + 2</pre>
<pre>syms x; f = cos(x) + i*sin(x);</pre>	<pre>simplify(f)  ans = cos(x) + sin(x)*i</pre>	<pre>g = simple(f)  g = exp(x*i)</pre>

In some cases, it is advantageous to apply `simple` twice to obtain the effect of two different simplification functions. For example:

```
syms x;
z = exp((cos(x)^2 - sin(x)^2)/(sin(x)*cos(x)))

z =
exp((cos(x)^2 - sin(x)^2)/(cos(x)*sin(x)))

z1 = simple(z)

z1 =
exp(cot(x) - tan(x))

z2 = simple(simple(z))

z2 =
exp(2/tan(2*x))
```

The `simple` function is particularly effective on expressions involving trigonometric functions:



<b>f</b>	<b>simple(f)</b>
<pre>syms x; f = cos(x)^2 + sin(x)^2;</pre>	<pre>f = simple(f)  f = 1</pre>
<pre>syms x; f = 2*cos(x)^2 - sin(x)^2;</pre>	<pre>f = simple(f)  f = 2 - 3*sin(x)^2</pre>
<pre>syms x; f = cos(x)^2 - sin(x)^2;</pre>	<pre>f = simple(f)  f = cos(2*x)</pre>
<pre>syms x; f = cos(x) + i*sin(x);</pre>	<pre>f = simple(f)  f = exp(x*i)</pre>
<pre>syms x; f = cos(3*acos(x));</pre>	<pre>f = simple(f)  f = 4*x^3 - 3*x</pre>

## Substitutions

There are two functions for symbolic substitution: `subexpr` and `subs`.

### **subexpr**

These commands

```
syms a x
s = solve(x^3 + a*x + 1)
```

solve the equation  $x^3 + ax + 1 = 0$  for the variable  $x$ :

```
s =
((a^3/27 + 1/4)^(1/2) - 1/2)^(1/3) - a/(3*((a^3/27 + 1/4)^(1/2) - 1/2)^(1/3))

(3^(1/2)*(a/(3*((a^3/27 + 1/4)^(1/2) - 1/2)^(1/3)) + ...
((a^3/27 + 1/4)^(1/2) - 1/2)^(1/3))*i)/2 + ...
a/(6*((a^3/27 + 1/4)^(1/2) - 1/2)^(1/3)) - ...
((a^3/27 + 1/4)^(1/2) - 1/2)^(1/3)/2

a/(6*((a^3/27 + 1/4)^(1/2) - 1/2)^(1/3)) - ...
(3^(1/2)*(a/(3*((a^3/27 + 1/4)^(1/2) - 1/2)^(1/3)) + ...
((a^3/27 + 1/4)^(1/2) - 1/2)^(1/3))*i)/2 - ...
((a^3/27 + 1/4)^(1/2) - 1/2)^(1/3)/2
```

This long expression has many repeated pieces, or subexpressions. The `subexpr` function allows you to save these common subexpressions as well as the symbolic object rewritten in terms of the subexpressions. The subexpressions are saved in a column vector called `sigma`.

Continuing with the example

```
r = subexpr(s)
```

returns

```
sigma =
(a^3/27 + 1/4)^(1/2) - 1/2

r =
sigma^(1/3) - a/(3*sigma^(1/3))
(3^(1/2)*(a/(3*sigma^(1/3)) + sigma^(1/3))*i)/2 + a/(6*sigma^(1/3)) - sigma^(1/3)/2
a/(6*sigma^(1/3)) - (3^(1/2)*(a/(3*sigma^(1/3)) + sigma^(1/3))*i)/2 - sigma^(1/3)/2
```

Notice that `subexpr` creates the variable `sigma` in the MATLAB workspace. You can verify this by typing `whos`, or the command

```
sigma
```

which returns

$$\text{sigma} = (a^{3/27} + 1/4)^{(1/2)} - 1/2$$

### subs

The following code finds the eigenvalues and eigenvectors of a circulant matrix A:

```
syms a b c
A = [a b c; b c a; c a b];
[v,E] = eig(A)

v =
[ (a^2 - a*b - a*c + b^2 - b*c + c^2)^(1/2)/(a - c) - (a - b)/(a - c),...
  - (a^2 - a*b - a*c + b^2 - b*c + c^2)^(1/2)/(a - c) - (a - b)/(a - c),...
  1]
[ - (a^2 - a*b - a*c + b^2 - b*c + c^2)^(1/2)/(a - c) - (b - c)/(a - c),...
  (a^2 - a*b - a*c + b^2 - b*c + c^2)^(1/2)/(a - c) - (b - c)/(a - c),...
  1]
[ 1, 1, 1]

E =
[ -(a^2-a*b-a*c+b^2-b*c+c^2)^(1/2), 0, 0]
[ 0, (a^2-a*b-a*c+b^2-b*c+c^2)^(1/2), 0]
[ 0, 0, a+b+c]
```

---

**Note** MATLAB might return the eigenvalues that appear on the diagonal of E in a different order. In this case, the corresponding eigenvectors, which are the columns of v, will also appear in a different order.

---

Suppose you want to replace the rather lengthy expression  $(a^2 - a*b - a*c + b^2 - b*c + c^2)^{(1/2)}$  throughout v and E. First, use subexpr:

```
E = subexpr(E, 'S')
```

which returns

```
S =
```

$$(a^2 - a*b - a*c + b^2 - b*c + c^2)^{(1/2)}$$

$$E =$$

$$\begin{bmatrix} -S, & 0, & & 0 \\ 0, & S, & & 0 \\ 0, & 0, & a + b + c & \end{bmatrix}$$

Next, substitute the symbol  $S$  into  $v$  with

```
v = simplify(subs(v, S, 'S'))
```

$$v =$$

$$\begin{bmatrix} (S - a + b)/(a - c), & -(S + a - b)/(a - c), & 1 \\ -(S + b - c)/(a - c), & (S - b + c)/(a - c), & 1 \\ & 1, & 1, & 1 \end{bmatrix}$$

Now suppose you want to evaluate  $v$  at  $a = 10$ . Use the `subs` command:

```
subs(v, a, 10)
```

This replaces all occurrences of  $a$  in  $v$  with 10:

```
ans =
```

$$\begin{bmatrix} -(S + b - 10)/(c - 10), & (S - b + 10)/(c - 10), & 1 \\ (S + b - c)/(c - 10), & -(S - b + c)/(c - 10), & 1 \\ & 1, & 1, & 1 \end{bmatrix}$$

Notice, however, that the symbolic expression that  $S$  represents is unaffected by this substitution. That is, the symbol  $a$  in  $S$  is not replaced by 10. The `subs` command is also a useful function for substituting in a variety of values for several variables in a particular expression. For example, suppose that in addition to substituting  $a = 10$  in  $S$ , you also want to substitute the values for 2 and 10 for  $b$  and  $c$ , respectively. The way to do this is to set values for  $a$ ,  $b$ , and  $c$  in the workspace. Then `subs` evaluates its input using the existing symbolic and double variables in the current workspace. In the example, you first set

```
a = 10; b = 2; c = 10;
subs(S)
```

```
ans =
      8
```

To look at the contents of the workspace, type:

```
whos
```

which gives

Name	Size	Bytes	Class	Attributes
A	3x3	622	sym	
E	3x3	1144	sym	
S	1x1	184	sym	
a	1x1	8	double	
ans	1x1	8	double	
b	1x1	8	double	
c	1x1	8	double	
v	3x3	1144	sym	

a, b, and c are now variables of class `double` while A, E, S, and v remain symbolic expressions (class `sym`).

If you want to preserve a, b, and c as symbolic variables, but still alter their value within S, use this procedure.

```
syms a b c
subs(S, {a, b, c}, {10, 2, 10})

ans =
     8
```

Typing `whos` reveals that a, b, and c remain 1-by-1 `sym` objects.

The `subs` command can be combined with `double` to evaluate a symbolic expression numerically. Suppose you have the following expressions

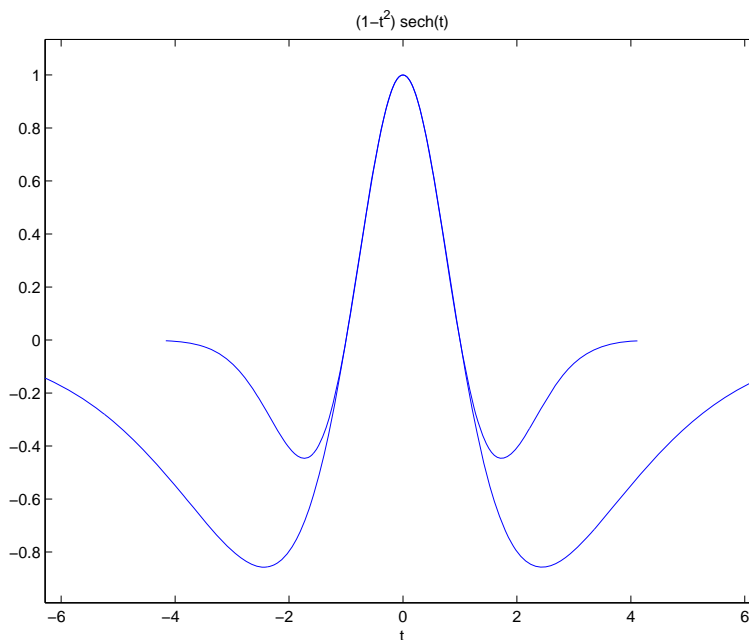
```
syms t
M = (1 - t^2)*exp(-1/2*t^2);
P = (1 - t^2)*sech(t);
```

and want to see how M and P differ graphically.

One approach is to type

```
ezplot(M);  
hold on;  
ezplot(P);  
hold off;
```

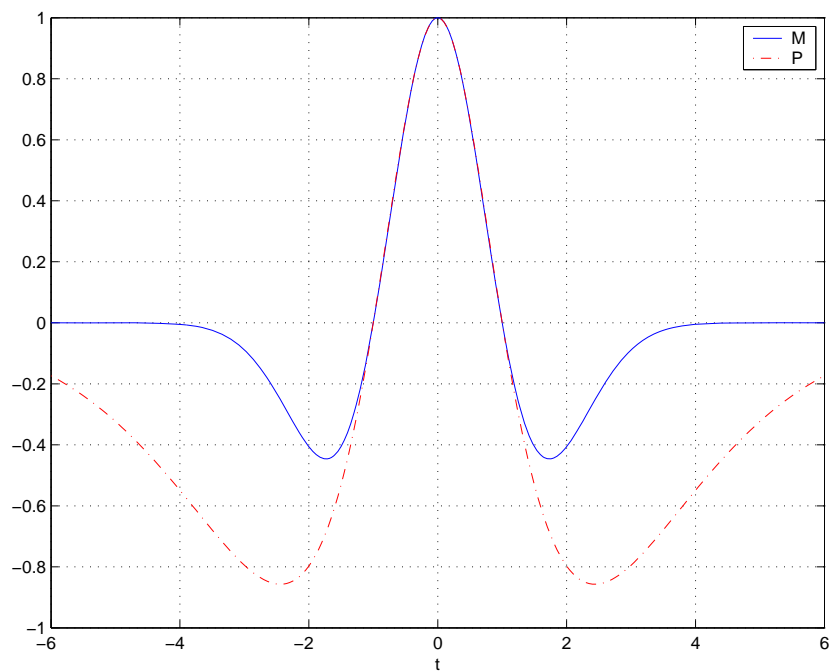
but this plot does not readily help you identify the curves.



Instead, combine `subs`, `double`, and `plot`:

```
T = -6:0.05:6;  
MT = double(subs(M, t, T));  
PT = double(subs(P, t, T));  
plot(T, MT, 'b', T, PT, 'r-.');  
title(' ');  
legend('M', 'P');  
xlabel('t'); grid;
```

to produce a multicolored graph that indicates the difference between  $M$  and  $P$ .



Finally the use of subs with strings greatly facilitates the solution of problems involving the Fourier, Laplace, or z-transforms. See “Integral Transforms and Z-Transforms” on page 3-91 for details.

## Variable-Precision Arithmetic

In this section...
“Overview” on page 3-48
“Example: Using the Different Kinds of Arithmetic” on page 3-49
“Another Example Using Different Kinds of Arithmetic” on page 3-52

### Overview

There are three different kinds of arithmetic operations in this toolbox.

Numeric	MATLAB floating-point arithmetic
Rational	MuPAD exact symbolic arithmetic
VPA	MuPAD variable-precision arithmetic

For example, the MATLAB statements

```
format long
1/2 + 1/3
```

use numeric computation to produce

```
ans =
0.8333333333333333
```

With Symbolic Math Toolbox software, the statement

```
sym(1/2) + 1/3
```

uses symbolic computation to yield

```
ans =
5/6
```

And, also with the toolbox, the statements

```
digits(25)
vpa('1/2 + 1/3')
```



use variable-precision arithmetic to return

```
ans =  
0.83333333333333333333333333333333
```

The floating-point operations used by numeric arithmetic are the fastest of the three, and require the least computer memory, but the results are not exact. The number of digits in the printed output of MATLAB double quantities is controlled by the `format` statement, but the internal representation is always the eight-byte floating-point representation provided by the particular computer hardware.

In the computation of the numeric result above, there are actually three roundoff errors, one in the division of 1 by 3, one in the addition of 1/2 to the result of the division, and one in the binary to decimal conversion for the printed output. On computers that use IEEE<sup>®</sup> floating-point standard arithmetic, the resulting internal value is the binary expansion of 5/6, truncated to 53 bits. This is approximately 16 decimal digits. But, in this particular case, the printed output shows only 15 digits.

The symbolic operations used by rational arithmetic are potentially the most expensive of the three, in terms of both computer time and memory. The results are exact, as long as enough time and memory are available to complete the computations.

Variable-precision arithmetic falls in between the other two in terms of both cost and accuracy. A global parameter, set by the function `digits`, controls the number of significant decimal digits. Increasing the number of digits increases the accuracy, but also increases both the time and memory requirements. The default value of `digits` is 32, corresponding roughly to floating-point accuracy.

## Example: Using the Different Kinds of Arithmetic

### Rational Arithmetic

By default, Symbolic Math Toolbox software uses rational arithmetic operations, i.e., MuPAD software's exact symbolic arithmetic. Rational arithmetic is invoked when you create symbolic variables using the `sym` function.

The `sym` function converts a double matrix to its symbolic form. For example, if the double matrix is

```
format short;  
A = [1.1,1.2,1.3;2.1,2.2,2.3;3.1,3.2,3.3]
```

```
A =  
    1.1000    1.2000    1.3000  
    2.1000    2.2000    2.3000  
    3.1000    3.2000    3.3000
```

its symbolic form is:

```
S = sym(A)  
  
S =  
[ 11/10,    6/5, 13/10]  
[ 21/10, 11/5, 23/10]  
[ 31/10, 16/5, 33/10]
```

For this matrix `A`, it is possible to discover that the elements are the ratios of small integers, so the symbolic representation is formed from those integers. On the other hand, the statement

```
E = [exp(1) (1 + sqrt(5))/2; log(3) rand]
```

returns a matrix

```
E =  
    2.7183    1.6180  
    1.0986    0.6324
```

whose elements are not the ratios of small integers, so

```
sym(E)
```

reproduces the floating-point representation in a symbolic form:

```
ans =  
[ 3060513257434037/1125899906842624,    910872158600853/562949953421312]  
[ 2473854946935173/2251799813685248, 1423946432832521/2251799813685248]
```

## Variable-Precision Numbers

Variable-precision numbers are distinguished from the exact rational representation by the presence of a decimal point. A power of 10 scale factor, denoted by 'e', is allowed. To use variable-precision instead of rational arithmetic, create your variables using the `vpa` function.

For matrices with purely double entries, the `vpa` function generates the representation that is used with variable-precision arithmetic. For example, if you apply `vpa` to the matrix `S` defined in the preceding section, with `digits(4)`, by entering

```
digits(4);  
vpa(S)
```

MATLAB returns the output

```
ans =  
[ 1.1, 1.2, 1.3]  
[ 2.1, 2.2, 2.3]  
[ 3.1, 3.2, 3.3]
```

Applying `vpa` to the matrix `E` defined in the preceding section, with `digits(25)`, by entering

```
digits(25)  
F = vpa(E)
```

returns

```
F =  
[ 2.718281828459045534884808, 1.618033988749894902525739]  
[ 1.098612288668109560063613, 0.6323592462254095103446616]
```

Restore the default `digits` setting:

```
digits(32);
```

## Converting to Floating-Point

To convert a rational or variable-precision number to its MATLAB floating-point representation, use the `double` function.

In the example, both `double(sym(E))` and `double(vpa(E))` return `E`.

### **Another Example Using Different Kinds of Arithmetic**

The next example is perhaps more interesting. Start with the symbolic expression

```
f = sym('exp(pi*sqrt(163))');
```

The statement

```
format long;  
double(f)
```

produces the printed floating-point value

```
ans =  
2.625374126407687e+017
```

Using the second argument of `vpa` to specify the number of digits,

```
vpa(f,18)
```

returns

```
ans =  
262537412640768744.0
```

and, too,

```
vpa(f,25)
```

returns

```
ans =  
262537412640768744.0
```

You might suspect that `f` actually has an integer value. However, the 40-digit value

```
vpa(f,40)
```

```
ans =
```

262537412640768743.999999999992500725972

shows that  $f$  is very close to, but not exactly equal to, an integer.

## Linear Algebra

### In this section...

“Basic Algebraic Operations” on page 3-54  
 “Linear Algebraic Operations” on page 3-55  
 “Eigenvalues” on page 3-60  
 “Jordan Canonical Form” on page 3-65  
 “Singular Value Decomposition” on page 3-67  
 “Eigenvalue Trajectories” on page 3-70

### Basic Algebraic Operations

Basic algebraic operations on symbolic objects are the same as operations on MATLAB objects of class `double`. This is illustrated in the following example.

The Givens transformation produces a plane rotation through the angle  $t$ . The statements

```
syms t;
G = [cos(t) sin(t); -sin(t) cos(t)]
```

create this transformation matrix.

```
G =
[ cos(t),  sin(t)]
[ -sin(t),  cos(t)]
```

Applying the Givens transformation twice should simply be a rotation through twice the angle. The corresponding matrix can be computed by multiplying  $G$  by itself or by raising  $G$  to the second power. Both

```
A = G*G
```

and

```
A = G^2
```

produce

```
A =
[ cos(t)^2 - sin(t)^2,    2*cos(t)*sin(t)]
[   -2*cos(t)*sin(t), cos(t)^2 - sin(t)^2]
```

The `simple` function

```
A = simple(A)
```

uses a trigonometric identity to return the expected form by trying several different identities and picking the one that produces the shortest representation.

```
A =
[ cos(2*t), sin(2*t)]
[ -sin(2*t), cos(2*t)]
```

The Givens rotation is an orthogonal matrix, so its transpose is its inverse. Confirming this by

```
I = G.' *G
```

which produces

```
I =
[ cos(t)^2 + sin(t)^2,    0]
[    0, cos(t)^2 + sin(t)^2]
```

and then

```
I = simple(I)
```

```
I =
[ 1, 0]
[ 0, 1]
```

## Linear Algebraic Operations

The following examples show how to do several basic linear algebraic operations using Symbolic Math Toolbox software.

The command

```
H = hilb(3)
```

generates the 3-by-3 Hilbert matrix. With `format short`, MATLAB prints

```
H =  
    1.0000    0.5000    0.3333  
    0.5000    0.3333    0.2500  
    0.3333    0.2500    0.2000
```

The computed elements of `H` are floating-point numbers that are the ratios of small integers. Indeed, `H` is a MATLAB array of class `double`. Converting `H` to a symbolic matrix

```
H = sym(H)
```

gives

```
H =  
[ 1, 1/2, 1/3]  
[ 1/2, 1/3, 1/4]  
[ 1/3, 1/4, 1/5]
```

This allows subsequent symbolic operations on `H` to produce results that correspond to the infinitely precise Hilbert matrix, `sym(hilb(3))`, not its floating-point approximation, `hilb(3)`. Therefore,

```
inv(H)
```

produces

```
ans =  
[ 9, -36, 30]  
[ -36, 192, -180]  
[ 30, -180, 180]
```

and

```
det(H)
```

yields



```
ans =
1/2160
```

You can use the backslash operator to solve a system of simultaneous linear equations. For example, the commands

```
% Solve Hx = b
b = [1; 1; 1];
x = H\b
```

produce the solution

```
x =
3
-24
30
```

All three of these results, the inverse, the determinant, and the solution to the linear system, are the exact results corresponding to the infinitely precise, rational, Hilbert matrix. On the other hand, using `digits(16)`, the command

```
digits(16);
V = vpa(hilb(3))
```

returns

```
V =
[ 1.0, 0.5, 0.3333333333333333]
[ 0.5, 0.3333333333333333, 0.25]
[ 0.3333333333333333, 0.25, 0.2]
```

The decimal points in the representation of the individual elements are the signal to use variable-precision arithmetic. The result of each arithmetic operation is rounded to 16 significant decimal digits. When inverting the matrix, these errors are magnified by the matrix condition number, which for `hilb(3)` is about 500. Consequently,

```
inv(V)
```

which returns

```
ans =
```

```
[ 9.0, -36.0, 30.0]
[-36.0, 192.0, -180.0]
[ 30.0, -180.0, 180.0]
```

shows the loss of two digits. So does

```
1/det(V)
```

which gives

```
ans =
2160.000000000018
```

and

```
V\b
```

which is

```
ans =
 3.0
-24.0
 30.0
```

Since H is nonsingular, calculating the null space of H with the command

```
null(H)
```

returns an empty matrix:

```
ans =
[ empty sym ]
```

Calculating the column space of H with

```
colspace(H)
```

returns a permutation of the identity matrix:

```
ans =
[ 1, 0, 0]
[ 0, 1, 0]
[ 0, 0, 1]
```

A more interesting example, which the following code shows, is to find a value  $s$  for  $H(1,1)$  that makes  $H$  singular. The commands

```
syms s
H(1,1) = s
Z = det(H)
sol = solve(Z)
```

produce

```
H =
[ s, 1/2, 1/3]
[ 1/2, 1/3, 1/4]
[ 1/3, 1/4, 1/5]
```

```
Z =
s/240 - 1/270
```

```
sol =
8/9
```

Then

```
H = subs(H, s, sol)
```

substitutes the computed value of  $sol$  for  $s$  in  $H$  to give

```
H =
[ 8/9, 1/2, 1/3]
[ 1/2, 1/3, 1/4]
[ 1/3, 1/4, 1/5]
```

Now, the command

```
det(H)
```

returns

```
ans =
0
```

and

```
inv(H)
```

produces the message

```
ans =  
FAIL
```

because H is singular. For this matrix, null space and column space are nontrivial:

```
Z = null(H)  
C = colspace(H)  
  
Z =  
3/10  
-6/5  
1  
C =  
[ 1, 0]  
[ 0, 1]  
[-3/10, 6/5]
```

It should be pointed out that even though H is singular, `vpa(H)` is not. For any integer value `d`, setting `digits(d)`, and then computing `inv(vpa(H))` results in an inverse with elements on the order of  $10^d$ .

## Eigenvalues

The symbolic eigenvalues of a square matrix A or the symbolic eigenvalues and eigenvectors of A are computed, respectively, using the commands `E = eig(A)` and `[V,E] = eig(A)`.

The variable-precision counterparts are `E = eig(vpa(A))` and `[V,E] = eig(vpa(A))`.

The eigenvalues of A are the zeros of the characteristic polynomial of A,  $\det(A-x*I)$ , which is computed by `poly(A)`.

The matrix H from the last section provides the first example:

```
H = sym([8/9 1/2 1/3; 1/2 1/3 1/4; 1/3 1/4 1/5])
```

$$\begin{aligned}
 H &= \\
 &[ 8/9, 1/2, 1/3] \\
 &[ 1/2, 1/3, 1/4] \\
 &[ 1/3, 1/4, 1/5]
 \end{aligned}$$

The matrix is singular, so one of its eigenvalues must be zero. The statement

$$[T, E] = \text{eig}(H)$$

produces the matrices T and E. The columns of T are the eigenvectors of H and the diagonal elements of E are the eigenvalues of H:

$$\begin{aligned}
 T &= \\
 &[ 218/285 - (4*12589^{(1/2)})/285, (4*12589^{(1/2)})/285 + 218/285, 3/10] \\
 &[ 292/285 - 12589^{(1/2)}/285, 12589^{(1/2)}/285 + 292/285, -6/5] \\
 &[ 1, 1, 1]
 \end{aligned}$$

$$\begin{aligned}
 E &= \\
 &[ 32/45 - 12589^{(1/2)}/180, 0, 0] \\
 &[ 0, 12589^{(1/2)}/180 + 32/45, 0] \\
 &[ 0, 0, 0]
 \end{aligned}$$

It may be easier to understand the structure of the matrices of eigenvectors, T, and eigenvalues, E, if you convert T and E to decimal notation. To do so, proceed as follows. The commands

$$\begin{aligned}
 Td &= \text{double}(T) \\
 Ed &= \text{double}(E)
 \end{aligned}$$

return

$$\begin{aligned}
 Td &= \\
 &-0.8098 \quad 2.3397 \quad 0.3000 \\
 &0.6309 \quad 1.4182 \quad -1.2000 \\
 &1.0000 \quad 1.0000 \quad 1.0000
 \end{aligned}$$

$$\begin{aligned}
 Ed &= \\
 &0.0878 \quad 0 \quad 0 \\
 &0 \quad 1.3344 \quad 0 \\
 &0 \quad 0 \quad 0
 \end{aligned}$$

The first eigenvalue is zero. The corresponding eigenvector (the first column of  $Td$ ) is the same as the basis for the null space found in the last section. The other two eigenvalues are the result of applying the quadratic formula to

$x^2 - \frac{64}{45}x + \frac{253}{2160}$  which is the quadratic factor in `factor(poly(H))`:

```
syms x
g = simple(factor(poly(H))/x);
solve(g)
```

```
ans =
    12589^(1/2)/180 + 32/45
    32/45 - 12589^(1/2)/180
```

Closed form symbolic expressions for the eigenvalues are possible only when the characteristic polynomial can be expressed as a product of rational polynomials of degree four or less. The Rosser matrix is a classic numerical analysis test matrix that illustrates this requirement. The statement

```
R = sym(rosser)
```

generates

```
R =
[ 611, 196, -192, 407, -8, -52, -49, 29]
[ 196, 899, 113, -192, -71, -43, -8, -44]
[ -192, 113, 899, 196, 61, 49, 8, 52]
[ 407, -192, 196, 611, 8, 44, 59, -23]
[ -8, -71, 61, 8, 411, -599, 208, 208]
[ -52, -43, 49, 44, -599, 411, 208, 208]
[ -49, -8, 8, 59, 208, 208, 99, -911]
[ 29, -44, 52, -23, 208, 208, -911, 99]
```

The commands

```
p = poly(R);
pretty(factor(p))
```

produce

$$x(x - 1020)(x^2 - 1040500)(x^2 - 1020x + 100)(x^2 - 1000)$$

The characteristic polynomial (of degree 8) factors nicely into the product of two linear terms and three quadratic terms. You can see immediately that four of the eigenvalues are 0, 1020, and a double root at 1000. The other four roots are obtained from the remaining quadratics. Use

```
eig(R)
```

to find all these values

```
ans =
      0
     1000
     1000
     1020
510 - 100*26^(1/2)
100*26^(1/2) + 510
-10*10405^(1/2)
10*10405^(1/2)
```

The Rosser matrix is not a typical example; it is rare for a full 8-by-8 matrix to have a characteristic polynomial that factors into such simple form. If you change the two “corner” elements of R from 29 to 30 with the commands

```
S = R; S(1,8) = 30; S(8,1) = 30;
```

and then try

```
p = poly(S)
```

you find

```
p =
x^8 - 4040*x^7 + 5079941*x^6 + 82706090*x^5...
- 5327831918568*x^4 + 4287832912719760*x^3...
- 1082699388411166000*x^2 + 51264008540948000*x...
+ 40250968213600000
```

You also find that `factor(p)` is `p` itself. That is, the characteristic polynomial cannot be factored over the rationals.

For this modified Rosser matrix

```
F = eig(S)
```

returns

```
F =
    1020.420188201504727818545749884
    1019.9935501291629257348091808173
    1019.5243552632016358324933278291
    1000.1206982933841335712817075454
    999.94691786044276755320289228602
    0.21803980548301606860857564424981
   -0.17053529728768998575200874607757
   -1020.05321425589151659318942526
```

Notice that these values are close to the eigenvalues of the original Rosser matrix. Further, the numerical values of `F` are a result of MuPAD software's floating-point arithmetic. Consequently, different settings of `digits` do not alter the number of digits to the right of the decimal place.

It is also possible to try to compute eigenvalues of symbolic matrices, but closed form solutions are rare. The Givens transformation is generated as the matrix exponential of the elementary matrix

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$

Symbolic Math Toolbox commands

```
syms t
A = sym([0 1; -1 0]);
G = expm(t*A)
```

return

```
G =
[      1/(2*exp(t*i)) + exp(t*i)/2,
```



$$\begin{bmatrix} i/(2\exp(t*i)) - (\exp(t*i)*i)/2 \\ - i/(2\exp(t*i)) + (\exp(t*i)*i)/2, \\ 1/(2\exp(t*i)) + \exp(t*i)/2 \end{bmatrix}$$

You can simplify this expression with the `simple` command:

```
[G,how] = simple(G)

G =
[ cos(t), sin(t)]
[ -sin(t), cos(t)]

how =
simplify
```

Next, the command

```
g = eig(G)
```

produces

```
g =
cos(t) - sin(t)*i
cos(t) + sin(t)*i
```

You can use `simple` to simplify this form of `g`:

```
[g,how] = simple(g)

g =
1/exp(t*i)
exp(t*i)

how =
rewrite(exp)
```

## Jordan Canonical Form

The Jordan canonical form results from attempts to diagonalize a matrix by a similarity transformation. For a given matrix  $A$ , find a nonsingular matrix  $V$ , so that  $\text{inv}(V) * A * V$ , or, more succinctly,  $J = V \backslash A * V$ , is “as close to diagonal as possible.” For almost all matrices, the Jordan canonical form is the diagonal matrix of eigenvalues and the columns of the transformation

matrix are the eigenvectors. This always happens if the matrix is symmetric or if it has distinct eigenvalues. Some nonsymmetric matrices with multiple eigenvalues cannot be diagonalized. The Jordan form has the eigenvalues on its diagonal, but some of the superdiagonal elements are one, instead of zero. The statement

```
J = jordan(A)
```

computes the Jordan canonical form of  $A$ . The statement

```
[V,J] = jordan(A)
```

also computes the similarity transformation. The columns of  $V$  are the generalized eigenvectors of  $A$ .

The Jordan form is extremely sensitive to perturbations. Almost any change in  $A$  causes its Jordan form to be diagonal. This makes it very difficult to compute the Jordan form reliably with floating-point arithmetic. It also implies that  $A$  must be known exactly (i.e., without roundoff error, etc.). Its elements must be integers, or ratios of small integers. In particular, the variable-precision calculation, `jordan(vpa(A))`, is not allowed.

For example, let

```
A = sym([12,32,66,116;-25,-76,-164,-294;  
        21,66,143,256;-6,-19,-41,-73])
```

```
A =  
[ 12, 32, 66, 116]  
[ -25, -76, -164, -294]  
[ 21, 66, 143, 256]  
[ -6, -19, -41, -73]
```

Then

```
[V,J] = jordan(A)
```

produces

```
V =  
[ 4, -2, 4, 3]  
[ -6, 8, -11, -8]
```

$$\begin{bmatrix} 4 & -7 & 10 & 7 \\ -1 & 2 & -3 & -2 \end{bmatrix}$$

$$J = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

Therefore  $A$  has a double eigenvalue at 1, with a single Jordan block, and a double eigenvalue at 2, also with a single Jordan block. The matrix has only two eigenvectors,  $V(:, 1)$  and  $V(:, 3)$ . They satisfy

$$\begin{aligned} A*V(:, 1) &= 1*V(:, 1) \\ A*V(:, 3) &= 2*V(:, 3) \end{aligned}$$

The other two columns of  $V$  are generalized eigenvectors of grade 2. They satisfy

$$\begin{aligned} A*V(:, 2) &= 1*V(:, 2) + V(:, 1) \\ A*V(:, 4) &= 2*V(:, 4) + V(:, 3) \end{aligned}$$

In mathematical notation, with  $v_j = v(:, j)$ , the columns of  $V$  and eigenvalues satisfy the relationships

$$(A - \lambda_1 I)v_2 = v_1$$

$$(A - \lambda_2 I)v_4 = v_3.$$

## Singular Value Decomposition

Only the variable-precision numeric computation of the complete singular vector decomposition is available in the toolbox. One reason for this is that the formulas that result from symbolic computation are usually too long and complicated to be of much use. If  $A$  is a symbolic matrix of floating-point or variable-precision numbers, then

$$S = \text{svd}(A)$$

computes the singular values of  $A$  to an accuracy determined by the current setting of `digits`. And

```
[U,S,V] = svd(A);
```

produces two orthogonal matrices, U and V, and a diagonal matrix, S, so that

```
A = U*S*V';
```

Consider the n-by-n matrix A with elements defined by  $A(i,j) = 1/(i - j + 1/2)$ . The most obvious way of generating this matrix is

```
n = 5;
for i=1:n
    for j=1:n
        A(i,j) = sym(1/(i-j+1/2));
    end
end
```

For n = 5, the matrix is

```
A
A =
[ 2, -2, -2/3, -2/5, -2/7]
[ 2/3, 2, -2, -2/3, -2/5]
[ 2/5, 2/3, 2, -2, -2/3]
[ 2/7, 2/5, 2/3, 2, -2]
[ 2/9, 2/7, 2/5, 2/3, 2]
```

It turns out many of the singular values of these matrices are close to  $\pi$ .

The most efficient way to generate the matrix is

```
n = 5;
[J,I] = meshgrid(1:n);
A = sym(1./(I - J+1/2));
```

Since the elements of A are the ratios of small integers, vpa(A) produces a variable-precision representation, which is accurate to digits precision. Hence

```
S = svd(vpa(A))
```

computes the desired singular values to full accuracy. With  $n = 16$  and `digits(30)`, the result is

```
S =
 3.14159265358979323846255035973
 3.14159265358979323843066846713
 3.14159265358979323325290142782
 3.14159265358979270342635559052
   3.1415926535897543920684990722
 3.14159265358767361712392612382
 3.14159265349961053143856838564
 3.14159265052654880815569479613
 3.14159256925492306470284863101
 3.14159075458605848728982577118
   3.1415575435991808369105065826
 3.14106044663470063805218371923
 3.13504054399744654843898901261
 3.07790297231119748658424727353
 2.69162158686066606774782763593
 1.20968137605668985332455685355
```

Compare `S` with `pi`, the floating-point representation of  $\pi$ . In the vector below, the first element is computed by subtraction with variable-precision arithmetic and then converted to a double:

```
format long;
double(pi*ones(16,1)-S)
```

The results are

```
ans =
 0.000000000000000
 0.000000000000000
 0.000000000000000
 0.000000000000001
 0.000000000000039
 0.000000000002120
 0.000000000090183
 0.000000003063244
 0.000000084334870
 0.000001899003735
```

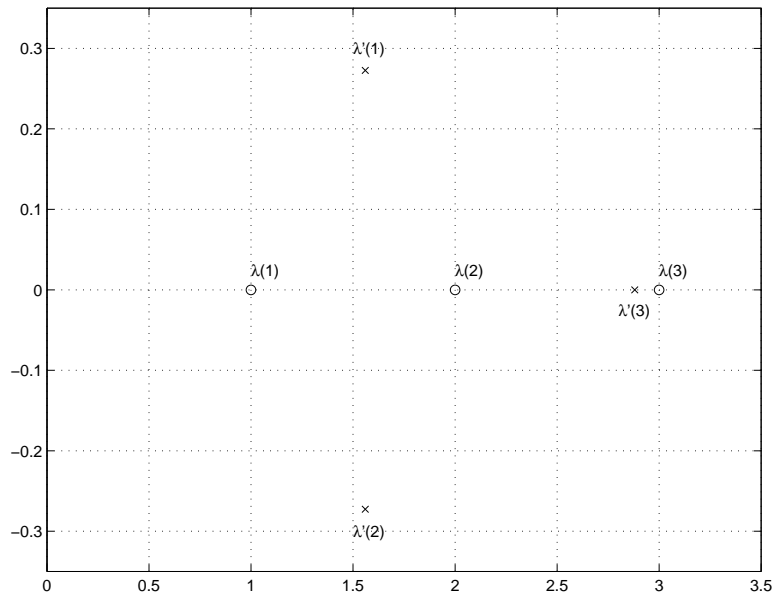
```
0.000035109990612
0.000532206955093
0.006552109592347
0.063689681278596
0.449971066729127
1.931911277533103
```

Since the relative accuracy of  $\pi$  is  $\pi \cdot \text{eps}$ , which is  $6.9757\text{e-}16$ , the result confirms the suspicion that four of the singular values of the 16-by-16 example equal  $\pi$  to floating-point accuracy.

## Eigenvalue Trajectories

This example applies several numeric, symbolic, and graphic techniques to study the behavior of matrix eigenvalues as a parameter in the matrix is varied. This particular setting involves numerical analysis and perturbation theory, but the techniques illustrated are more widely applicable.

In this example, you consider a 3-by-3 matrix  $A$  whose eigenvalues are 1, 2, 3. First, you perturb  $A$  by another matrix  $E$  and parameter  $t : A \rightarrow A + tE$ . As  $t$  increases from 0 to  $10^{-6}$ , the eigenvalues  $\lambda_1 = 1$ ,  $\lambda_2 = 2$ ,  $\lambda_3 = 3$  change to  $\lambda_1' = 1.5596 + 0.2726i$ ,  $\lambda_2' = 1.5596 - 0.2726i$ ,  $\lambda_3' = 2.8808$ .



This, in turn, means that for some value of  $t = \tau$ ,  $0 < \tau < 10^{-6}$ , the perturbed matrix  $A(t) = A + tE$  has a double eigenvalue  $\lambda_1 = \lambda_2$ . The example shows how to find the value of  $\tau$ , called  $\tau$ , where this happens.

The starting point is a MATLAB test example, known as `gallery(3)`.

```
A = gallery(3)
```

```
A =
-149    -50   -154
 537   180   546
 -27    -9   -25
```

This is an example of a matrix whose eigenvalues are sensitive to the effects of roundoff errors introduced during their computation. The actual computed eigenvalues may vary from one machine to another, but on a typical workstation, the statements

```
format long
e = eig(A)

produce

e =
    1.000000000010722
    1.99999999991790
    2.99999999997399
```

Of course, the example was created so that its eigenvalues are actually 1, 2, and 3. Note that three or four digits have been lost to roundoff. This can be easily verified with the toolbox. The statements

```
B = sym(A);
e = eig(B) '
p = poly(B)
f = factor(p)

produce

e =
    [1, 2, 3]

p =
    x^3 - 6*x^2 + 11*x - 6

f =
    (x - 3)*(x - 1)*(x - 2)
```

Are the eigenvalues sensitive to the perturbations caused by roundoff error because they are “close together”? Ordinarily, the values 1, 2, and 3 would be regarded as “well separated.” But, in this case, the separation should be viewed on the scale of the original matrix. If  $A$  were replaced by  $A/1000$ , the eigenvalues, which would be .001, .002, .003, would “seem” to be closer together.

But eigenvalue sensitivity is more subtle than just “closeness.” With a carefully chosen perturbation of the matrix, it is possible to make two of its eigenvalues coalesce into an actual double root that is extremely sensitive to roundoff and other errors.



One good perturbation direction can be obtained from the outer product of the left and right eigenvectors associated with the most sensitive eigenvalue. The following statement creates the perturbation matrix:

$$E = [130, -390, 0; 43, -129, 0; 133, -399, 0]$$

$$E = \begin{bmatrix} 130 & -390 & 0 \\ 43 & -129 & 0 \\ 133 & -399 & 0 \end{bmatrix}$$

The perturbation can now be expressed in terms of a single, scalar parameter  $t$ . The statements

```
syms x t
A = A + t*E
```

replace  $A$  with the symbolic representation of its perturbation:

$$A = \begin{bmatrix} 130*t - 149, & -390*t - 50, & -154 \\ 43*t + 537, & 180 - 129*t, & 546 \\ 133*t - 27, & -399*t - 9, & -25 \end{bmatrix}$$

Computing the characteristic polynomial of this new  $A$

```
p = simple(poly(A))
```

gives

$$p = 11*x^3 - 1221271*t*x^2 - x^2*(t + 6) + 492512*t*x + x^3 - 6$$

$p$  is a cubic in  $x$  whose coefficients vary linearly with  $t$ .

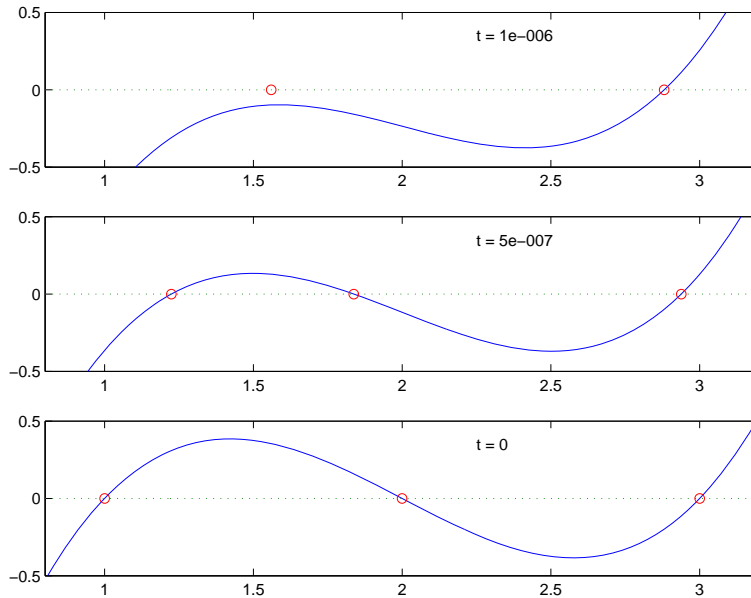
It turns out that when  $t$  is varied over a very small interval, from 0 to  $1.0e-6$ , the desired double root appears. This can best be seen graphically. The first figure shows plots of  $p$ , considered as a function of  $x$ , for three different values of  $t$ :  $t = 0$ ,  $t = 0.5e-6$ , and  $t = 1.0e-6$ . For each value, the eigenvalues are computed numerically and also plotted:

```
x = .8:.01:3.2;
```

```

for k = 0:2
    c = sym2poly(subs(p,t,k*0.5e-6));
    y = polyval(c,x);
    lambda = eig(double(subs(A,t,k*0.5e-6)));
    subplot(3,1,3-k)
    plot(x,y,'-',x,0*x,':',lambda,0*lambda,'o')
    axis([.8 3.2 -.5 .5])
    text(2.25,.35,['t = ' num2str( k*0.5e-6 )]);
end

```



The bottom subplot shows the unperturbed polynomial, with its three roots at 1, 2, and 3. The middle subplot shows the first two roots approaching each other. In the top subplot, these two roots have become complex and only one real root remains.

The next statements compute and display the actual eigenvalues

```
e = eig(A);
```

```

ee = subexpr(e);

sigma =
(1221271*t)/2 + (t + 6)^3/27 - ((492512*t + 11)*(t + 6))/6 + ...
(((492512*t)/3 - (t + 6)^2/9 + 11/3)^3 + ((1221271*t)/2 + ...
(t + 6)^3/27 - ((492512*t + 11)*(t + 6))/6 + 3)^2)^(1/2) + 3

pretty(ee)

```

showing that e(2) and e(3) form a complex conjugate pair:

$$\begin{array}{|c}
 \text{+-} \\
 | \quad t \quad \quad \quad 1/3 \\
 | \quad - + \text{sigma} \quad - \#3 + 2 \\
 | \quad 3 \\
 | \\
 | \quad \quad \quad 1/3 \\
 | \quad t \quad \text{sigma} \\
 | \quad - - \text{-----} + \#1 + 2 - \#2 \\
 | \quad 3 \quad \quad 2 \\
 | \\
 | \quad \quad \quad 1/3 \\
 | \quad t \quad \text{sigma} \\
 | \quad - - \text{-----} + \#1 + 2 + \#2 \\
 | \quad 3 \quad \quad 2 \\
 | \\
 \text{+-} \\
 \text{-+}
 \end{array}$$

where

$$\begin{aligned}
 \#1 &= \frac{\frac{492512 t}{3} - \frac{(t + 6)^2}{9} + 11/3}{2 \text{sigma}^{1/3}} \\
 \#2 &= \frac{1/2}{3} \frac{1/3 (\text{sigma} + \#3) i}{\text{-----}}
 \end{aligned}$$

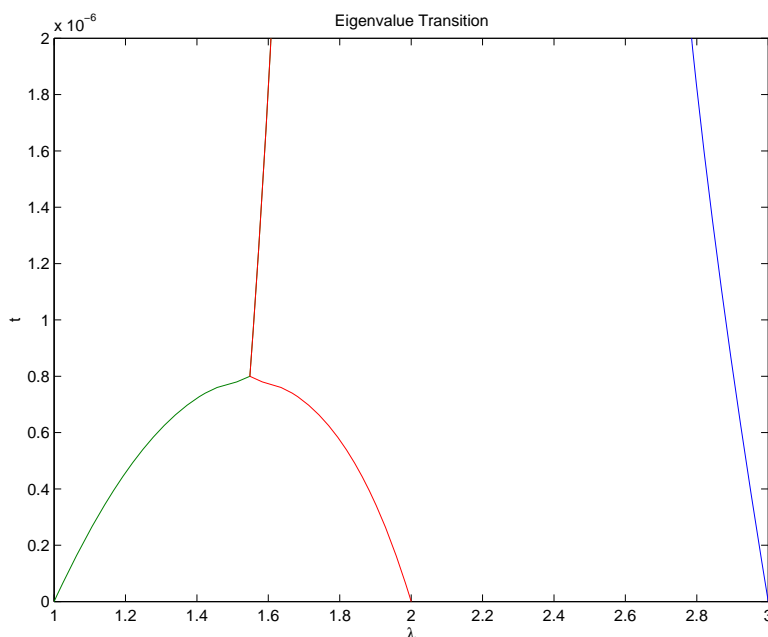
$$\lambda_3 = \frac{492512 t^2 (t + 6)^2}{3^3 \cdot 9} + 11/3$$

sigma

Next, the symbolic representations of the three eigenvalues are evaluated at many values of t

```
tvals = (2:-.02:0)' * 1.e-6;
r = size(tvals,1);
c = size(e,1);
lambda = zeros(r,c);
for k = 1:c
    lambda(:,k) = double(subs(e(k),t,tvals));
end
plot(lambda,tvals)
xlabel('\lambda'); ylabel('t');
title('Eigenvalue Transition')
```

to produce a plot of their trajectories.



Above  $\tau = 0.8e^{-6}$ , the graphs of two of the eigenvalues intersect, while below  $\tau = 0.8e^{-6}$ , two real roots become a complex conjugate pair. What is the precise value of  $\tau$  that marks this transition? Let  $\tau$  denote this value of  $\tau$ .

One way to find the *exact* value of  $\tau$  involves polynomial discriminants. The discriminant of a quadratic polynomial is the familiar quantity under the square root sign in the quadratic formula. When it is negative, the two roots are complex.

There is no `discrim` function in the toolbox, but there is one in the MuPAD language. The statement

```
doc(symengine, 'discrim')
```

gives the MuPAD help for the function.

## polylib::discrim – discriminant of a polynomial

`polylib::discrim(p, x)` returns the discriminant of the polynomial `p` with respect to the variable `x`.

→ Examples

**Call:**

```
polylib::discrim(p, x)
```

This shows that the `discrim` function is in the `polylib` library. Use these commands

```
syms a b c x
evalin(symengine, 'polylib::discrim(a*x^2+b*x+c, x)')
```

to show the generic quadratic's discriminant,  $b^2 - 4ac$ :

```
ans =
b^2 - 4*a*c
```

The discriminant for the perturbed cubic characteristic polynomial is obtained, using

```
discrim = feval(symengine, 'polylib::discrim', p, x)
```

which produces

```
discrim =
242563185060*t^4 - 477857003880091920*t^3 + ...
1403772863224*t^2 - 5910096*t + 4
```

The quantity  $\tau$  is one of the four roots of this quartic. You can find a numeric value for  $\tau$  with the following code.

```
s = solve(discrim);
tau = vpa(s)
```

```
tau =
1970031.04061804553618913725474883634597991201389
0.000000783792490596794010485879469854518820556090553664
0.00000107692481604921513807537160160597784208236311263 - 0.00000308544636502289065492747*i
0.00000308544636502289065492746538275636180217710757295*i + 0.00000107692481604921513807537160160597784249167873707
```

Of the four solutions, you know that

```
tau = tau(2)
```

is the transition point

```
tau =
0.00000078379249059679401048084
```

because it is closest to the previous estimate.

A more generally applicable method for finding  $\tau$  is based on the fact that, at a double root, both the function and its derivative must vanish. This results in two polynomial equations to be solved for two unknowns. The statement

```
sol = solve(p,diff(p,'x'))
```

solves the pair of algebraic equations  $p = 0$  and  $dp/dx = 0$  and produces

```
sol =
t: [4x1 sym]
x: [4x1 sym]
```

Find  $\tau$  now by

```
format short
tau = double(sol.t(2))
```

which reveals that the second element of `sol.t` is the desired value of  $\tau$ :

```
tau =
7.8379e-007
```

Therefore, the second element of `sol.x`

```
sigma = double(sol.x(2))
```

is the double eigenvalue

```
sigma =  
    1.5476
```

To verify that this value of  $\tau$  does indeed produce a double eigenvalue at  $\sigma = 1.5476$ , substitute  $\tau$  for  $t$  in the perturbed matrix  $A(t) = A + tE$  and find the eigenvalues of  $A(t)$ . That is,

```
e = eig(double(subs(A, t, tau)))
```

```
e =  
    1.5476  
    1.5476  
    2.9048
```

confirms that  $\sigma = 1.5476$  is a double eigenvalue of  $A(t)$  for  $t = 7.8379e-07$ .



## Solving Equations

### In this section...

“Solving Algebraic Equations” on page 3-81

“Several Algebraic Equations” on page 3-82

“Single Differential Equation” on page 3-85

“Several Differential Equations” on page 3-88

### Solving Algebraic Equations

If  $S$  is a symbolic expression,

```
solve(S)
```

attempts to find values of the symbolic variable in  $S$  (as determined by `symvar`) for which  $S$  is zero. For example,

```
syms a b c x
S = a*x^2 + b*x + c;
solve(S)
```

uses the familiar quadratic formula to produce

```
ans =
- (b + (b^2 - 4*a*c)^(1/2)) / (2*a)
- (b - (b^2 - 4*a*c)^(1/2)) / (2*a)
```

This is a symbolic vector whose elements are the two solutions.

If you want to solve for a specific variable, you must specify that variable as an additional argument. For example, if you want to solve  $S$  for  $b$ , use the command

```
b = solve(S,b)
```

which returns

```
b =
-(a*x^2 + c)/x
```

Note that these examples assume equations of the form  $f(x) = 0$ . If you need to solve equations of the form  $f(x) = q(x)$ , you must use quoted strings. In particular, the command

```
s = solve('cos(2*x) + sin(x) = 1')
```

returns a vector with three solutions.

```
s =
      0
     pi/6
    (5*pi)/6
```

There are also solutions at each of these results plus  $k\pi$  for integer  $k$ , as you can see in the MuPAD solution:

```
[ solve(cos(2*x) + sin(x) = 1, x)
  { pi*k | k in Z } union { pi/6 + 2*pi*k | k in Z } union { 5*pi/6 + 2*pi*k | k in Z }
```

## Several Algebraic Equations

This section explains how to solve systems of equations using Symbolic Math Toolbox software. As an example, suppose you have the system

$$\begin{aligned}x^2y^2 &= 0 \\ x - \frac{y}{2} &= \alpha,\end{aligned}$$

and you want to solve for  $x$  and  $y$ . First, create the necessary symbolic objects.

```
syms x y;
alpha = sym('alpha');
```

There are several ways to address the output of `solve`. One is to use a two-output call

```
[x, y] = solve(x^2*y^2, x-y/2 - alpha)
```

which returns

```

x =
  alpha
    0

y =
    0
 -2*alpha

```

Modify the first equation to  $x^2y^2 = 1$  and there are more solutions.

```

eqs1 = 'x^2*y^2=1, x-y/2-alpha';
[x,y] = solve(eqs1)

```

produces four distinct solutions:

```

x =
  alpha/2 + (alpha^2 + 2)^(1/2)/2
  alpha/2 + (alpha^2 - 2)^(1/2)/2
  alpha/2 - (alpha^2 + 2)^(1/2)/2
  alpha/2 - (alpha^2 - 2)^(1/2)/2

y =
  (alpha^2 + 2)^(1/2) - alpha
  (alpha^2 - 2)^(1/2) - alpha
 - alpha - (alpha^2 + 2)^(1/2)
 - alpha - (alpha^2 - 2)^(1/2)

```

Since you did not specify the dependent variables, `solve` uses `symvar` to determine the variables.

This way of assigning output from `solve` is quite successful for “small” systems. Plainly, if you had, say, a 10-by-10 system of equations, typing

```
[x1,x2,x3,x4,x5,x6,x7,x8,x9,x10] = solve(...)
```

is both awkward and time consuming. To circumvent this difficulty, `solve` can return a structure whose fields are the solutions. In particular, consider the system  $u^2 - v^2 = a^2$ ,  $u + v = 1$ ,  $a^2 - 2*a = 3$ . The command

```
S = solve('u^2 - v^2 = a^2', 'u + v = 1', 'a^2 - 2*a = 3')
```

returns

```
S =  
  a: [2x1 sym]  
  u: [2x1 sym]  
  v: [2x1 sym]
```

The solutions for *a* reside in the “a-field” of *S*. That is,

```
S.a
```

produces

```
ans =  
 -1  
  3
```

Similar comments apply to the solutions for *u* and *v*. The structure *S* can now be manipulated by field and index to access a particular portion of the solution. For example, if you want to examine the second solution, you can use the following statement

```
s2 = [S.a(2), S.u(2), S.v(2)]
```

to extract the second component of each field.

```
s2 =  
 [ 3, 5, -4]
```

The following statement

```
M = [S.a, S.u, S.v]
```

creates the solution matrix *M*

```
M =  
 [ -1, 1, 0]  
 [  3, 5, -4]
```

whose rows comprise the distinct solutions of the system.

Linear systems of simultaneous equations can also be solved using matrix division. For example,

```
clear u v x y
```

```
syms u v x y
S = solve(x + 2*y - u, 4*x + 5*y - v);
sol = [S.x; S.y]
```

```
A = [1 2; 4 5];
b = [u; v];
z = A\b
```

results in

```
sol =
(2*v)/3 - (5*u)/3
(4*u)/3 - v/3

z =
(2*v)/3 - (5*u)/3
(4*u)/3 - v/3
```

Thus `s` and `z` produce the same solution, although the results are assigned to different variables.

## Single Differential Equation

The function `dsolve` computes symbolic solutions to ordinary differential equations. The equations are specified by symbolic expressions containing the letter `D` to denote differentiation. The symbols `D2`, `D3`, ... `DN`, correspond to the second, third, ..., `N`th derivative, respectively. Thus, `D2y` is the toolbox equivalent of  $d^2y/dt^2$ . The dependent variables are those preceded by `D` and the default independent variable is `t`. Note that names of symbolic variables should not contain `D`. The independent variable can be changed from `t` to some other symbolic variable by including that variable as the last input argument.

Initial conditions can be specified by additional equations. If initial conditions are not specified, the solutions contain constants of integration, `C1`, `C2`, etc.

The output from `dsolve` parallels the output from `solve`. That is, you can call `dsolve` with the number of output variables equal to the number of dependent variables or place the output in a structure whose fields contain the solutions of the differential equations.

**Example 1**

The following call to `dsolve`

```
dsolve('Dy = t*y')
```

uses `y` as the dependent variable and `t` as the default independent variable.

The output of this command is

```
ans =  
C2*exp(t^2/2)
```

$y = C \cdot \exp(t^2/2)$  is a solution to the equation for any constant `C`.

To specify an initial condition, use

```
y = dsolve('Dy = t*y', 'y(0) = 2')
```

This produces

```
y =  
2*exp(t^2/2)
```

Notice that `y` is in the MATLAB workspace, but the independent variable `t` is not. Thus, the command `diff(y,t)` returns an error. To place `t` in the workspace, enter `syms t`.

**Example 2**

Nonlinear equations may have multiple solutions, even when initial conditions are given:

```
x = dsolve('(Dx + x)^2 = 1', 'x(0) = 0')
```

results in

```
x =  
1/exp(t) - 1  
1 - 1/exp(t)
```

**Example 3**

Here is a second-order differential equation with two initial conditions, and the default independent variable changed to  $x$ . The commands

```
y = dsolve('D2y = cos(2*x) - y', 'y(0) = 1',
'Dy(0) = 0', 'x');
simplify(y)
```

produce

```
ans =
1 - (8*(cos(x)/2 - 1/2)^2)/3
```

**Example 4**

The key issues in this example are the order of the equation and the initial conditions. To solve the ordinary differential equation

$$\frac{d^3u}{dx^3} = u$$

$$u(0) = 1, u'(0) = -1, u''(0) = \pi,$$

with  $x$  as the independent variable, type

```
u = dsolve('D3u = u', ...
'u(0) = 1', 'Du(0) = -1', 'D2u(0) = pi', 'x')
```

Use  $D3u$  to represent  $d^3u/dx^3$  and  $D2u(0)$  for  $u''(0)$ .

```
u =
(pi*exp(x))/3 - (cos((3^(1/2)*x)/2)*(pi/3 - 1))/exp(x/2) ...
- (3^(1/2)*sin((3^(1/2)*x)/2)*(pi + 1))/(3*exp(x/2))
```

**Further ODE Examples**

This table shows a few more examples of differential equations and their Symbolic Math Toolbox syntax. The final entry in the table is the Airy differential equation, whose solution is referred to as the Airy function.

Differential Equation	MATLAB Command
$\frac{dy}{dt} + 4y(t) = e^{-t}$ $y(0) = 1$	<code>y = dsolve('Dy+4*y = exp(-t)', 'y(0) = 1')</code>
$2x^2y'' + 3xy' - y = 0$ $(' = d/dx)$	<code>y = dsolve('2*x^2*D2y + 3*x*Dy - y = 0', 'x')</code>
$\frac{d^2y}{dx^2} = xy(x)$ $y(0) = 0, y(3) = \frac{1}{\pi} K_{1/3}(2\sqrt{3})$ <p>(The Airy equation)</p>	<code>y = dsolve('D2y = x*y', 'y(0) = 0', 'y(3) = bessellk(1/3, 2*sqrt(3))/pi', 'x')</code>

## Several Differential Equations

The function `dsolve` can handle several ordinary differential equations in several variables, with or without initial conditions. For example, here is a pair of linear, first-order equations.

$$S = \text{dsolve}('Df = 3*f + 4*g', 'Dg = -4*f + 3*g')$$

The toolbox returns the computed solutions in the structure `S`. You can determine the values of `f` and `g` by typing

$$f = S.f$$

$$g = S.g$$

$$f =$$

$$C2*\cos(4*t)*\exp(3*t) + C1*\sin(4*t)*\exp(3*t)$$

$$g =$$

$$C1*\cos(4*t)*\exp(3*t) - C2*\sin(4*t)*\exp(3*t)$$

If you prefer to recover `f` and `g` directly, as well as include initial conditions, type

$$[f, g] = \text{dsolve}('Df = 3*f + 4*g, Dg = -4*f + 3*g', \dots)$$



```
'f(0) = 0, g(0) = 1')
```

```
f =
sin(4*t)*exp(3*t)
```

```
g =
cos(4*t)*exp(3*t)
```

Now, suppose you are solving a system of differential equations in matrix form. For example, solve the system  $Y' = AY + B$ , where  $A$ ,  $B$ , and  $Y$  represent the following matrices:

```
syms t x y;
A = [1 2; -1 1];
B = [1; t];
Y = [x; y];
sys = A*Y + B
```

```
sys =
x + 2*y + 1
t - x + y
```

The `dsolve` function does not accept matrices. To be able to use this solver, extract the components of the matrix and convert them to strings:

```
eq1 = char(sys(1))
eq2 = char(sys(2))
```

```
eq1 =
x + 2*y + 1
```

```
eq2 =
t - x + y
```

Use the `strcat` function to concatenate the left and right sides of the equations. Use the `dsolve` function to solve the system:

```
[x, y] = dsolve(strcat('Dx = ',eq1), strcat('Dy = ',eq2))

x =
2^(1/2)*exp(t)*cos(2^(1/2)*t)*(C6 + (4*sin(2^(1/2)*t) + ...
2^(1/2)*cos(2^(1/2)*t) + 6*t*sin(2^(1/2)*t) + ...
```

```

6*2^(1/2)*t*cos(2^(1/2)*t)/(18*exp(t)) +...
2^(1/2)*exp(t)*sin(2^(1/2)*t)*(C5 - (4*cos(2^(1/2)*t) -...
2^(1/2)*sin(2^(1/2)*t) + 6*t*cos(2^(1/2)*t) -...
6*2^(1/2)*t*sin(2^(1/2)*t))/(18*exp(t))

```

```

y =
exp(t)*cos(2^(1/2)*t)*(C5 - (4*cos(2^(1/2)*t) -...
2^(1/2)*sin(2^(1/2)*t) + 6*t*cos(2^(1/2)*t) -...
6*2^(1/2)*t*sin(2^(1/2)*t))/(18*exp(t)) -...
exp(t)*sin(2^(1/2)*t)*(C6 + (4*sin(2^(1/2)*t) +...
2^(1/2)*cos(2^(1/2)*t) + 6*t*sin(2^(1/2)*t) +...
6*2^(1/2)*t*cos(2^(1/2)*t))/(18*exp(t))

```

## Integral Transforms and Z-Transforms

### In this section...

“Fourier and Inverse Fourier Transforms” on page 3-91

“Laplace and Inverse Laplace Transforms” on page 3-98

“Z-Transforms and Inverse Z-Transforms” on page 3-104

### Fourier and Inverse Fourier Transforms

The Fourier transform of a function  $f(x)$  is defined as

$$F[f](w) = \int_{-\infty}^{\infty} f(x)e^{-iwx} dx,$$

and the inverse Fourier transform (IFT) as

$$F^{-1}[f](x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(w)e^{iwx} dw.$$

This documentation refers to this formulation as the Fourier transform of  $f$  with respect to  $x$  as a function of  $w$ . Or, more concisely, the Fourier transform of  $f$  with respect to  $x$  at  $w$ . Mathematicians often use the notation  $F[f]$  to denote the Fourier transform of  $f$ . In this setting, the transform is taken with respect to the independent variable of  $f$  (if  $f = f(t)$ , then  $t$  is the independent variable;  $f = f(x)$  implies that  $x$  is the independent variable, etc.) at the default variable  $w$ . This documentation refers to  $F[f]$  as the Fourier transform of  $f$  at  $w$  and  $F^{-1}[f]$  is the IFT of  $f$  at  $x$ . See `fourier` and `ifourier` in the reference pages for tables that show the Symbolic Math Toolbox commands equivalent to various mathematical representations of the Fourier and inverse Fourier transforms.

For example, consider the Fourier transform of the Cauchy density function,  $(\pi(1 + x^2))^{-1}$ :

```
syms x
cauchy = 1/(pi*(1+x^2));
```

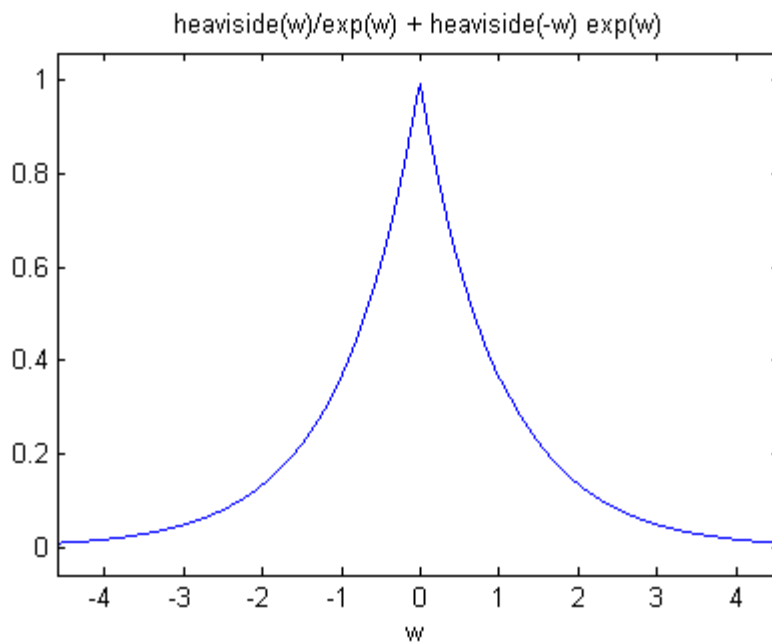
```
fcauchy = fourier(cauchy)

fcauchy =
((pi*heaviside(w))/exp(w) + pi*heaviside(-w)*exp(w))/pi

fcauchy = expand(fcauchy)

fcauchy =
heaviside(w)/exp(w) + heaviside(-w)*exp(w)

ezplot(fcauchy)
```



The Fourier transform is symmetric, since the original Cauchy density function is symmetric.

To recover the Cauchy density function from the Fourier transform, call `ifourier`:

```
finvfcauchy = ifourier(fcauchy)
```

```

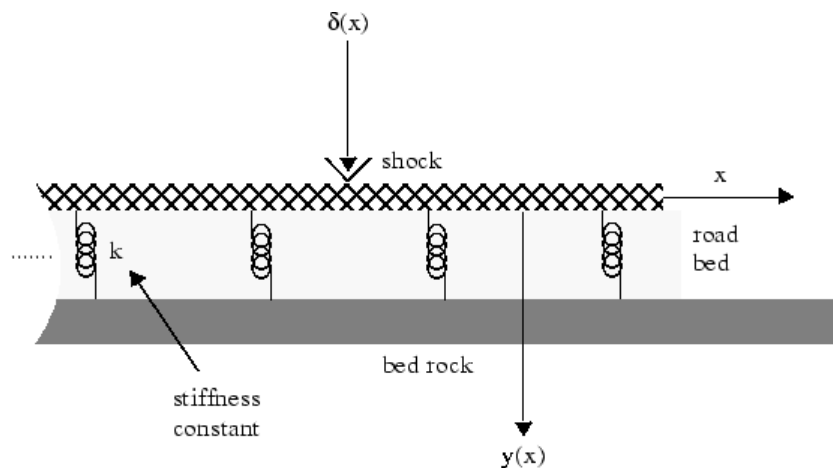
finvfcauchy =
-(1/(x*i - 1) - 1/(x*i + 1))/(2*pi)

simplify(finvfcauchy)

ans =
1/(pi*(x^2 + 1))

```

An application of the Fourier transform is the solution of ordinary and partial differential equations over the real line. Consider the deformation of an infinitely long beam resting on an elastic foundation with a shock applied to it at a point. A “real world” analogy to this phenomenon is a set of railroad tracks atop a road bed.



The shock could be induced by a pneumatic hammer blow.

The differential equation idealizing this physical setting is

$$\frac{d^4 y}{dx^4} + \frac{k}{EI} y = \frac{1}{EI} \delta(x), \quad -\infty < x < \infty.$$

Here,  $E$  represents elasticity of the beam (railroad track),  $I$  is the “beam constant,” and  $k$  is the spring (road bed) stiffness. The shock force on the right

side of the differential equation is modeled by the Dirac Delta function  $\delta(x)$ . The Dirac Delta function has the following important property:

$$\int_{-\infty}^{\infty} f(x-y)\delta(y)dy = f(x).$$

A definition of the Dirac Delta function is

$$\delta(x) = \lim_{n \rightarrow \infty} n\chi_{(-1/2n, 1/2n)}(x),$$

where

$$\chi_{(-1/2n, 1/2n)}(x) = \begin{cases} 1 & \text{for } -\frac{1}{2n} < x < \frac{1}{2n} \\ 0 & \text{otherwise.} \end{cases}$$

You can evaluate the Dirac Delta function at a point (say)  $x = 3$ , using the commands

```
syms x
del = sym('dirac(x)');
vpa(subs(del, x, 3))
```

which return

```
ans =
0.0
```

Returning to the differential equation, let  $Y(w) = F[y(x)](w)$  and  $\Delta(w) = F[\delta(x)](w)$ . Indeed, try the command `fourier(del, x, w)`. The Fourier transform turns differentiation into exponentiation, and, in particular,

$$F\left[\frac{d^4 y}{dx^4}\right](w) = w^4 Y(w).$$

To see a demonstration of this property, try this

```
syms w x
fourier(diff(sym('y(x)'), x, 4), x, w)
```

which returns

```
ans =
w^4*transform::fourier(y(x), x, -w)
```

Note that you can call the `fourier` command with one, two, or three inputs (see the reference pages for `fourier`). With a single input argument, `fourier(f)` returns a function of the default variable  $w$ . If the input argument is a function of  $w$ , `fourier(f)` returns a function of  $t$ . All inputs to `fourier` must be symbolic objects.

Applying the Fourier transform to the differential equation above yields the algebraic equation

$$\left(w^4 + \frac{k}{EI}\right)Y(w) = \Delta(w),$$

or

$$Y(w) = \Delta(w)G(w),$$

where

$$G(w) = \frac{1}{w^4 + \frac{k}{EI}} = F[g(x)](w)$$

for some function  $g(x)$ . That is,  $g$  is the inverse Fourier transform of  $G$ :

$$g(x) = F^{-1}[G(w)](x)$$

The Symbolic Math Toolbox counterpart to the IFT is `ifourier`. This behavior of `ifourier` parallels `fourier` with one, two, or three input arguments (see the reference pages for `ifourier`).

Continuing with the solution of the differential equation, observe that the ratio

$$\frac{K}{EI}$$

is a relatively “large” number since the road bed has a high stiffness constant  $k$  and a railroad track has a low elasticity  $E$  and beam constant  $I$ . Make the simplifying assumption that

$$\frac{K}{EI} = 1024.$$

This is done to ease the computation of  $F^{-1}[G(w)](x)$ . Now type

```
G = 1/(w^4 + 1024);
g = ifourier(G, w, x);
g = simplify(g);
pretty(g)
```

and see

```

1/2      / pi      \
2      sin| -- + 4 x | heaviside(x)
          \ 4      /
-----
512 exp(4 x)

1/2      / pi \
2      heaviside(-x) sin| 4 x - -- | exp(4 x)
          \      4 /
-----
512
```

Notice that  $g$  contains the Heaviside distribution

$$H(x) = \begin{cases} 1 & \text{for } x > 0 \\ 0 & \text{for } x < 0 \\ 1/2 & \text{for } x = 0. \end{cases}$$



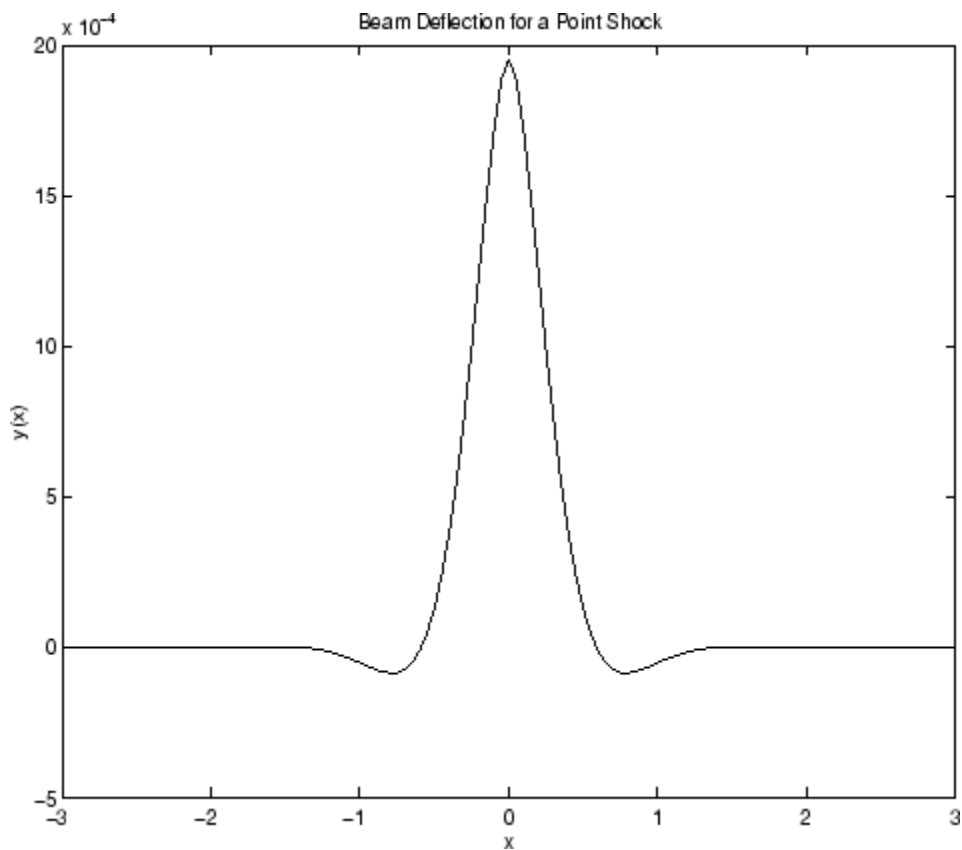
Since  $Y$  is the product of Fourier transforms,  $y$  is the convolution of the transformed functions. That is,  $F[y] = Y(w) = \Delta(w) G(w) = F[\delta] F[g]$  implies

$$y(x) = (\delta * g)(x) = \int_{-\infty}^{\infty} g(x-y)\delta(y)dy = g(x).$$

by the special property of the Dirac Delta function. To plot this function, substitute the domain of  $x$  into  $y(x)$ , using the `subs` command.

```
XX = -3:0.05:3;  
YY = double(subs(g, x, XX));  
plot(XX, YY)  
title('Beam Deflection for a Point Shock')  
xlabel('x'); ylabel('y(x)');
```

The resulting graph



shows that the impact of a blow on a beam is highly localized; the greatest deflection occurs at the point of impact and falls off sharply from there.

## Laplace and Inverse Laplace Transforms

The Laplace transform of a function  $f(t)$  is defined as

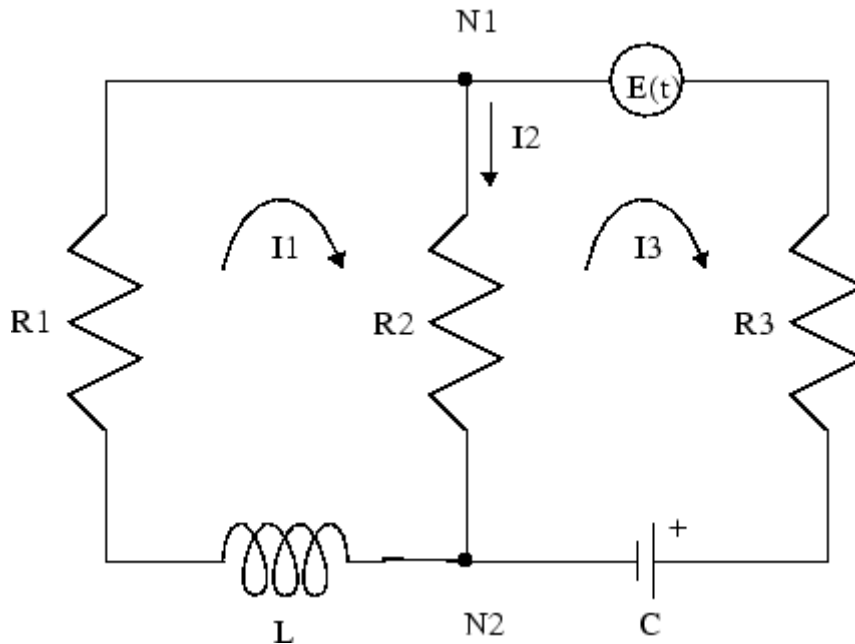
$$L[f](s) = \int_0^{\infty} f(t)e^{-ts} dt,$$

while the inverse Laplace transform (ILT) of  $f(s)$  is

$$L^{-1}[f](t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} f(s)e^{st} ds,$$

where  $c$  is a real number selected so that all singularities of  $f(s)$  are to the left of the line  $s = c$ . The notation  $L[f]$  denotes the Laplace transform of  $f$  at  $s$ . Similarly,  $L^{-1}[f]$  is the ILT of  $f$  at  $t$ .

The Laplace transform has many applications including the solution of ordinary differential equations/initial value problems. Consider the resistance-inductor-capacitor (RLC) circuit below.



Let  $R_j$  and  $I_j$ ,  $j = 1, 2, 3$  be resistances (measured in ohms) and currents (amperes), respectively;  $L$  be inductance (henrys), and  $C$  be capacitance (farads);  $E(t)$  be the electromotive force, and  $Q(t)$  be the charge.

By applying Kirchhoff's voltage and current laws, Ohm's Law, and Faraday's Law, you can arrive at the following system of simultaneous ordinary differential equations.

$$\frac{dI_1}{dt} + \frac{R_2}{L} \frac{dQ}{dt} = \frac{R_1 - R_2}{L} I_1, I_1(0) = I_0.$$

$$\frac{dQ}{dt} = \frac{1}{R_3 + R_2} \left( E(t) - \frac{1}{C} Q(t) \right) + \frac{R_2}{R_3 + R_2} I_1, Q(0) = Q_0.$$

Solve this system of differential equations using `laplace`. First treat the  $R_j$ ,  $L$ , and  $C$  as (unknown) real constants and then supply values later on in the computation.

```
syms R1 R2 R3 L C real
dI1 = sym('diff(I1(t),t)'); dQ = sym('diff(Q(t),t)');
I1 = sym('I1(t)'); Q = sym('Q(t)');
syms t s
E = sin(t); % Voltage
eq1 = dI1 + R2*dQ/L - (R2 - R1)*I1/L;
eq2 = dQ - (E - Q/C)/(R2 + R3) - R2*I1/(R2 + R3);
```

At this point, you have constructed the equations in the MATLAB workspace. An approach to solving the differential equations is to apply the Laplace transform, which you will apply to `eq1` and `eq2`.

```
L1 = laplace(eq1,t,s)
L2 = laplace(eq2,t,s)
```

returns

```
L1 =
s*laplace(I1(t), t, s) - I1(0)
+ ((R1 - R2)*laplace(I1(t), t, s))/L
- (R2*(Q(0) - s*laplace(Q(t), t, s)))/L

L2 =
s*laplace(Q(t), t, s) - Q(0)
- (R2*laplace(I1(t), t, s))/(R2 + R3) - (C/(s^2 + 1)
- laplace(Q(t), t, s))/(C*(R2 + R3))
```

Now you need to solve the system of equations  $L1 = 0$ ,  $L2 = 0$  for `laplace(I1(t),t,s)` and `laplace(Q(t),t,s)`, the Laplace transforms of  $I_1$  and  $Q$ , respectively. To do this, make a series of substitutions. For the

purposes of this example, use the quantities  $R1 = 4 \Omega$  (ohms),  $R2 = 2 \Omega$ ,  $R3 = 3 \Omega$ ,  $C = 1/4$  farads,  $L = 1.6$  H (henrys),  $I1(0) = 15$  A (amperes), and  $Q(0) = 2$  A/sec. Substituting these values in L1

```
syms LI1 LQ
NI1 = subs(L1,{R1,R2,R3,L,C,'I1(0)','Q(0)'}, ...
        {4,2,3,1.6,1/4,15,2})
```

returns

```
NI1 =
s*laplace(I1(t), t, s) + (5*s*laplace(Q(t), t, s))/4
+ (5*laplace(I1(t), t, s))/4 - 35/2
```

The substitution

```
NQ =
subs(L2,{R1,R2,R3,L,C,'I1(0)','Q(0)'},{4,2,3,1.6,1/4,15,2})
```

returns

```
NQ =
s*laplace(Q(t), t, s) - 1/(5*(s^2 + 1))
+ (4*laplace(Q(t), t, s))/5 - (2*laplace(I1(t), t, s))/5 - 2
```

To solve for  $\text{laplace}(I1(t), t, s)$  and  $\text{laplace}(Q(t), t, s)$ , make a final pair of substitutions. First, replace the strings ' $\text{laplace}(I1(t), t, s)$ ' and ' $\text{laplace}(Q(t), t, s)$ ' by the syms LI1 and LQ, using

```
NI1 = ...
subs(NI1,{'laplace(I1(t),t,s)','laplace(Q(t),t,s)'},{LI1,LQ})
```

to obtain

```
NI1 =
(5*LI1)/4 + LI1*s + (5*LQ*s)/4 - 35/2
```

Collecting terms

```
NI1 = collect(NI1,LI1)
```

gives

$$\begin{aligned} \text{NI1} = \\ (s + 5/4)*\text{LI1} + (5*\text{LQ}*s)/4 - 35/2 \end{aligned}$$

A similar string substitution

$$\begin{aligned} \text{NQ} = \dots \\ \text{subs}(\text{NQ}, \{ 'laplace(\text{I1}(t), t, s) ', 'laplace(\text{Q}(t), t, s) ' \}, \{ \text{LI1}, \text{LQ} \}) \end{aligned}$$

yields

$$\begin{aligned} \text{NQ} = \\ (4*\text{LQ})/5 - (2*\text{LI1})/5 + \text{LQ}*s - 1/(5*(s^2 + 1)) - 2 \end{aligned}$$

which, after collecting terms,

$$\text{NQ} = \text{collect}(\text{NQ}, \text{LQ})$$

gives

$$\begin{aligned} \text{NQ} = \\ (s + 4/5)*\text{LQ} - (2*\text{LI1})/5 - 1/(5*(s^2 + 1)) - 2 \end{aligned}$$

Now, solving for LI1 and LQ

$$[\text{LI1}, \text{LQ}] = \text{solve}(\text{NI1}, \text{NQ}, \text{LI1}, \text{LQ})$$

you obtain

$$\begin{aligned} \text{LI1} = \\ (5*(60*s^3 + 56*s^2 + 59*s + 56))/((s^2 + 1)*(20*s^2 + 51*s + 20)) \end{aligned}$$

$$\begin{aligned} \text{LQ} = \\ (40*s^3 + 190*s^2 + 44*s + 195)/((s^2 + 1)*(20*s^2 + 51*s + 20)) \end{aligned}$$

To recover I1 and Q, compute the inverse Laplace transform of LI1 and LQ.

Inverting LI1

$$\text{I1} = \text{ilaplace}(\text{LI1}, s, t)$$

produces

$$\begin{aligned} \text{I1} = \\ (15*(\cosh((1001^{1/2})*t))/40) \end{aligned}$$

$$\begin{aligned}
 & - (293*1001^{(1/2)}*\sinh((1001^{(1/2)}*t)/40))/21879)/\exp((51*t)/40) \\
 & - (5*\sin(t))/51
 \end{aligned}$$

Inverting LQ

$$Q = \text{ilaplace}(LQ, s, t)$$

yields

$$\begin{aligned}
 Q = & \\
 & (4*\sin(t))/51 - (5*\cos(t))/51 + (107*(\cosh((1001^{(1/2)}*t)/40) \\
 & + (2039*1001^{(1/2)}*\sinh((1001^{(1/2)}*t)/40))/15301))/(51*\exp((51*t)/40))
 \end{aligned}$$

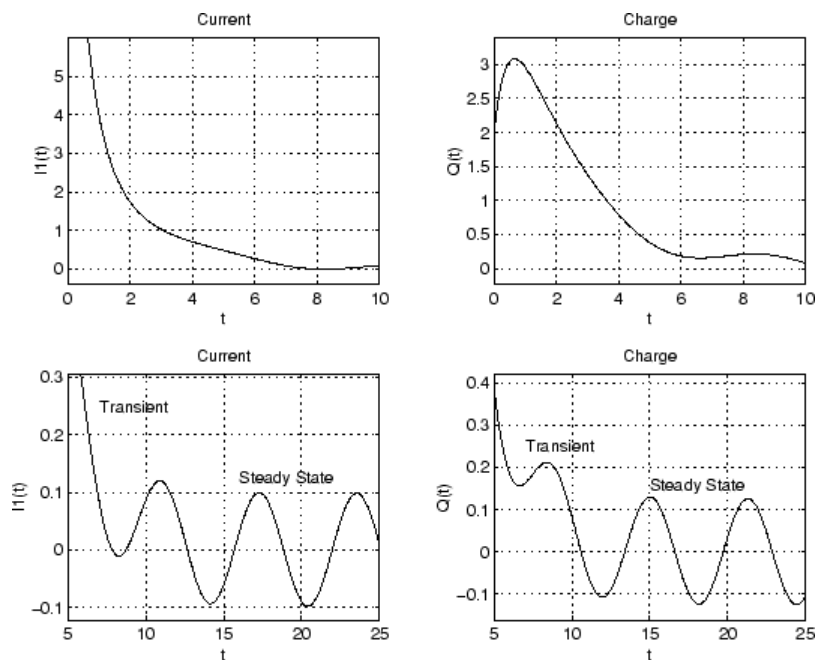
Now plot the current  $I_1(t)$  and charge  $Q(t)$  in two different time domains,  $0 \leq t \leq 10$  and  $5 \leq t \leq 25$ . The statements

```

subplot(2,2,1); ezplot(I1,[0,10]);
title('Current'); ylabel('I1(t)'); grid
subplot(2,2,2); ezplot(Q,[0,10]);
title('Charge'); ylabel('Q(t)'); grid
subplot(2,2,3); ezplot(I1,[5,25]);
title('Current'); ylabel('I1(t)'); grid
text(7,0.25,'Transient'); text(16,0.125,'Steady State');
subplot(2,2,4); ezplot(Q,[5,25]);
title('Charge'); ylabel('Q(t)'); grid
text(7,0.25,'Transient'); text(15,0.16,'Steady State');

```

generate the desired plots



Note that the circuit’s behavior, which appears to be exponential decay in the short term, turns out to be oscillatory in the long term. The apparent discrepancy arises because the circuit’s behavior actually has two components: an exponential part that decays rapidly (the “transient” component) and an oscillatory part that persists (the “steady-state” component).

## Z-Transforms and Inverse Z-Transforms

The (one-sided)  $z$ -transform of a function  $f(n)$  is defined as

$$Z[f](z) = \sum_{n=0}^{\infty} f(n)z^{-n}.$$

The notation  $Z[f]$  refers to the  $z$ -transform of  $f$  at  $z$ . Let  $R$  be a positive number so that the function  $g(z)$  is analytic on and outside the circle  $|z| = R$ . Then the inverse  $z$ -transform (IZT) of  $g$  at  $n$  is defined as



$$Z^{-1}[g](n) = \frac{1}{2\pi i} \oint_{|z|=R} g(z)z^{n-1} dz, \quad n = 1, 2, \dots$$

The notation  $Z^{-1}[f]$  means the IZT of  $f$  at  $n$ . The Symbolic Math Toolbox commands `ztrans` and `iztrans` apply the  $z$ -transform and IZT to symbolic expressions, respectively. See `ztrans` and `iztrans` for tables showing various mathematical representations of the  $z$ -transform and inverse  $z$ -transform and their Symbolic Math Toolbox counterparts.

The  $z$ -transform is often used to solve difference equations. In particular, consider the famous “Rabbit Problem.” That is, suppose that rabbits reproduce only on odd birthdays (1, 3, 5, 7, ...). If  $p(n)$  is the rabbit population at year  $n$ , then  $p$  obeys the difference equation

$$p(n+2) = p(n+1) + p(n), \quad p(0) = 1, \quad p(1) = 2.$$

You can use `ztrans` to find the population each year  $p(n)$ . First, apply `ztrans` to the equations

```
pn = sym('p(n)');
pn1 = sym('p(n+1)');
pn2 = sym('p(n+2)');
syms n z
eq = pn2 - pn1 - pn;
Zeq = ztrans(eq, n, z)
```

to obtain

```
Zeq =
z*p(0) - z*ztrans(p(n), n, z) - z*p(1) + z^2*ztrans(p(n), n, z)
- z^2*p(0) - ztrans(p(n), n, z)
```

Next, replace `'ztrans(p(n), n, z)'` with `Pz` and insert the initial conditions for  $p(0)$  and  $p(1)$ .

```
syms Pz
Zeq = subs(Zeq, {'ztrans(p(n), n, z)', 'p(0)',
'p(1)'}, {Pz, 1, 2})
```

to obtain

```
Zeq =
Pz*z^2 - z - Pz*z - Pz - z^2
```

Collecting terms

```
eq = collect(Zeq, Pz)
```

yields

```
eq =
(z^2 - z - 1)*Pz - z^2 - z
```

Now solve for Pz

```
P = solve(eq, Pz)
```

to obtain

```
P =
-(z^2 + z)/(- z^2 + z + 1)
```

To recover  $p(n)$ , take the inverse  $z$ -transform of  $P$ .

```
p = iztrans(P, z, n);
p = simple(p)
```

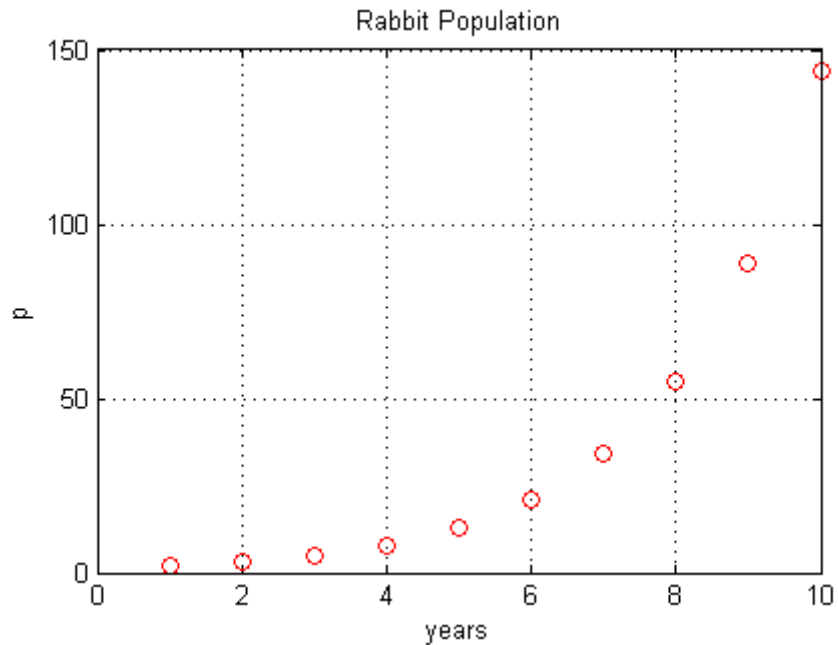
The result is a bit complicated, but explicit:

```
p =
(3*5^(1/2)*(1/2 - 5^(1/2)/2)^(n - 1))/5 -
(3*5^(1/2)*(5^(1/2)/2 + 1/2)^(n - 1))/5 +
(4*(-1)^n*cos(n*(pi/2 + asinh(1/2)*i)))/i^n
```

Finally, plot  $p$ :

```
m = 1:10;
y = double(subs(p,n,m));
plot(m,y,'r0')
title('Rabbit Population');
xlabel('years'); ylabel('p');
grid on
```

to show the growth in rabbit population over time.



## References

- [1] Andrews, L.C., Shivamoggi, B.K., *Integral Transforms for Engineers and Applied Mathematicians*, Macmillan Publishing Company, New York, 1986
- [2] Crandall, R.E., *Projects in Scientific Computation*, Springer-Verlag Publishers, New York, 1994
- [3] Strang, G., *Introduction to Applied Mathematics*, Wellesley-Cambridge Press, Wellesley, MA, 1986

## Special Functions of Applied Mathematics

### In this section...

“Numerical Evaluation of Special Functions Using `mfun`” on page 3-108

“Syntax and Definitions of `mfun` Special Functions” on page 3-109

“Diffraction Example” on page 3-114

### Numerical Evaluation of Special Functions Using `mfun`

Over 50 of the special functions of classical applied mathematics are available in the toolbox. These functions are accessed with the `mfun` function, which numerically evaluates special functions for the specified parameters. This allows you to evaluate functions that are not available in standard MATLAB software, such as the Fresnel cosine integral. In addition, you can evaluate several MATLAB special functions in the complex plane, such as the error function `erf`.

For example, suppose you want to evaluate the hyperbolic cosine integral at the points  $2 + i$ ,  $0$ , and  $4.5$ . Look in the tables in “Syntax and Definitions of `mfun` Special Functions” on page 3-109 to find the available functions and their syntax. You can also enter the command

```
mfunlist
```

to see the list of functions available for `mfun`. This list provides a brief mathematical description of each function, its `mfun` name, and the parameters it needs. From the tables or list, you can see that the hyperbolic cosine integral is called `Chi`, and it takes one complex argument.

Type

```
z = [2 + i 0 4.5];
w = mfun('Chi', z)
```

which returns

```
w =
    2.0303 + 1.7227i    NaN    13.9658
```

`mfun` returns the special value NaN where the function has a singularity. The hyperbolic cosine integral has a singularity at  $z = 0$ .

---

**Note** `mfun` functions perform numerical, not symbolic, calculations. The input parameters should be scalars, vectors, or matrices of type double, or complex doubles, not symbolic variables.

---

## Syntax and Definitions of `mfun` Special Functions

The following conventions are used in the next table, unless otherwise indicated in the **Arguments** column.

$x, y$	real argument
$z, z1, z2$	complex argument
$m, n$	integer argument

### `mfun` Special Functions

Function Name	Definition	<code>mfun</code> Name	Arguments
Bernoulli numbers and polynomials	Generating functions: $\frac{e^{xt}}{e^t - 1} = \sum_{n=0}^{\infty} B_n(x) \cdot \frac{t^{n-1}}{n!}$	<code>bernoulli(n)</code> <code>bernoulli(n,t)</code>	$n \geq 0$ $0 <  t  < 2\pi$
Bessel functions	<code>BesselI</code> , <code>BesselJ</code> —Bessel functions of the first kind. <code>BesselK</code> , <code>BesselY</code> —Bessel functions of the second kind.	<code>BesselJ(v,x)</code> <code>BesselY(v,x)</code> <code>BesselI(v,x)</code> <code>BesselK(v,x)</code>	$v$ is real.
Beta function	$B(x,y) = \frac{\Gamma(x) \cdot \Gamma(y)}{\Gamma(x+y)}$	<code>Beta(x,y)</code>	

**mfun Special Functions (Continued)**

Function Name	Definition	mfun Name	Arguments
Binomial coefficients	$\binom{m}{n} = \frac{m!}{n!(m-n)!}$ $= \frac{\Gamma(m+1)}{\Gamma(n+1)\Gamma(m-n+1)}$	binomial(m,n)	
Complete elliptic integrals	<p>Legendre's complete elliptic integrals of the first, second, and third kind. This definition uses modulus <math>k</math>. The numerical <code>ellipke</code> function and the MuPAD functions for computing elliptic integrals use the parameter <math>m = k^2 = \sin^2 \alpha</math>.</p>	EllipticK(k) EllipticE(k) EllipticPi(a,k)	<p><math>a</math> is real,  <math>-\infty &lt; a &lt; \infty</math>.</p> <p><math>k</math> is real,  <math>0 &lt; k &lt; 1</math>.</p>
Complete elliptic integrals with complementary modulus	<p>Associated complete elliptic integrals of the first, second, and third kind using complementary modulus. This definition uses modulus <math>k</math>. The numerical <code>ellipke</code> function and the MuPAD functions for computing elliptic integrals use the parameter <math>m = k^2 = \sin^2 \alpha</math>.</p>	EllipticCK(k) EllipticCE(k) EllipticCPi(a,k)	<p><math>a</math> is real,  <math>-\infty &lt; a &lt; \infty</math>.</p> <p><math>k</math> is real,  <math>0 &lt; k &lt; 1</math>.</p>
Complementary error function and its iterated integrals	$\operatorname{erfc}(z) = \frac{2}{\sqrt{\pi}} \cdot \int_z^{\infty} e^{-t^2} dt = 1 - \operatorname{erf}(z)$ $\operatorname{erfc}(-1, z) = \frac{2}{\sqrt{\pi}} \cdot e^{-z^2}$ $\operatorname{erfc}(n, z) = \int_z^{\infty} \operatorname{erfc}(n-1, t) dt$	erfc(z) erfc(n, z)	$n > 0$

**mfun Special Functions (Continued)**

Function Name	Definition	mfun Name	Arguments
Dawson's integral	$F(x) = e^{-x^2} \cdot \int_0^x e^{t^2} dt$	dawson(x)	
Digamma function	$\Psi(x) = \frac{d}{dx} \ln(\Gamma(x)) = \frac{\Gamma'(x)}{\Gamma(x)}$	Psi(x)	
Dilogarithm integral	$f(x) = \int_1^x \frac{\ln(t)}{1-t} dt$	dilog(x)	$x > 1$
Error function	$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt$	erf(z)	
Euler numbers and polynomials	Generating function for Euler numbers: $\frac{1}{\cosh(t)} = \sum_{n=0}^{\infty} E_n \frac{t^n}{n!}$	euler(n) euler(n, z)	$n \geq 0$ $ t  < \frac{\pi}{2}$
Exponential integrals	$Ei(n, z) = \int_1^{\infty} \frac{e^{-zt}}{t^n} dt$ $Ei(x) = PV \left( - \int_{-\infty}^x \frac{e^t}{t} dt \right)$	Ei(n, z) Ei(x)	$n \geq 0$ $\operatorname{Real}(z) > 0$
Fresnel sine and cosine integrals	$C(x) = \int_0^x \cos\left(\frac{\pi}{2} t^2\right) dt$ $S(x) = \int_0^x \sin\left(\frac{\pi}{2} t^2\right) dt$	FresnelC(x) FresnelS(x)	

**mfun Special Functions (Continued)**

Function Name	Definition	mfun Name	Arguments
Gamma function	$\Gamma(z) = \int_0^{\infty} t^{z-1} e^{-t} dt$	GAMMA(z)	
Harmonic function	$h(n) = \sum_{k=1}^n \frac{1}{k} = \Psi(n+1) + \gamma$	harmonic(n)	$n > 0$
Hyperbolic sine and cosine integrals	$Shi(z) = \int_0^z \frac{\sinh(t)}{t} dt$ $Chi(z) = \gamma + \ln(z) + \int_0^z \frac{\cosh(t) - 1}{t} dt$	Shi(z) Chi(z)	
(Generalized) hypergeometric function	$F(n, d, z) = \sum_{k=0}^{\infty} \frac{\prod_{i=1}^j \frac{\Gamma(n_i + k)}{\Gamma(n_i)} \cdot z^k}{\prod_{i=1}^m \frac{\Gamma(d_i + k)}{\Gamma(d_i)} \cdot k!}$ <p>where <math>j</math> and <math>m</math> are the number of terms in <math>n</math> and <math>d</math>, respectively.</p>	hypergeom(n,d,x) where $n = [n1, n2, \dots]$ $d = [d1, d2, \dots]$	$n1, n2, \dots$ are real. $d1, d2, \dots$ are real and nonnegative.
Incomplete elliptic integrals	Legendre's incomplete elliptic integrals of the first, second, and third kind. This definition uses modulus $k$ . The numerical <code>ellipke</code> function and the MuPAD functions for computing elliptic integrals use the parameter $m = k^2 = \sin^2 \alpha$ .	EllipticF(x,k) EllipticE(x,k) EllipticPi(x,a,k)	$0 < x \leq \infty$ . $a$ is real, $-\infty < a < \infty$ . $k$ is real, $0 < k < 1$ .
Incomplete gamma function	$\Gamma(a, z) = \int_z^{\infty} e^{-t} \cdot t^{a-1} dt$	GAMMA(z1, z2) $z1 = a$ $z2 = z$	



**mfun Special Functions (Continued)**

Function Name	Definition	mfun Name	Arguments
Logarithm of the gamma function	$\ln\text{GAMMA}(z) = \ln(\Gamma(z))$	$\ln\text{GAMMA}(z)$	
Logarithmic integral	$Li(x) = PV \left\{ \int_0^x \frac{dt}{\ln t} \right\} = Ei(\ln x)$	$Li(x)$	$x > 1$
Polygamma function	$\Psi^{(n)}(z) = \frac{d^n}{dz} \Psi(z)$ where $\Psi(z)$ is the Digamma function.	$\Psi(n, z)$	$n \geq 0$
Shifted sine integral	$Ssi(z) = Si(z) - \frac{\pi}{2}$	$Ssi(z)$	

The following orthogonal polynomials are available using mfun. In all cases,  $n$  is a nonnegative integer and  $x$  is real.

**Orthogonal Polynomials**

Polynomial	mfun Name	Arguments
Chebyshev of the first and second kind	$T(n, x)$ $U(n, x)$	
Gegenbauer	$G(n, a, x)$	$a$ is a nonrational algebraic expression or a rational number greater than $-1/2$ .
Hermite	$H(n, x)$	
Jacobi	$P(n, a, b, x)$	$a, b$ are nonrational algebraic expressions or rational numbers greater than $-1$ .
Laguerre	$L(n, x)$	

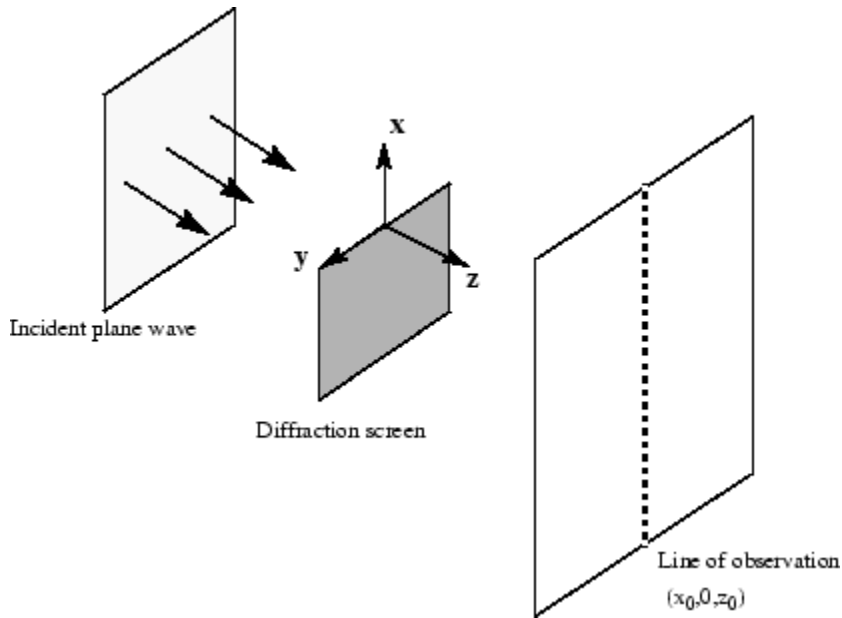
**Orthogonal Polynomials (Continued)**

Polynomial	mfun Name	Arguments
Generalized Laguerre	$L(n, a, x)$	$a$ is a nonrational algebraic expression or a rational number greater than $-1$ .
Legendre	$P(n, x)$	

**Diffraction Example**

This example is from diffraction theory in classical electrodynamics. (J.D. Jackson, *Classical Electrodynamics*, John Wiley & Sons, 1962).

Suppose you have a plane wave of intensity  $I_0$  and wave number  $k$ . Assume that the plane wave is parallel to the  $xy$ -plane and travels along the  $z$ -axis as shown below. This plane wave is called the *incident wave*. A perfectly conducting flat diffraction screen occupies half of the  $xy$ -plane, that is  $x < 0$ . The plane wave strikes the diffraction screen, and you observe the diffracted wave from the line whose coordinates are  $(x, 0, z_0)$ , where  $z_0 > 0$ .



The intensity of the diffracted wave is given by

$$I = \frac{I_0}{2} \left[ \left( C(\zeta) + \frac{1}{2} \right)^2 + \left( S(\zeta) + \frac{1}{2} \right)^2 \right],$$

where

$$\zeta = \sqrt{\frac{k}{2z_0}} \cdot x,$$

and  $C(\zeta)$  and  $S(\zeta)$  are the Fresnel cosine and sine integrals:

$$C(\zeta) = \int_0^\zeta \cos\left(\frac{\pi}{2} t^2\right) dt$$

$$S(\zeta) = \int_0^\zeta \sin\left(\frac{\pi}{2} t^2\right) dt.$$

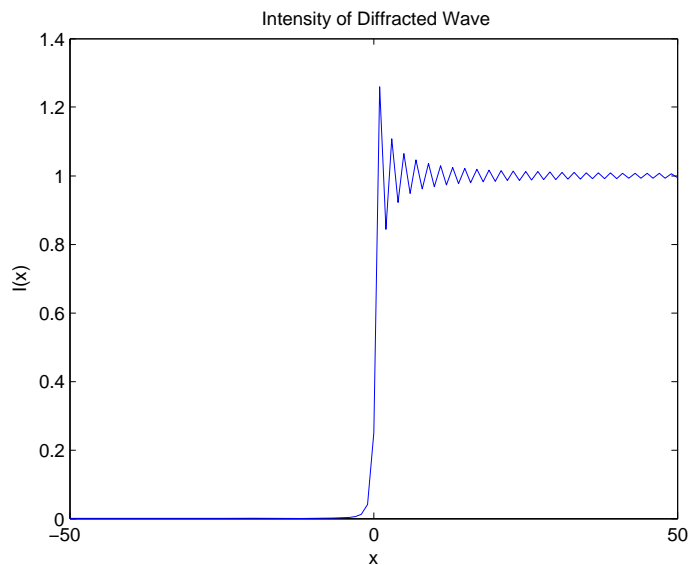
How does the intensity of the diffracted wave behave along the line of observation? Since  $k$  and  $z_0$  are constants independent of  $x$ , you set

$$\sqrt{\frac{k}{2z_0}} = 1,$$

and assume an initial intensity of  $I_0 = 1$  for simplicity.

The following code generates a plot of intensity as a function of  $x$ :

```
x = -50:50;
C = mfun('FresnelC',x);
S = mfun('FresnelS',x);
I0 = 1;
T = (C+1/2).^2 + (S+1/2).^2;
I = (I0/2)*T;
plot(x,I);
xlabel('x');
ylabel('I(x)');
title('Intensity of Diffracted Wave');
```



You see from the graph that the diffraction effect is most prominent near the edge of the diffraction screen ( $x = 0$ ), as you expect.

Note that values of  $x$  that are large and positive correspond to observation points far away from the screen. Here, you would expect the screen to have no effect on the incident wave. That is, the intensity of the diffracted wave should be the same as that of the incident wave. Similarly,  $x$  values that are large and negative correspond to observation points under the screen that are far away from the screen edge. Here, you would expect the diffracted wave to have zero intensity. These results can be verified by setting

$$x = [\text{Inf} \text{ -Inf}]$$

in the code to calculate  $I$ .

## Using Graphics

In this section...
“Creating Plots” on page 3-118
“Exploring Function Plots” on page 3-129
“Editing Graphs” on page 3-131
“Saving Graphs” on page 3-132

### Creating Plots

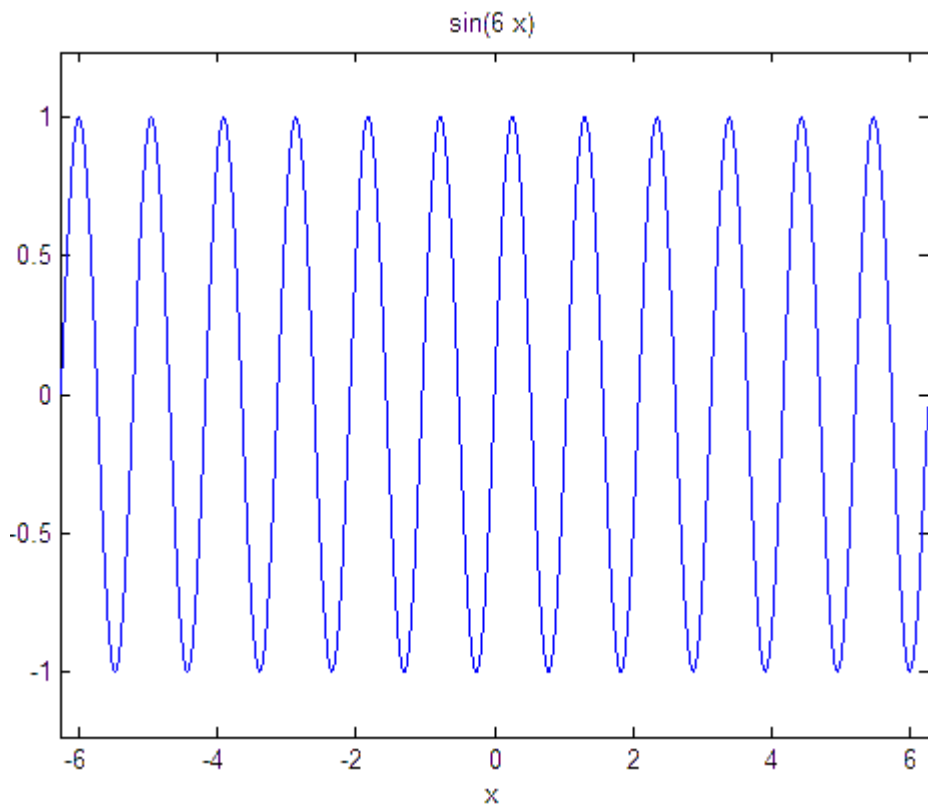
#### Using Symbolic Plotting Functions

MATLAB provides a wide variety of techniques for plotting numerical data. Graphical capabilities of MATLAB include plotting tools, standard plotting functions, graphic manipulation and data exploration tools, and tools for printing and exporting graphics to standard formats. The Symbolic Math Toolbox features expand the graphic capabilities of MATLAB and enable you to plot symbolic functions. The toolbox provides the following plotting functions:

- `ezplot` that creates 2-D plots of explicit and implicit symbolic functions in Cartesian coordinates.
- `ezplot3` that creates 3-D parametric function plots. The `animate` option of `ezplot3` lets you create animated function plots.
- `ezpolar` that creates symbolic function plots in polar coordinates.
- `ezsurf` that creates surface plots of symbolic functions. The `ezsurf` plotting function creates combined surface and contour function plots.
- `ezcontour` that creates contour plots of symbolic functions. The `ezcontourf` function creates filled contour plots.
- `ezmesh` that creates mesh plots of symbolic functions. The `ezmeshc` function creates combined mesh and contour function plots.

For example, plot the symbolic function  $\sin(6x)$  in Cartesian coordinates. By default, `ezplot` uses the range  $-2\pi < x < 2\pi$  :

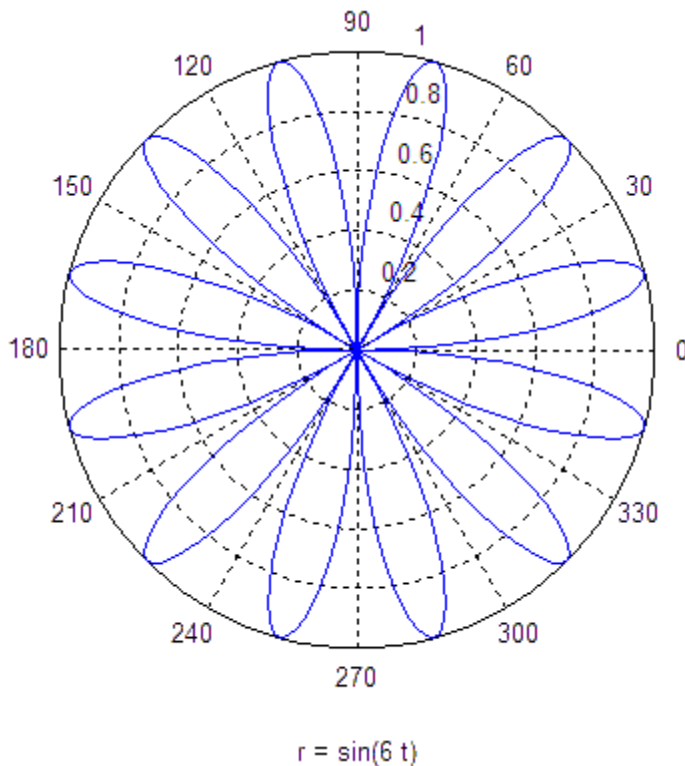
```
syms x;  
ezplot(sin(6*x))
```



When plotting a symbolic function, `ezplot` uses the default 60-by-60 grid (mesh setting). The plotting function does not adapt the mesh setting around steep parts of a function plot or around singularities. (These parts are typically less smooth than the rest of a function plot.) Also, `ezplot` does not allow you to change the mesh setting.

To plot a symbolic function in polar coordinates  $r$  (radius) and  $\theta$  (polar angle), use the `ezpolar` plotting function. By default, `ezpolar` plots a symbolic function over the domain  $0 < \theta < 2\pi$ . For example, plot the function  $\sin(6t)$  in polar coordinates:

```
syms t;
ezpolar(sin(6*t))
```



### Using MATLAB Plotting Functions

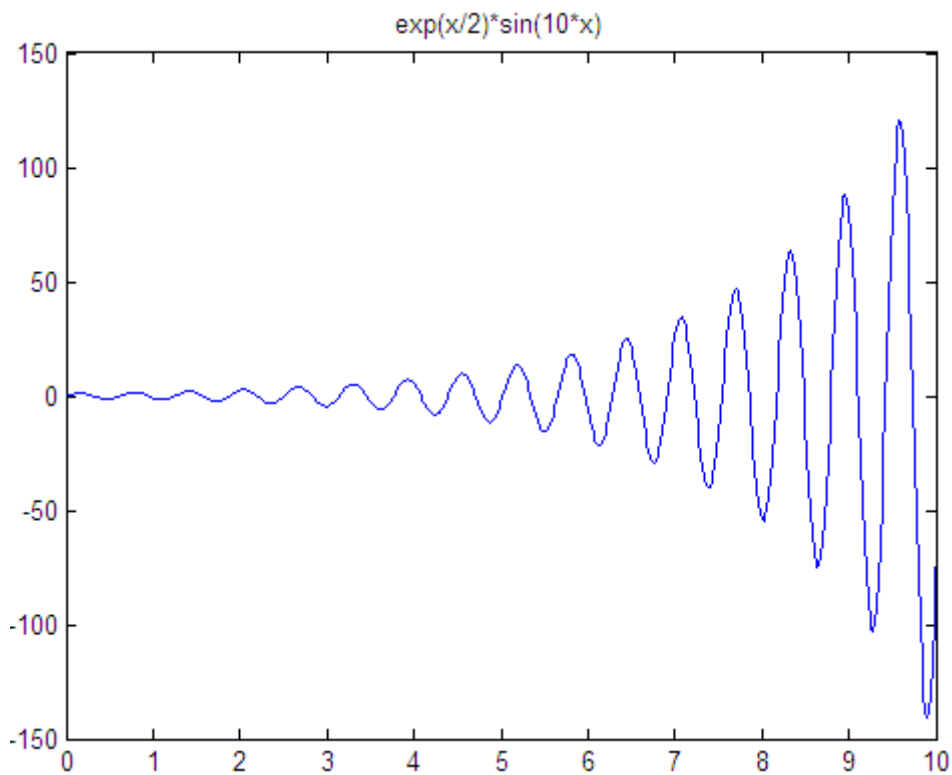
When plotting a symbolic expression, you also can use the plotting functions provided by MATLAB. For example, plot the symbolic expression  $e^{x/2} \sin(10x)$ . First, use `matlabFunction` to convert the symbolic expression to a MATLAB function. The result is a function handle `h` that points to the resulting MATLAB function:

```
syms x;
h = matlabFunction(exp(x/2)*sin(10*x))
```



Now, plot the resulting MATLAB function by using one of the standard plotting functions that accept function handles as arguments. For example, use the `fplot` function:

```
fplot(h, [0 10]);  
hold on;  
title('exp(x/2)*sin(10*x)');  
hold off
```



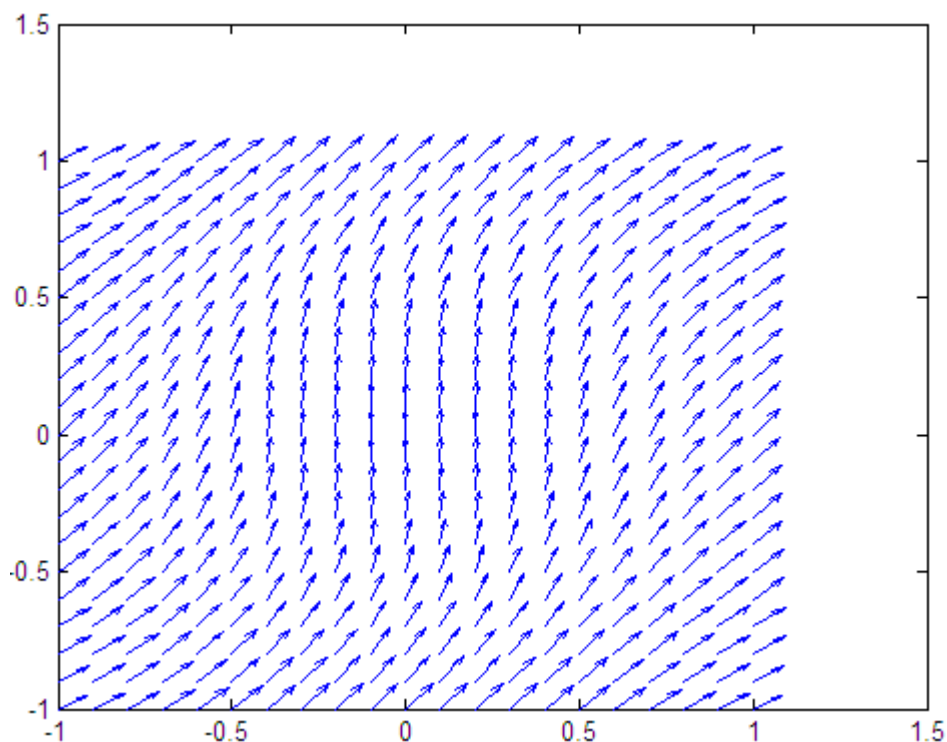
An alternative approach is to replace symbolic variables in an expression with numeric values by using the `subs` function. For example, in the following expressions  $u$  and  $v$ , substitute the symbolic variables  $x$  and  $y$  with the numeric values defined by `meshgrid`:

```
syms x y
```

```
u = sin(x^2 + y^2); v = cos(x*y);  
[X, Y] = meshgrid(-1:.1:1, -1:.1:1);  
U = subs(u, [x y], {X,Y}); V = subs(v, [x y], {X,Y});
```

Now, you can use standard MATLAB plotting functions to plot the expressions  $U$  and  $V$ . For example, create the plot of a vector field defined by the functions  $U(X, Y)$  and  $V(X, Y)$ :

```
quiver(X, Y, U, V)
```

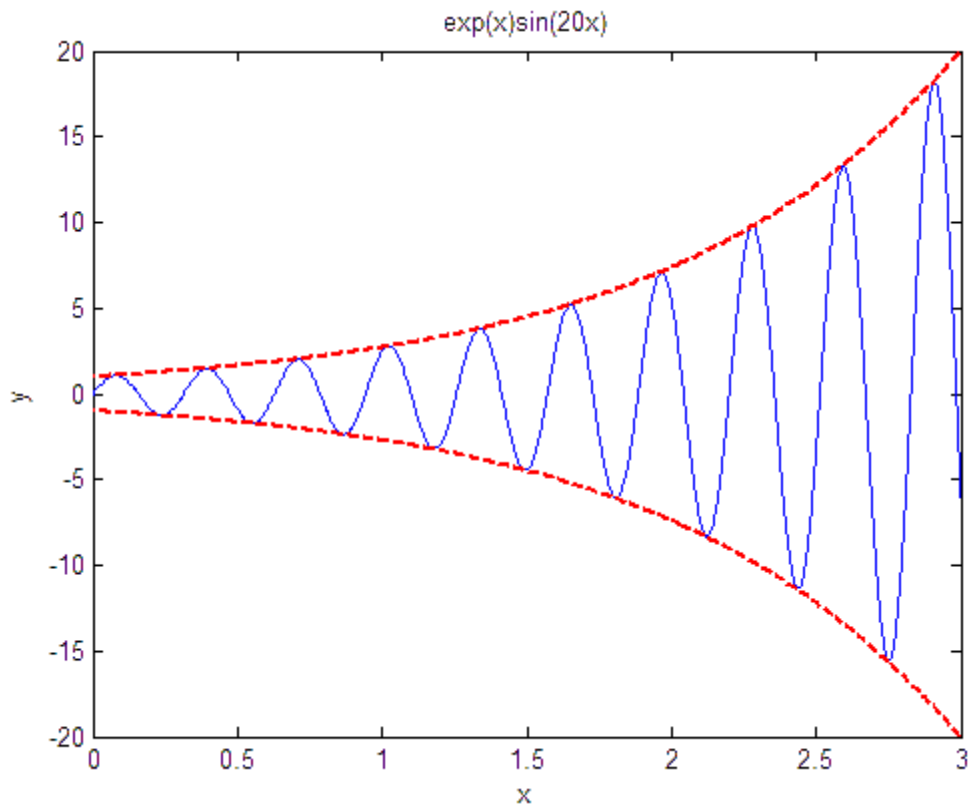


### Plotting Multiple Symbolic Functions in One Graph

To plot several symbolic functions in one graph, add them to the graph sequentially. To be able to add a new function plot to the graph that already contains a function plot, use the `hold on` command. This command retains the first function plot in the graph. Without this command, the system

replaces the existing plot with the new one. Now, add new plots. Each new plot appears on top of the existing plots. While you use the `hold on` command, you also can change the elements of the graph (such as colors, line styles, line widths, titles) or add new elements. When you finish adding new function plots to a graph and modifying the graph elements, enter the `hold off` command:

```
syms x y;
ezplot(exp(x)*sin(20*x) - y, [0, 3, -20, 20]);
hold on;
p1 = ezplot(exp(x) - y, [0, 3, -20, 20]);
set(p1,'Color','red', 'LineStyle', '--', 'LineWidth', 2);
p2 = ezplot(-exp(x) - y, [0, 3, -20, 20]);
set(p2,'Color','red', 'LineStyle', '--', 'LineWidth', 2);
title('exp(x)sin(20x)');
hold off
```



### Plotting Multiple Symbolic Functions in One Figure

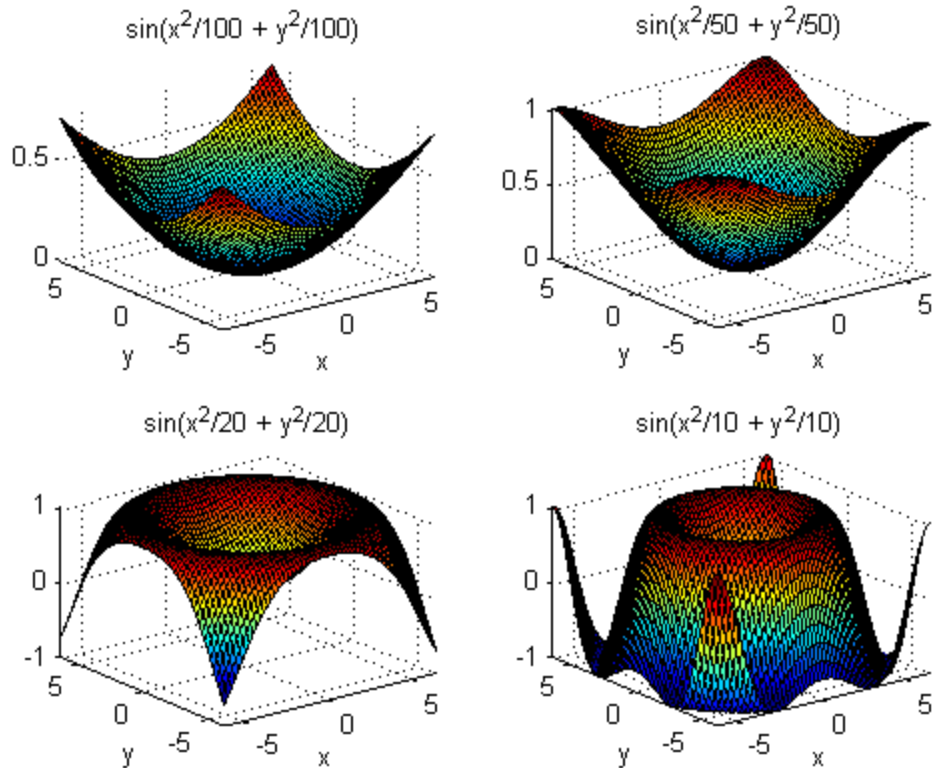
To display several function plots in one figure without overlapping, divide a figure window into several rectangular panes (tiles). Then, you can display each function plot in its own pane. For example, you can assign different values to symbolic parameters of a function, and plot the function for each value of a parameter. Collecting such plots in one figure can help you compare the plots. To display multiple plots in the same window, use the `subplot` command:

```
subplot(m,n,p)
```

This command partitions the figure window into an  $m$ -by- $n$  matrix of small subplots and selects the subplot  $p$  for the current plot. MATLAB numbers the

subplots along the first row of the figure window, then the second row, and so on. For example, plot the expression  $\sin(x^2 + y^2)/a$  for the following four values of the symbolic parameter  $a$ :

```
syms x y;
z = x^2 + y^2;
subplot(2, 2, 1); ezsurf(sin(z/100));
subplot(2, 2, 2); ezsurf(sin(z/50));
subplot(2, 2, 3); ezsurf(sin(z/20));
subplot(2, 2, 4); ezsurf(sin(z/10));
```

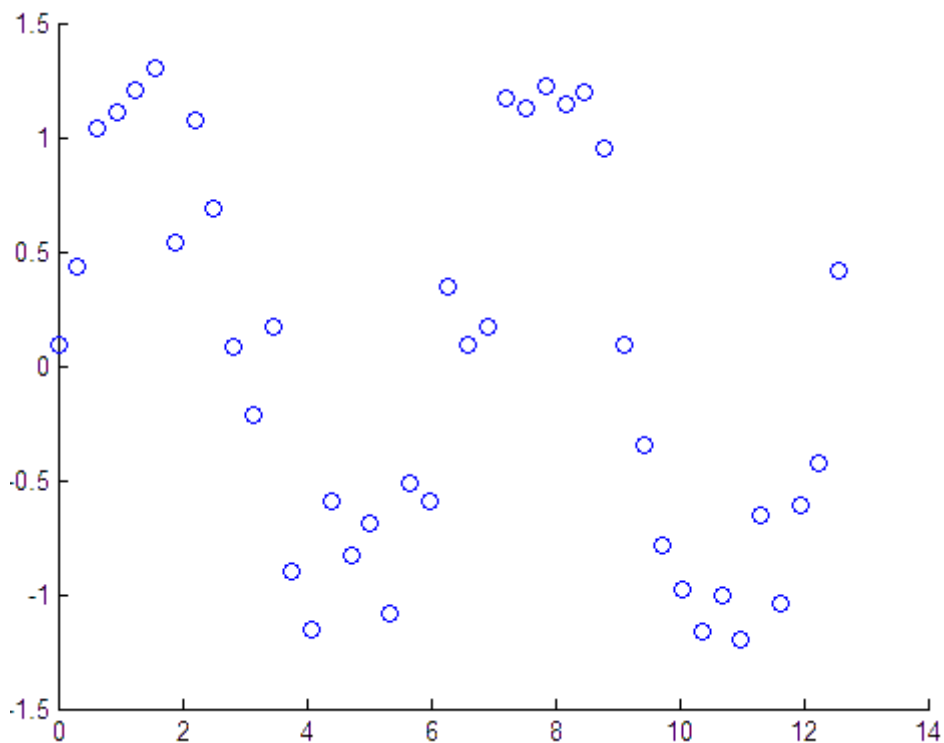


### Combining Symbolic Function Plots and Numeric Data Plots

The combined graphical capabilities of MATLAB and the Symbolic Math Toolbox software let you plot numeric data and symbolic functions in one

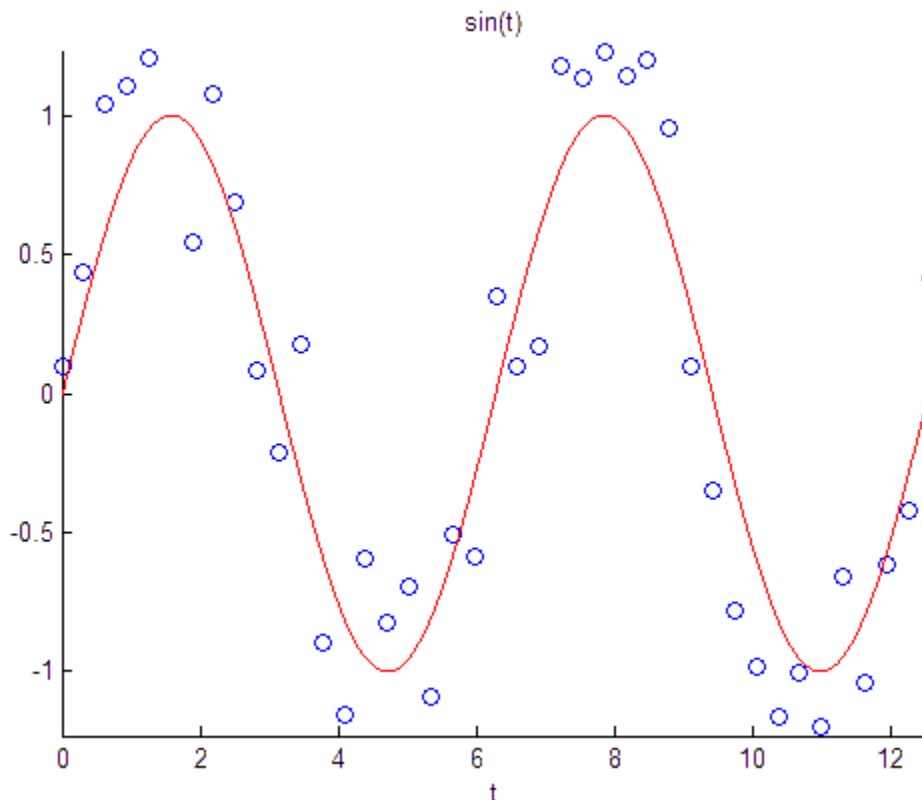
graph. Suppose, you have two discrete data sets,  $x$  and  $y$ . Use the `scatter` plotting function to plot these data sets as a collection of points with coordinates  $(x_1, y_1)$ ,  $(x_2, y_2)$ , ...,  $(x_3, y_3)$ :

```
x = 0:pi/10:4*pi;  
y = sin(x) + (-1).^randi(10, 1, 41).*rand(1, 41)./2;  
scatter(x, y)
```



Now, suppose you want to plot the sine function on top of the scatter plot in the same graph. First, use the `hold` on command to retain the current plot in the figure. (Without this command, the symbolic plot that you are about to create replaces the numeric data plot.) Then, use `ezplot` to plot the sine function. By default, MATLAB does not use a different color for a new function; the sine function appears in blue. To change the color or any other property of the plot, create the handle for the `ezplot` function call, and then use the `set` function:

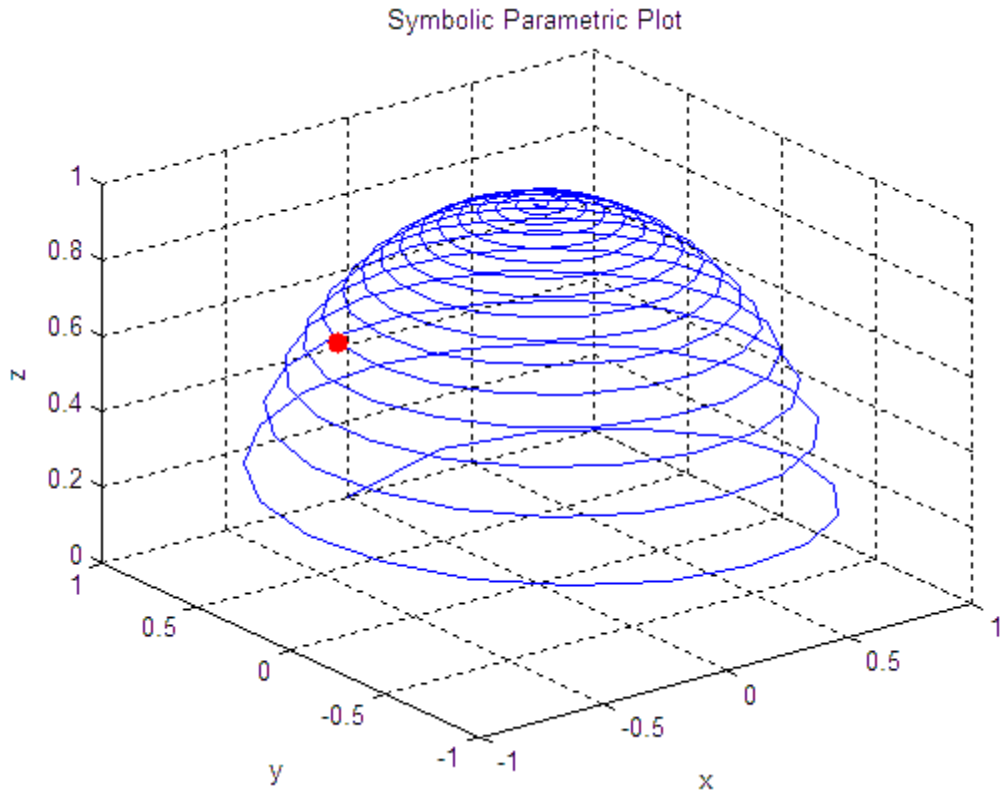
```
hold on;  
syms t;  
p = ezplot(sin(t), [0 4*pi]);  
set(p,'Color','red');
```



MATLAB provides the plotting functions that simplify the process of generating spheres, cylinders, ellipsoids, and so on. The Symbolic Math Toolbox software lets you create a symbolic function plot in the same graph with these volumes. For example, use the following commands to generate the spiral function plot wrapped around the top hemisphere. The `animate` option switches the `ezplot3` function to animation mode. The red dot on the resulting graph moves along the spiral:

```
syms t;
```

```
x = (1-t)*sin(100*t);
y = (1-t)*cos(100*t);
z = sqrt(1 - x^2 - y^2);
ezplot3(x, y, z, [0 1], 'animate');
title('Symbolic Parametric Plot');
```

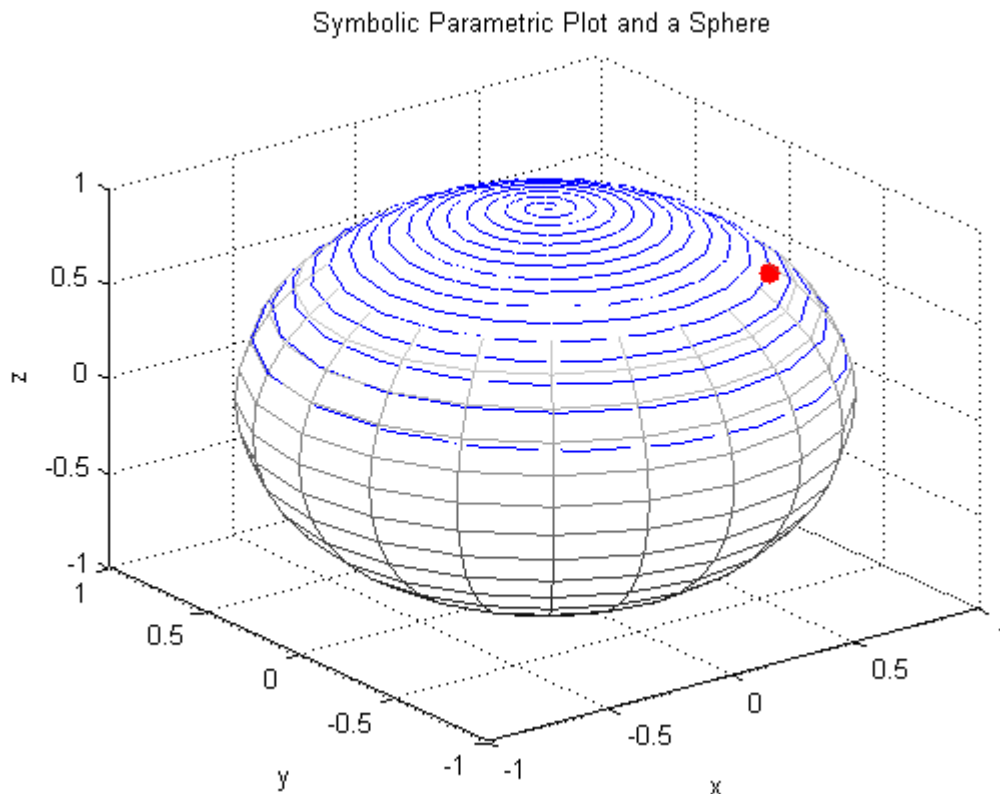


Add the sphere with radius 1 and the center at (0, 0, 0) to this graph. The `sphere` function generates the required sphere, and the `mesh` function creates a mesh plot for that sphere. Combining the plots clearly shows that the symbolic parametric function plot is wrapped around the top hemisphere:

```
hold on;
[X,Y,Z] = sphere;
mesh(X, Y, Z);
```

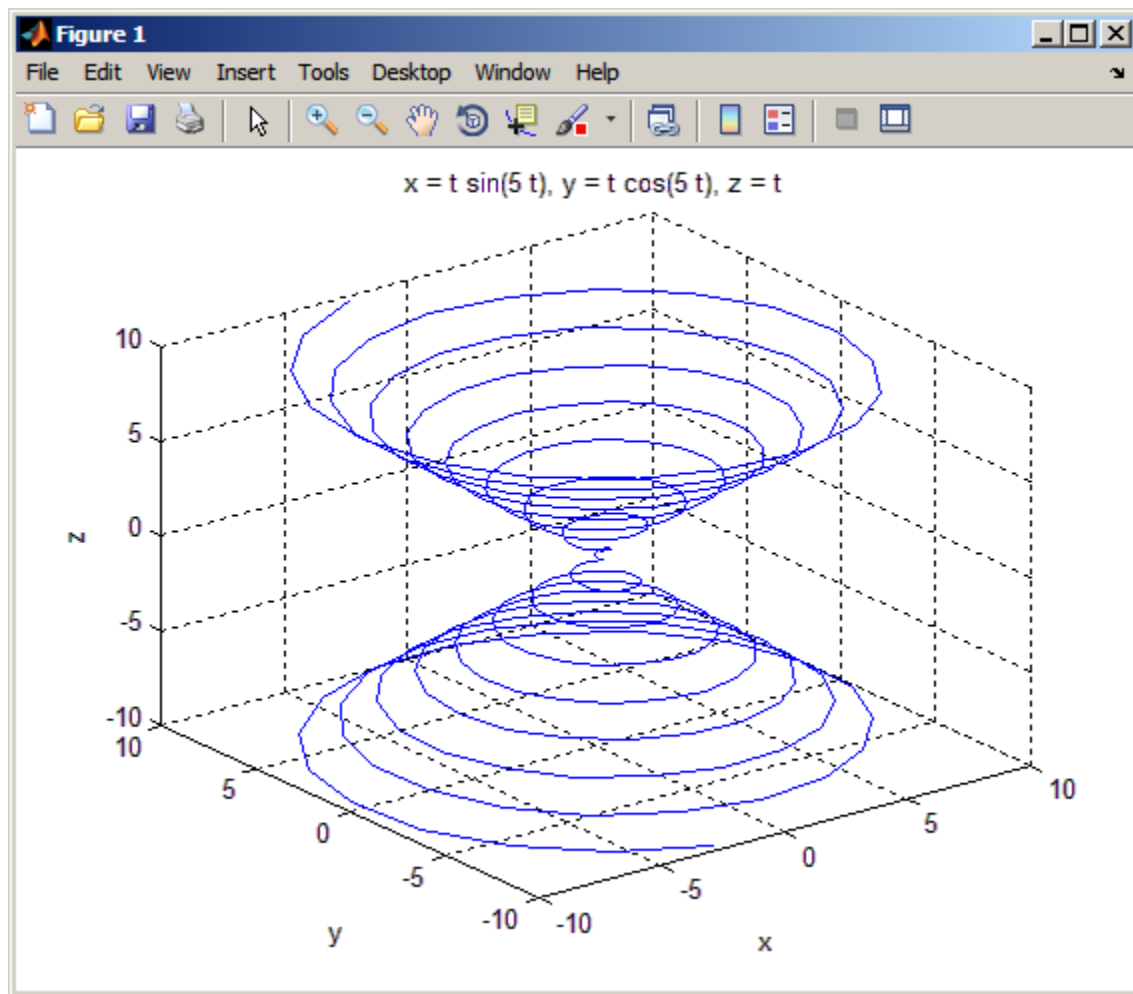


```
colormap(gray);  
title('Symbolic Parametric Plot and a Sphere');
```







## Exploring Function Plots

Plotting a symbolic function can help you visualize and explore the features of the function. Graphical representation of a symbolic function can also help you communicate your ideas or results. MATLAB displays a graph in a special window called a *figure* window. This window provides interactive tools for further exploration of a function or data plot.



Interactive data exploration tools are available in the figure toolbar and also from the **Tools** menu. By default, a figure window displays one toolbar that provides shortcuts to the most common operations. You can enable two other toolbars from the **View** menu. When exploring symbolic function plots, use the same operations as you would for the numeric data plots. For example:

- Zoom in and out on particular parts of a graph (). Zooming allows you to see small features of a function plot. Zooming behaves differently for 2-D or 3-D views. For more information, see “Enlarging the View”.
- Shift the view of the graph with the pan tool (). Panning is useful when you have zoomed in on a graph and want to move around the plot to view different portions. For more information, see “Panning — Shifting Your View of the Graph”.
- Rotate 3-D graphs (). Rotating 3-D graphs allows you to see more features of the surface and mesh function plots. For more information, see “Rotate 3D — Interactive Rotation of 3-D Views”.
- Display particular data values on a graph and export them to MATLAB workspace variables (). For more information, see “Data Cursor — Displaying Data Values Interactively”.

For more information about data exploration tools available in MATLAB, see “Ways to Explore Graphical Data”.


## Editing Graphs

MATLAB supports the following two approaches for editing graphs:

- Interactive editing lets you use the mouse to select and edit objects on a graph.
- Command-line editing lets you use MATLAB commands to edit graphs.

These approaches work for graphs that display numeric data plots, symbolic function plots, or combined plots.

To enable the interactive plot editing mode in the MATLAB figure window,

click the Edit Plot button () or select **Tools > Edit Plot** from the main menu. If you enable plot editing mode in the MATLAB figure window, you can perform point-and-click editing of your graph. In this mode, you can modify the appearance of a graphics object by double-clicking the object and

changing the values of its properties. For more information about interactive editing, see “Working in Plot Edit Mode”.

The complete collection of properties is accessible through a graphical user interface called the Property Editor. To open a graph in the Property Editor window:

- 1** Enable plot editing mode in the MATLAB figure window.
- 2** Double-click any element on the graph.

For information about editing object properties in plot editing mode, see “The Property Editor”.

If you prefer to work from the MATLAB command line or if you want to create a code file, you can edit graphs by using MATLAB commands. For information about command-line graph editing, see “Understanding Handle Graphics® Objects”.

Also, you can combine the interactive and command-line editing approaches to achieve the look you want for the graphs you create.

## **Saving Graphs**

After you create, edit, and explore a function plot, you might want to save the result. MATLAB provides three different ways to save graphs:

- Save a graph as a MATLAB FIG-file (a binary format). The FIG-file stores all information about a graph, including function plots, graph data, annotations, data tips, menus and other controls. You can open the FIG-file only with MATLAB. For more information, see “Saving a Graph in FIG-File Format”.
- Export a graph to a different file format. When saving a graph, you can choose a file format other than FIG. For example, you can export your graphs to EPS, JPEG, PNG, BMP, TIFF, PDF, and other file formats. You can open the exported file in an appropriate application. For more information, see “Saving to a Different Format — Exporting Figures”.

- Print a graph on paper or print it to file. To ensure the correct plot size, position, alignment, paper size and orientation, use Print Preview. For more information, see “Printing Figures”.
- Generate a MATLAB file from a graph. You can use the generated code to reproduce the same graph or create a similar graph using different data. This approach is useful for generating MATLAB code for work that you have performed interactively with the plotting tools. For more information, see “Generating a MATLAB File to Recreate a Graph”.

## Generating Code from Symbolic Expressions

### In this section...

“Generating C or Fortran Code” on page 3-134

“Generating MATLAB Functions” on page 3-135

“Generating MATLAB Function Blocks” on page 3-140

“Generating Simscape Equations” on page 3-144

### Generating C or Fortran Code

You can generate C or Fortran code fragments from a symbolic expression, or generate files containing code fragments, using the `ccode` and `fortran` functions. These code fragments calculate numerical values as if substituting numbers for variables in the symbolic expression.

To generate code from a symbolic expression `g`, enter either `ccode(g)` or `fortran(g)`.

For example:

```
syms x y
z = 30*x^4/(x*y^2 + 10) - x^3*(y^2 + 1)^2;
fortran(z)

ans =
    t0 = (x**4*3.0D1)/(x*y**2+1.0D1)-x**3*(y**2+1.0D0)**2

ccode(z)

ans =
    t0 =
    ((x*x*x*x)*3.0E1)/(x*(y*y)+1.0E1)-(x*x*x)*pow(y*y+1.0,2.0);
```

To generate a file containing code, either enter `ccode(g, 'file', 'filename')` or `fortran(g, 'file', 'filename')`. For the example above,

```
fortran(z, 'file', 'fortrantest')
```

generates a file named `fortrantest` in the current folder. `fortrantest` consists of the following:

```
t12 = x**2
t13 = y**2
t14 = t13+1
t0 = (t12**2*30)/(t13*x+10)-t12*t14**2*x
```

Similarly, the command

```
ccode(z, 'file', 'ccodetest')
```

generates a file named `ccodetest` that consists of the lines

```
t16 = x*x;
t17 = y*y;
t18 = t17+1.0;
t0 = ((t16*t16)*3.0E1)/(t17*x+1.0E1)-t16*(t18*t18)*x;
```

`ccode` and `fortran` generate many intermediate variables. This is called *optimized* code. MATLAB generates intermediate variables as a lowercase letter `t` followed by an automatically generated number, for example `t32`. Intermediate variables can make the resulting code more efficient by reusing intermediate expressions (such as `t12` in `fortrantest`, and `t16` in `ccodetest`). They can also make the code easier to read by keeping expressions short.

If you work in the MuPAD notebook interface, see the `generate::C` and `generate::fortran` function help pages in the MuPAD documentation.

## Generating MATLAB Functions

You can use `matlabFunction` to generate a MATLAB function handle that calculates numerical values as if you were substituting numbers for variables in a symbolic expression. Also, `matlabFunction` can create a file that accepts numeric arguments and evaluates the symbolic expression applied to the arguments. The generated file is available for use in any MATLAB calculation, whether or not the computer running the file has a license for Symbolic Math Toolbox functions.

If you work in the MuPAD notebook interface, see “Creating MATLAB Functions from MuPAD Expressions” on page 4-56.

### Generating a Function Handle

`matlabFunction` can generate a function handle from any symbolic expression. For example:

```
syms x y
r = sqrt(x^2 + y^2);
ht = matlabFunction(tanh(r))

ht =
    @(x,y)tanh(sqrt(x.^2+y.^2))
```

You can use this function handle to calculate numerically:

```
ht(.5,.5)

ans =
    0.6089
```

You can pass the usual MATLAB double-precision numbers or matrices to the function handle. For example:

```
cc = [.5,3];
dd = [-.5,.5];
ht(cc, dd)

ans =
    0.6089    0.9954
```

### Controlling the Order of Variables

`matlabFunction` generates input variables in alphabetical order from a symbolic expression. That is why the function handle in “Generating a Function Handle” on page 3-136 has `x` before `y`:

```
ht = @(x,y)tanh((x.^2 + y.^2).^(1./2))
```

You can specify the order of input variables in the function handle using the `vars` option. You specify the order by passing a cell array of strings or symbolic arrays, or a vector of symbolic variables. For example:

```
syms x y z
r = sqrt(x^2 + 3*y^2 + 5*z^2);
```



```

ht1 = matlabFunction(tanh(r), 'vars', [y x z])

ht1 =
    @(y,x,z)tanh(sqrt(x.^2+y.^2.*3.0+z.^2.*5.0))

ht2 = matlabFunction(tanh(r), 'vars', {'x', 'y', 'z'})

ht2 =
    @(x,y,z)tanh(sqrt(x.^2+y.^2.*3.0+z.^2.*5.0))

ht3 = matlabFunction(tanh(r), 'vars', {'x', [y z]})

ht3 =
    @(x,in2)tanh(sqrt(x.^2+in2(:,1).^2.*3.0+in2(:,2).^2.*5.0))

```

## Generating a File

You can generate a file from a symbolic expression, in addition to a function handle. Specify the file name using the `file` option. Pass a string containing the file name or the path to the file. If you do not specify the path to the file, `matlabFunction` creates this file in the current folder.

This example generates a file that calculates the value of the symbolic matrix `F` for double-precision inputs `t`, `x`, and `y`:

```

syms x y t
z = (x^3 - tan(y))/(x^3 + tan(y));
w = z/(1 + t^2);
F = [w, (1 + t^2)*x/y; (1 + t^2)*x/y, 3*z - 1];
matlabFunction(F, 'file', 'testMatrix.m')

```

The file `testMatrix.m` contains the following code:

```

function F = testMatrix(t,x,y)
%TESTMATRIX
%    F = TESTMATRIX(T,X,Y)

t2 = x.^2;
t3 = tan(y);
t4 = t2.*x;
t5 = t.^2;
t6 = t5 + 1;

```

```

t7 = 1./y;
t8 = t6.*t7.*x;
t9 = t3 + t4;
t10 = 1./t9;
F = [-(t10.*(t3 - t4))./t6,t8; t8,- t10.*(3.*t3 - 3.*t2.*x) - 1];

```

`matlabFunction` generates many intermediate variables. This is called *optimized* code. MATLAB generates intermediate variables as a lowercase letter `t` followed by an automatically generated number, for example `t32`. Intermediate variables can make the resulting code more efficient by reusing intermediate expressions (such as `t4`, `t6`, `t8`, `t9`, and `t10` in the calculation of `F`). Using intermediate variables can make the code easier to read by keeping expressions short.

If you don't want the default alphabetical order of input variables, use the `vars` option to control the order. Continuing the example,

```
matlabFunction(F,'file','testMatrix.m','vars',[x y t])
```

generates a file equivalent to the previous one, with a different order of inputs:

```
function F = testMatrix(x,y,t)
...
```

### Naming Output Variables

By default, the names of the output variables coincide with the names you use calling `matlabFunction`. For example, if you call `matlabFunction` with the variable `F`

```

syms x y t
z = (x^3 - tan(y))/(x^3 + tan(y));
w = z/(1 + t^2);
F = [w, (1 + t^2)*x/y; (1 + t^2)*x/y,3*z - 1];
matlabFunction(F,'file','testMatrix.m','vars',[x y t])

```

the generated name of an output variable is also `F`:

```
function F = testMatrix(x,y,t)
...
```

If you call `matlabFunction` using an expression instead of individual variables

```

syms x y t
z = (x^3 - tan(y))/(x^3 + tan(y));
w = z/(1 + t^2);
F = [w, (1 + t^2)*x/y; (1 + t^2)*x/y, 3*z - 1];
matlabFunction(w + z + F, 'file', 'testMatrix.m', ...
'vars', [x y t])

```

the default names of output variables consist of the word `out` followed by the number, for example:

```

function out1 = testMatrix(x,y,t)
...

```

To customize the names of output variables, use the `output` option:

```

syms x y z
r = x^2 + y^2 + z^2;
q = x^2 - y^2 - z^2;
f = matlabFunction(r, q, 'file', 'new_function', ...
'outputs', {'name1', 'name2'})

```

The generated function returns *name1* and *name2* as results:

```

function [name1,name2] = new_function(x,y,z)
...

```

## Converting MuPAD Expressions

You can convert a MuPAD expression or function to a MATLAB function:

```

syms x y;
f = evalin(symengine, 'arcsin(x) + arccos(y)');
matlabFunction(f, 'file', 'new_function');

```

The created file contains the same expressions written in the MATLAB language:

```

function f = new_function(x,y)
%NEW_FUNCTION
%   F = NEW_FUNCTION(X,Y)

f = asin(x) + acos(y);

```

---

**Tip** `matlabFunction` cannot correctly convert some MuPAD expressions to MATLAB functions. These expressions do not trigger an error message. When converting a MuPAD expression or function that is not on the MATLAB vs. MuPAD Expressions list, always check the results of conversion. To verify the results, execute the resulting function.

---

## Generating MATLAB Function Blocks

Using `emlBlock`, you can generate a MATLAB Function block. The generated block is available for use in Simulink® models, whether or not the computer running the simulations has a license for Symbolic Math Toolbox.

If you work in the MuPAD notebook interface, see “Creating MATLAB Function Blocks from MuPAD Expressions” on page 4-59.

## Generating and Editing a Block

Suppose, you want to create a model involving the van der Pol equation. Before you can convert a symbolic expression to a MATLAB Function block, create an empty model or open an existing one:

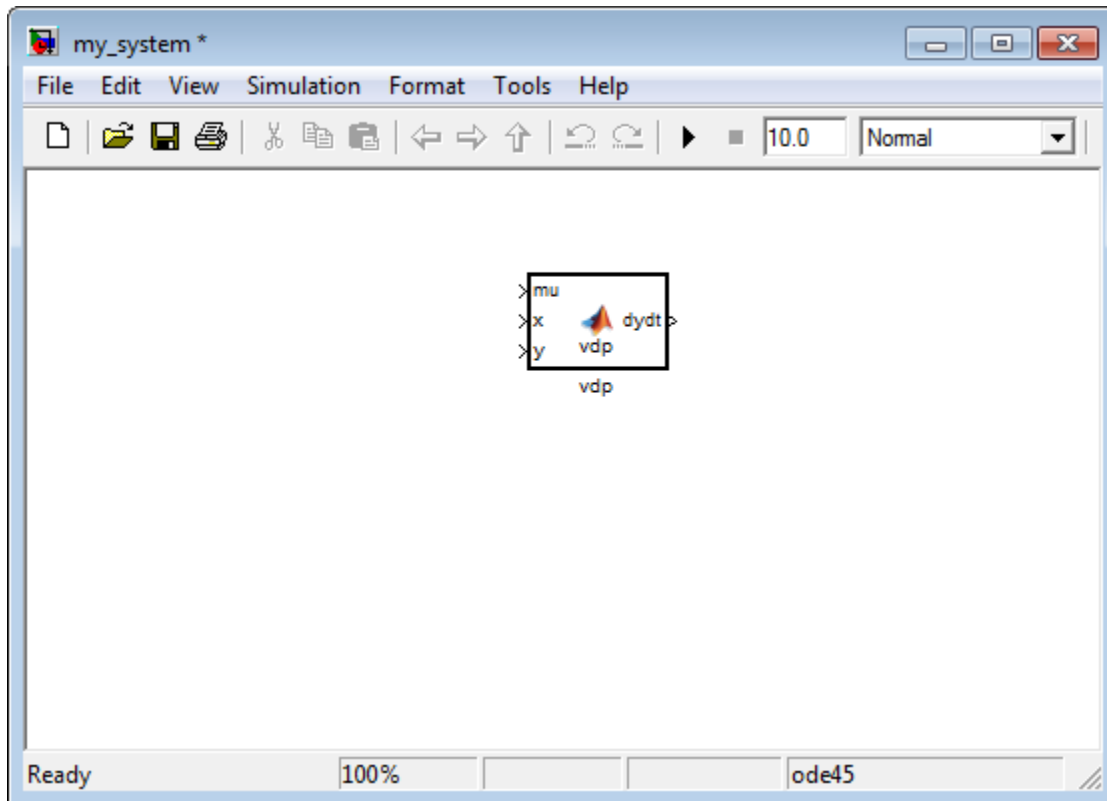
```
new_system('my_system');  
open_system('my_system');
```

Create a symbolic expression and pass it to the `emlBlock` command. Also specify the block name:

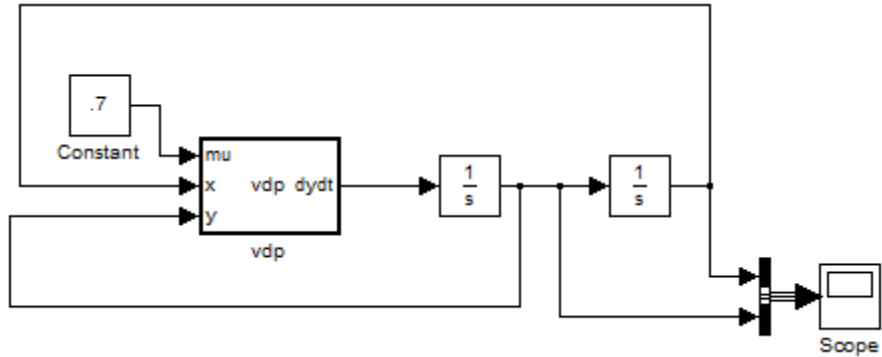
```
syms x y;  
mu = sym('mu');  
dydt = -x - mu*y*(x^2 - 1);  
emlBlock('my_system/vdp', dydt);
```

If you use the name of an existing block, the `emlBlock` command replaces the definition of an existing block with the converted symbolic expression.

The model `my_system` contains the generated block.



Add other Simulink blocks and wiring to properly define the system.



You can open and edit the generated block. To open a block, select **Edit > Open Block** or use the context menu.

```

1  function dydt = vdp(mu,x,y)
2  %#codegen
3
4  -  dydt = -x-mu.*y.*(x.^2-1.0);

```

### Controlling the Order of Input Ports

emlBlock generates input variables and the corresponding input ports in alphabetical order from a symbolic expression. To change the order of input variables, use the vars option:

```

syms x y;
mu = sym('mu');
dydt = -x - mu*y*(x^2 - 1);
emlBlock('my_system/vdp', dydt,...
'vars', [y mu x]);

```

## Naming the Output Ports

By default, `emlBlock` generates the names of the output ports as the word `out` followed by the output port number, for example, `out3`. The `output` option allows you to use the custom names of the output ports:

```
syms x y;
mu = sym('mu');
dydt = -x - mu*y*(x^2 - 1);
emlBlock('my_system/vdp', dydt, ...
'outputs', {'name1'});
```

## Converting MuPAD Expressions

You can convert a MuPAD expression or function to a MATLAB Function block:

```
syms x y;
f = evalin(symengine, 'arcsin(x) + arccos(y)');
emlBlock('my_system/my_block', f);
```

The resulting block contains the same expressions written in the MATLAB language:

```
function f = my_block(x,y)
%#codegen

f = asin(x) + acos(y);
```

---

**Tip** Some MuPAD expressions cannot be correctly converted to a block. These expressions do not trigger an error message. When converting a MuPAD expression or function that is not on the MATLAB vs. MuPAD Expressions list, always check the results of conversion. To verify the results, you can:

- Run the simulation containing the resulting block.
  - Open the block and verify that all the functions are defined in MATLAB Function Library Reference.
-

## Generating Simscape Equations

Simscape™ software extends the Simulink product line with tools for modeling and simulating multidomain physical systems, such as those with mechanical, hydraulic, pneumatic, thermal, and electrical components. Unlike other Simulink blocks, which represent mathematical operations or operate on signals, Simscape blocks represent physical components or relationships directly. With Simscape blocks, you build a model of a system just as you would assemble a physical system. For more information about Simscape software see [www.mathworks.com/products/simscape/](http://www.mathworks.com/products/simscape/).

You can extend the Simscape modeling environment by creating custom components. When you define a component, use the equation section of the component file to establish the mathematical relationships among a component's variables, parameters, inputs, outputs, time, and the time derivatives of each of these entities. The Symbolic Math Toolbox and Simscape software let you perform symbolic computations and use the results of these computations in the equation section. The `simscapeEquation` function translates the results of symbolic computations to Simscape language equations.

If you work in the MuPAD notebook interface, see “Creating Simscape Equations from MuPAD Expressions” on page 4-61.

## Converting Algebraic and Differential Equations

Suppose, you want to generate a Simscape equation from the solution of the following ordinary differential equation. As a first step, use the `dsolve` function to solve the equation:

```
s = dsolve('D2y = -a^2*y', 'y(0) = 1', 'Dy(pi/a) = 0');  
s = simplify(s)
```

The solution is:

```
s =  
cos(a*t)
```

Then, use the `simscapeEquation` function to rewrite the solution in the Simscape language:

```
simscapeEquation(s)
```



`simscapeEquation` generates the following code:

```
ans =
s == cos(a*time);
```

The variable *time* replaces all instances of the variable *t* except for derivatives with respect to *t*. To use the generated equation, copy the equation and paste it to the equation section of the Simscape component file. Do not copy the automatically generated variable `ans` and the equal sign that follows it.

`simscapeEquation` converts any derivative with respect to the variable *t* to the Simscape notation, `X.der`, where *X* is the time-dependent variable. For example, convert the following differential equation to a Simscape equation. Also, here you explicitly specify the left and the right sides of the equation by using the syntax `simscapeEquation(LHS, RHS)`:

```
syms a;
x = sym('x(t)');
simscapeEquation(diff(x), -a^2*x)

ans =
x.der == -a^2*x;
```

`simscapeEquation` also translates piecewise expressions to the Simscape language. For example, the result of the following Fourier transform is a piecewise function:

```
syms v u;
syms x real;
f = exp(-x^2*abs(v))*sin(v)/v;
s = fourier(f, v, u)

s =
piecewise([x <> 0, atan((u + 1)/x^2) - atan((u - 1)/x^2)])
```

From this symbolic piecewise equation, `simscapeEquation` generates valid code for the equation section of a Simscape component file:

```
simscapeEquation(s)

ans =
s == if (x ~= 0.0),
```

```
-atan(1.0/x^2*(u-1.0))+atan(1.0/x^2*(u+1.0));  
else  
    NaN;  
end;
```

Clear the assumption that  $x$  is real:

```
syms x clear
```

### Converting MuPAD Equations

If you perform symbolic computations in the MuPAD Notebook Interface and want to convert the results to Simscape equations, use the `generate::Simscape` function in MuPAD.

### Limitations

The equation section of a Simscape component file supports a limited number of functions. See the list of Supported Functions for more information. If a symbolic equation contains the functions that the equation section of a Simscape component file does not support, `simscapeEquation` cannot correctly convert these equations to Simscape equations. Such expressions do not trigger an error message. The following types of expressions are prone to invalid conversion:

- Expressions with infinities
- Expressions returned by `evalin` and `feval`

# MuPAD in Symbolic Math Toolbox

---

- “Understanding MuPAD” on page 4-2
- “MuPAD for MATLAB Users” on page 4-10
- “Integration of MuPAD and MATLAB” on page 4-28
- “Integrating Symbolic Computations in Other Toolboxes and Simulink”  
on page 4-56

## Understanding MuPAD

### In this section...

“Introduction to MuPAD” on page 4-2

“MuPAD Engines and MATLAB Workspace” on page 4-2

“Introductory Example Using a MuPAD Notebook from MATLAB” on page 4-3

### Introduction to MuPAD

Starting with Version 4.9, Symbolic Math Toolbox is powered by the MuPAD symbolic engine.

- MuPAD notebooks provide an additional interface for performing symbolic calculations, variable-precision calculations, plotting, and animations. “Introductory Example Using a MuPAD Notebook from MATLAB” on page 4-3 shows how to use this interface.
- Symbolic Math Toolbox functions let you copy variables and expressions between the MATLAB workspace and MuPAD notebooks. For details, see “Copying Variables and Expressions Between the MATLAB Workspace and MuPAD Notebooks” on page 4-32.
- You can call MuPAD functions and procedures, including custom procedures, from the MATLAB environment. For details, see “Calling Built-In MuPAD Functions from the MATLAB Command Window” on page 4-40.
- You can convert the results of symbolic computations into MATLAB functions, Simulink blocks, or use them in equation sections when building new components in Simscape.

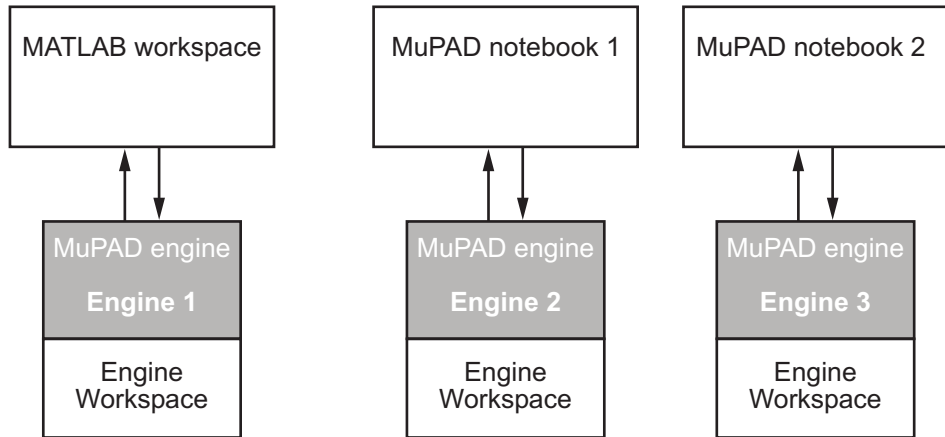
### MuPAD Engines and MATLAB Workspace

A MuPAD engine is a separate process that runs on your computer in addition to a MATLAB process. A MuPAD engine starts when you first call a function that needs a symbolic engine, such as `syms`. Symbolic Math Toolbox functions that use the symbolic engine use standard MATLAB syntax, such as `y = int(x^2)`.

Conceptually, each MuPAD notebook has its own symbolic engine, with an associated workspace. You can have any number of MuPAD notebooks open simultaneously.

One engine exists for use by  
Symbolic Math Toolbox.

Each MuPAD notebook also  
has its own engine.



The engine workspace associated with the MATLAB workspace is generally empty, except for assumptions you make about variables. For details, see “Clearing Assumptions and Resetting the Symbolic Engine” on page 4-51.

## Introductory Example Using a MuPAD Notebook from MATLAB

This example shows how to use a MuPAD notebook to calculate symbolically the mean and variance of a normal random variable that is restricted to be positive. For details on using a MuPAD notebook, see “Calculating in a MuPAD Notebook” on page 4-14.

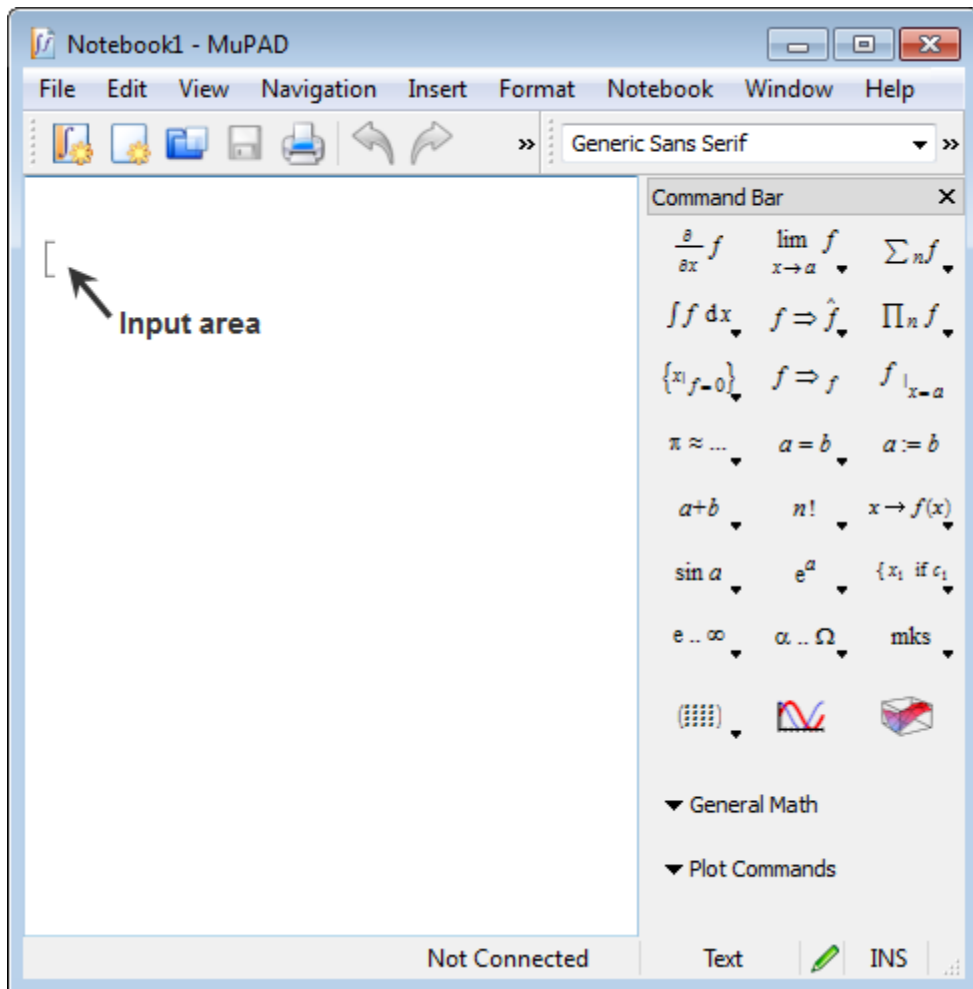
The density function of the normal and positive random variable is

$$f(x) = \begin{cases} e^{-x^2/2} \sqrt{2/\pi} & \text{if } x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

- 1 At the MATLAB command line, enter the command

mupad

A blank MuPAD notebook opens.



- 2 Type commands in the input area, indicated by a left bracket. For example, type the following commands and press **Enter**:

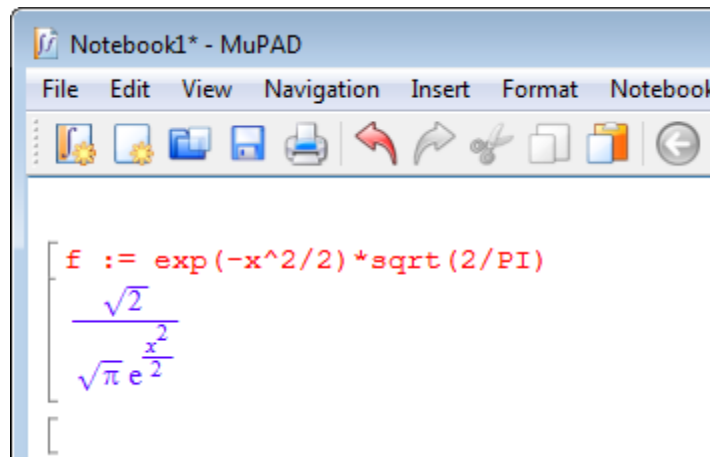
```
f := exp(-x^2/2)*sqrt(2/PI)
```

---

**Note** Assignment in a MuPAD notebook uses :=, not the MATLAB syntax =. Also, the MuPAD syntax for the mathematical constant  $\pi$  is PI, not the MATLAB syntax pi. For more information on common syntax differences, see “Differences Between MATLAB and MuPAD Syntax” on page 4-28.

---

The MuPAD notebook displays results in real math notation. For example, now your notebook appears as follows.



**3** The mean of the random variable is

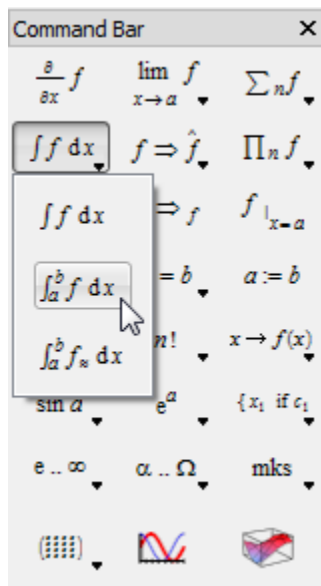
$$\text{mean} = \int_0^{\infty} x \cdot f dx.$$

To calculate the mean of the random variable:

**a** Type

```
mean :=
```

- b To place an integral in the correct syntax, click the integral button in the Command Bar (by default, it appears on the right), and select definite limits as shown.



The correct syntax for integration appears in the input area.

```
[ mean := int(#f, #x=#a..#b)
```

- c Press **Tab** to select the replaceable fields #f, #x, and so on. Press **Ctrl+space bar** to autocomplete inputs. For example, type infi and press **Ctrl+space bar** to enter infinity.
- d Replace #f with x\*f, #x with x, #a with 0, and #b with infinity.
- e Press **Enter** when your input area reads:

```
mean := int(x*f, x = 0..infinity)
```



**Note** The syntax for integration and infinity differ from the MATLAB versions.

```

Notebook1* - MuPAD
File Edit View Navigation Insert Format Notebook
[ ]
f := exp(-x^2/2)*sqrt(2/PI)
[
  sqrt(2)
  -----
  sqrt(pi) e^(x^2/2)
]
mean := int(x*f, x = 0..infinity)
[
  sqrt(2)
  -----
  sqrt(pi)
]
[

```

4 The variance of the random variable is

$$\text{variance} = \int_0^{\infty} (x - \text{mean})^2 \cdot f dx.$$

To calculate the variance of the random variable, type the following command and press **Enter**:

```
variance := int((x - mean)^2*f, x = 0..infinity)
```

$$\left[ \begin{array}{l} \text{variance} := \text{int}((x-\text{mean})^2*f, x=0..\text{infinity}) \\ \int_0^{\infty} \frac{\sqrt{2} \left(x - \frac{\sqrt{2}}{\sqrt{\pi}}\right)^2}{\sqrt{\pi} e^{\frac{x^2}{2}}} dx \end{array} \right.$$

- 5 The result of evaluating `variance` is a complicated expression. Try to simplify it using the `simplify` command:

```
simplify(variance)
```

The result is indeed simpler:

$$\left[ \begin{array}{l} \text{simplify}(\text{variance}) \\ \frac{\pi - 2}{\pi} \end{array} \right.$$

- 6 Another expression for the variance of the random variable is

$$\text{variance} = \int_0^{\infty} x^2 \cdot f dx - \text{mean}^2.$$

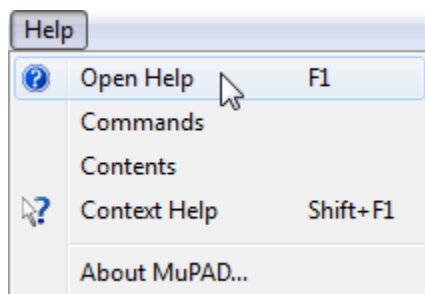
To calculate the variance of the random variable using this definition, type the following command and press **Enter**:

```
variance2 := int(x^2*f, x = 0..infinity) - mean^2
```

$$\left[ \begin{array}{l} \text{variance2} := \text{int}(x^2*f, x=0..\text{infinity}) - \text{mean}^2 \\ 1 - \frac{2}{\pi} \end{array} \right.$$

The two expressions for variance, `variance` and `variance2`, are obviously equivalent.

For details on working in MuPAD notebooks, select **Help > Open Help** or press **F1** to open the MuPAD Help Browser.



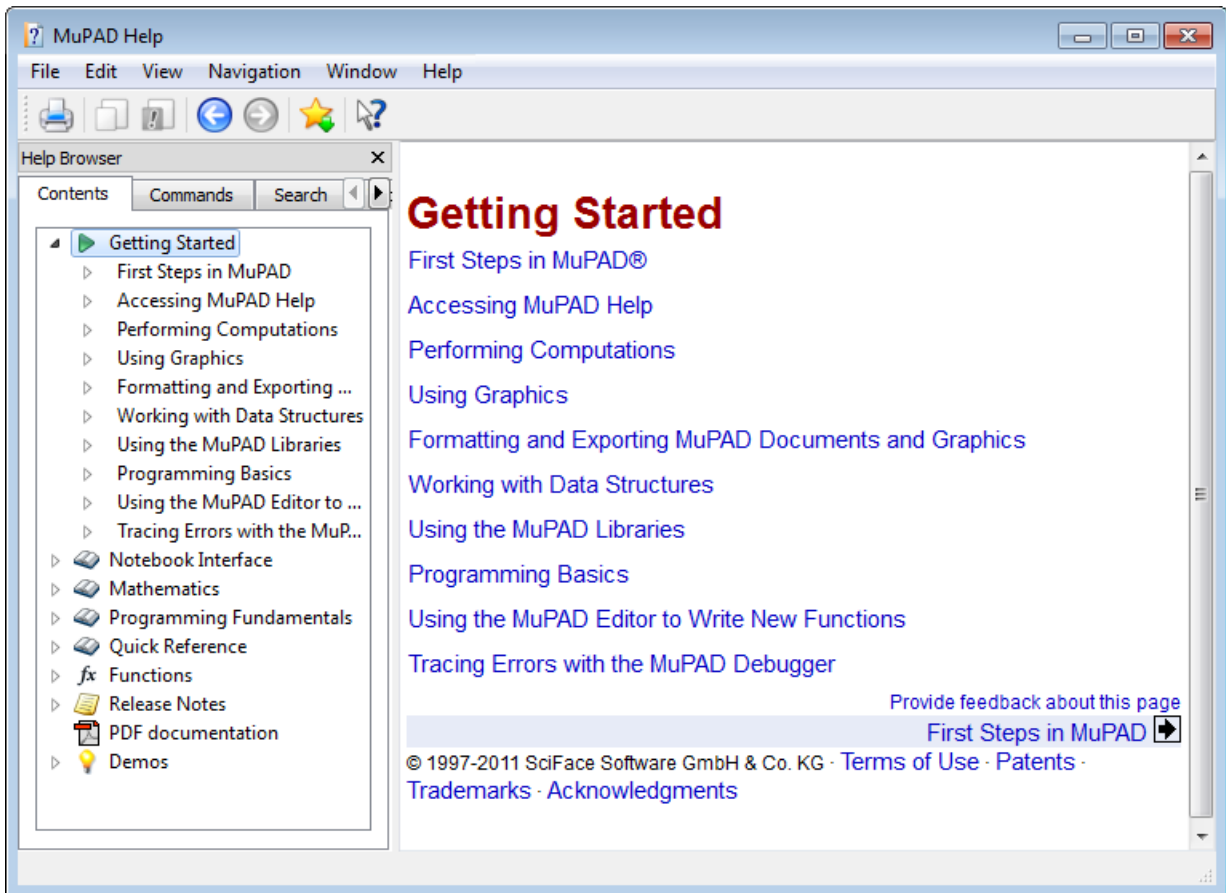
## MuPAD for MATLAB Users

In this section...
“Getting Help for MuPAD” on page 4-10
“Creating, Opening, and Saving MuPAD Notebooks” on page 4-11
“Calculating in a MuPAD Notebook” on page 4-14
“Other MuPAD Interfaces: Editor and Debugger” on page 4-21
“Notebook Files and Program Files” on page 4-26
“Source Code of the MuPAD Library Functions” on page 4-27

### Getting Help for MuPAD

Extensive online help is available for MuPAD. To access the MuPAD Help Browser from the MATLAB workspace, use one of the following methods:

- Enter `doc(symengine)` at the MATLAB Command Window.



MuPAD Help contains complete documentation of the MuPAD language. It also explains how to use MuPAD interfaces, such as notebooks and the editor.

- For help on a specific MuPAD function, enter `doc(symengine, 'functionName')` at the MATLAB command line to display MuPAD Help at the `functionName` function.

## Creating, Opening, and Saving MuPAD Notebooks

To create a new MuPAD notebook from the MATLAB command line, enter

```
nb = mupad
```

You can use any variable name instead of `nb`. This syntax opens a blank MuPAD notebook.

The variable `nb` is a handle to the notebook. The toolbox uses this handle only for communication between the MATLAB workspace and the MuPAD notebook. Use handles as described in “Copying Variables and Expressions Between the MATLAB Workspace and MuPAD Notebooks” on page 4-32.

You also can open an existing MuPAD notebook file named *file\_name* from the MATLAB command line by entering

```
nb2 = mupad('file_name')
```

where *file\_name* must be a full path unless the notebook is in the current folder. This command is useful if you lose the handle to a notebook, in which case, you can save the notebook file and then reopen it with a fresh handle.

---

**Caution** You can lose data when saving a MuPAD notebook. A notebook saves its inputs and outputs, but not the state of its engine. In particular, MuPAD does not save variables copied into a notebook using `setVar(nb, ...)`.

---

To open a notebook and automatically jump to a particular location, create a link target at that location inside a notebook and refer to it when opening a notebook. For information about creating link targets, see “Formatting and Exporting MuPAD Documents and Graphics” in the “Getting Started” chapter of the MuPAD documentation. To refer to a link target when opening a notebook, enter:

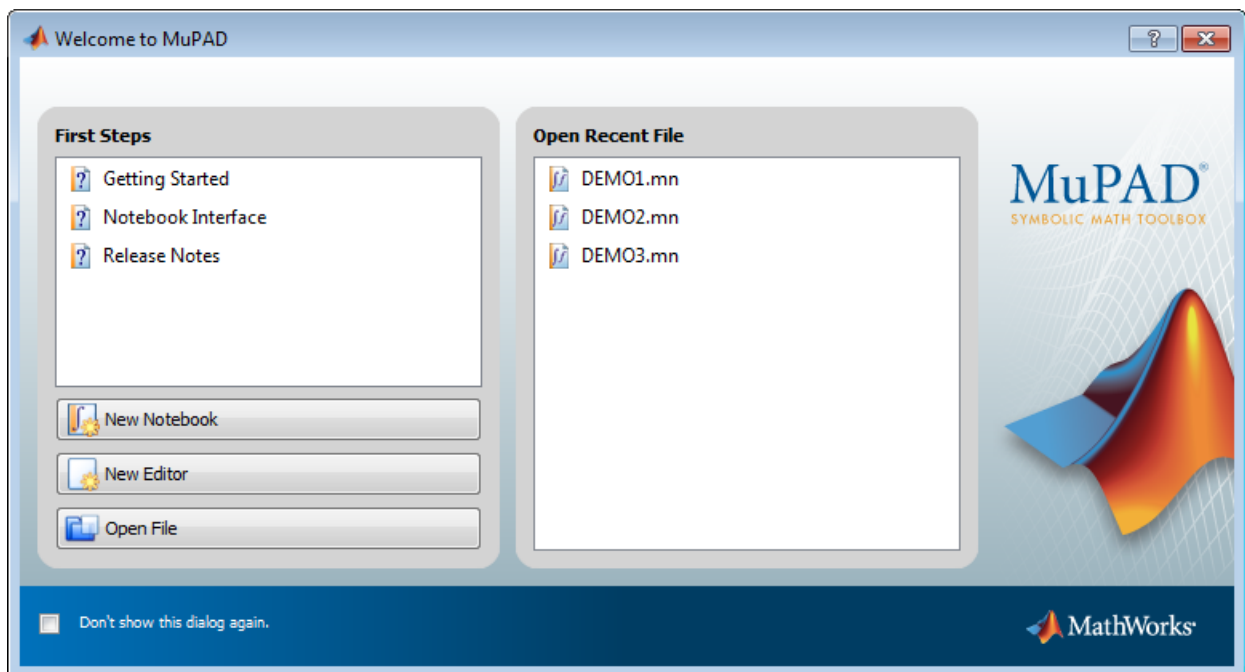
```
nb2 = mupad('file_name#linktarget_name')
```

You also can open and save MuPAD notebook files using the usual file system commands, and by using the MATLAB or MuPAD **File** menu. However, to be able to use a handle to a notebook, you must open the notebook using the `mupad` command at the MATLAB command line.

**Tip** MuPAD notebook files open in an unevaluated state. In other words, the notebook is not synchronized with its engine when it opens. To synchronize a notebook with its engine, select **Notebook > Evaluate All**. For details, see “Synchronizing a Notebook and its Engine” on page 4-19.

You also can use the Welcome to MuPAD dialog box to access various MuPAD interfaces. To open this dialog box, enter:

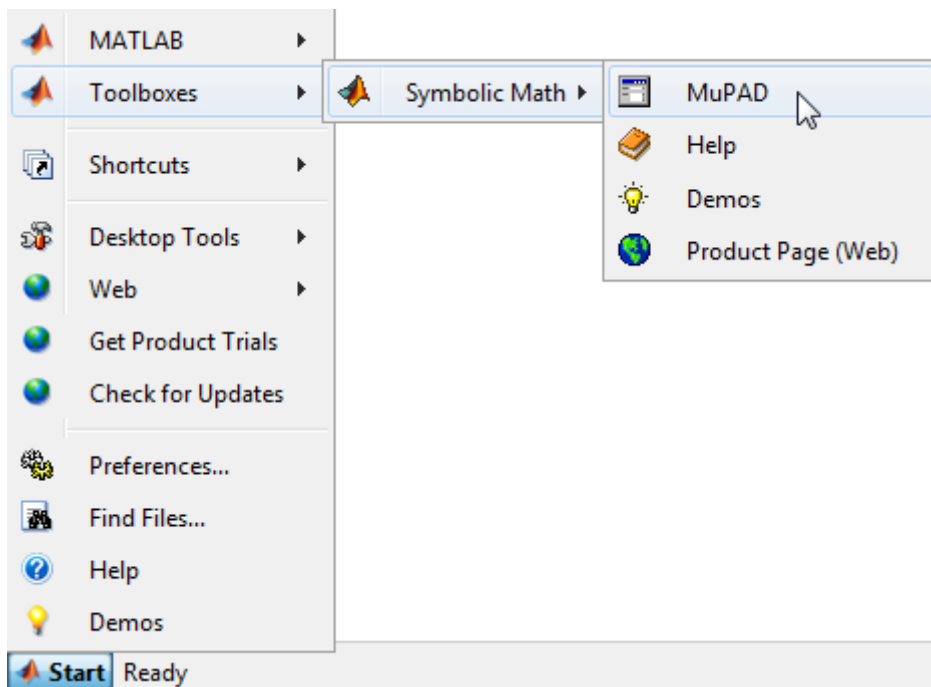
```
mupadwelcome
```



- To access MuPAD Help, click one of the three options in the **First Steps** pane.
- To open an existing file, click its name in the **Open Recent File** pane.
- To open a new notebook, click the **New Notebook** button.

- To open a new program file in the MuPAD Editor, click the **New Editor** button. For information on this interface and its associated debugger, see “Other MuPAD Interfaces: Editor and Debugger” on page 4-21.
- To open an existing MuPAD notebook or program file, click **Open File** and navigate to the file.

Alternatively, you can open the MuPAD welcome dialog box from the MATLAB **Start** menu.



## Calculating in a MuPAD Notebook

### Visual Elements of a Notebook

A MuPAD notebook has the following main components.



The screenshot shows the MuPAD Notebook1\* interface. The window title is "Notebook1\* - MuPAD". The menu bar includes File, Edit, View, Navigation, Insert, Format, Notebook, Window, and Help. Below the menu is a toolbar with various icons. A "Command Bar" is located at the top right, containing a font style dropdown set to "Generic Sans Serif".

The notebook content is divided into regions:

- Text region:** Contains the text "A MuPAD notebook has three types of regions: input regions, output regions, and text regions."
- Input region:** Contains the code:
 

```
y := x^3;
plot(y, x = -2..2)
```
- Output region:** Contains the plot of the function  $y = x^3$  for  $x$  in the range  $[-2, 2]$ . The plot shows a blue curve passing through the origin, with x and y axes ranging from -2 to 2 and -8 to 8 respectively.

The Command Bar on the right contains various mathematical symbols and operators, such as  $\frac{\partial}{\partial x} f$ ,  $\lim_{x \rightarrow a} f$ ,  $\sum_n f$ ,  $\int f dx$ ,  $f \Rightarrow \hat{f}$ ,  $\prod_n f$ ,  $\{x|_{f=0}\}$ ,  $f \Rightarrow f$ ,  $f|_{x=a}$ ,  $\pi \approx \dots$ ,  $a = b$ ,  $a := b$ ,  $a + b$ ,  $n!$ ,  $x \rightarrow f(x)$ ,  $\sin a$ ,  $e^a$ ,  $\{x_i \text{ if } c_i\}$ ,  $e \dots \infty$ ,  $\alpha \dots \Omega$ , and  $\text{mks}$ . It also includes a "General Math" section and a "Plot Commands" section.

At the bottom of the notebook, there is a status bar with "Undo last operation.", "Mem 7 MB, T 0 s", and a "Text" button.

- Enter commands for execution, evaluation, or plotting in input regions.
- Enter comments in text regions. You can type and format text in a notebook similar to working in any word processing application.

- Use the **Command Bar** to help you enter commands into input regions with the proper syntax.
- Use the **Insert** menu to add a text area (called **Text Paragraph**) or input regions (called **Calculation**).
- Use the **Notebook** menu to evaluate expressions in input regions.

### Working in a Notebook

The MuPAD notebook interface differs from the MATLAB interface. Things to remember when working in a MuPAD notebook are:

- Commands typed in an input area are not evaluated until you press **Enter**.
- You can edit the commands typed in *any* input area. For example, you can change a command, correct syntax, or try different values of parameters simply by selecting the area you want to change and typing over it. Press **Enter** to reevaluate the result.
- Results do not automatically cascade or propagate through a notebook, as described in “Cascading Calculations” on page 4-16.
- The MATLAB method of recalling a previous command by pressing an up arrow key does not have the same effect in a MuPAD notebook. Instead, you use arrow keys for navigation in MuPAD notebooks, similar to most word processors.

### Cascading Calculations

If you change a variable in a notebook, the change does not automatically propagate throughout the notebook. For example, consider the following set of MuPAD commands:

```

z := sin(x)
sin(x)

y := z / (1 + z^2)
sin(x)
sin(x)^2 + 1

w := simplify(y / (1 - y))
sin(x)
sin(x)^2 - sin(x) + 1

```

Now change the definition of  $z$  in the first line of the notebook from  $\sin(x)$  to  $\cos(x)$  and press **Enter**:

```

z := cos(x)
cos(x)

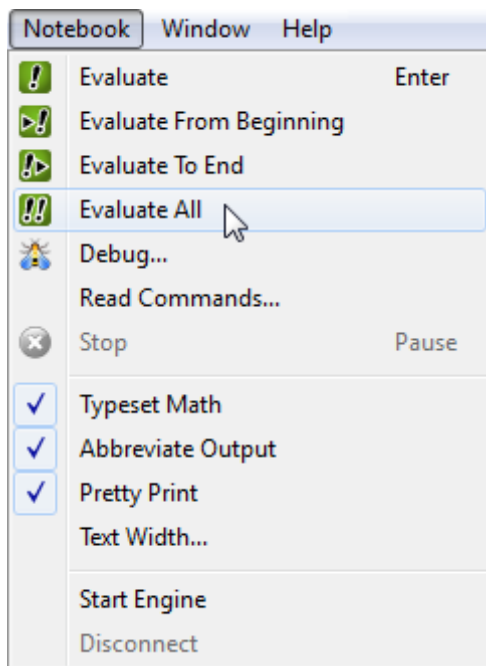
y := z / (1 + z^2)
sin(x)
sin(x)^2 + 1

w := simplify(y / (1 - y))
sin(x)
sin(x)^2 - sin(x) + 1

```

Only the first line was reevaluated. Therefore  $y$  and  $z$  are no longer synchronized. The notebook is in an inconsistent state.

To have the changes cascade to all parts of the notebook, select **Notebook** > **Evaluate All**.



The engine evaluates all the expressions in the notebook from top to bottom, and the notebook becomes consistent:

```

z := cos(x)
cos(x)

y := z/(1 + z^2)
cos(x)
-----
cos(x)^2 + 1

w := simplify(y/(1 - y))
cos(x)
-----
cos(x)^2 - cos(x) + 1

```

### Synchronizing a Notebook and its Engine

When you open a saved MuPAD notebook file, the notebook display is not synchronized with its engine. For example, suppose you saved the notebook pictured in the start of “Cascading Calculations” on page 4-16:

```

z := sin(x)
sin(x)

y := z/(1 + z^2)
sin(x)
-----
sin(x)^2 + 1

w := simplify(y/(1 - y))
sin(x)
-----
sin(x)^2 - sin(x) + 1

```

If you open that file and immediately try to work in it without synchronizing the notebook with its engine, the expressions in the notebook display are unavailable for calculations. For example, try to calculate  $u := (1+w)/w$ :

$$\begin{array}{l} z := \sin(x) \\ \sin(x) \\ y := z / (1 + z^2) \\ \frac{\sin(x)}{\sin(x)^2 + 1} \\ w := \text{simplify}(y / (1 - y)) \\ \frac{\sin(x)}{\sin(x)^2 - \sin(x) + 1} \\ u := (1 + w) / w \\ \frac{w + 1}{w} \end{array}$$

The variable  $w$  has no definition as far as the engine is concerned.

To remedy this situation, select **Notebook > Evaluate All**. The variable  $u$  changes to reflect the value of  $w$ :

$$\begin{array}{l} u := (1 + w) / w \\ \frac{\left( \frac{\sin(x)}{\sin(x)^2 - \sin(x) + 1} + 1 \right) (\sin(x)^2 - \sin(x) + 1)}{\sin(x)} \end{array}$$

## Other MuPAD Interfaces: Editor and Debugger

Besides the notebook interface, MuPAD provides separate interfaces for editing and debugging your code.

The MuPAD Editor is an interface for creating and editing your own procedures. The Editor automatically formats the code and, therefore, helps you avoid errors, or at least reduce their number. The main techniques are:

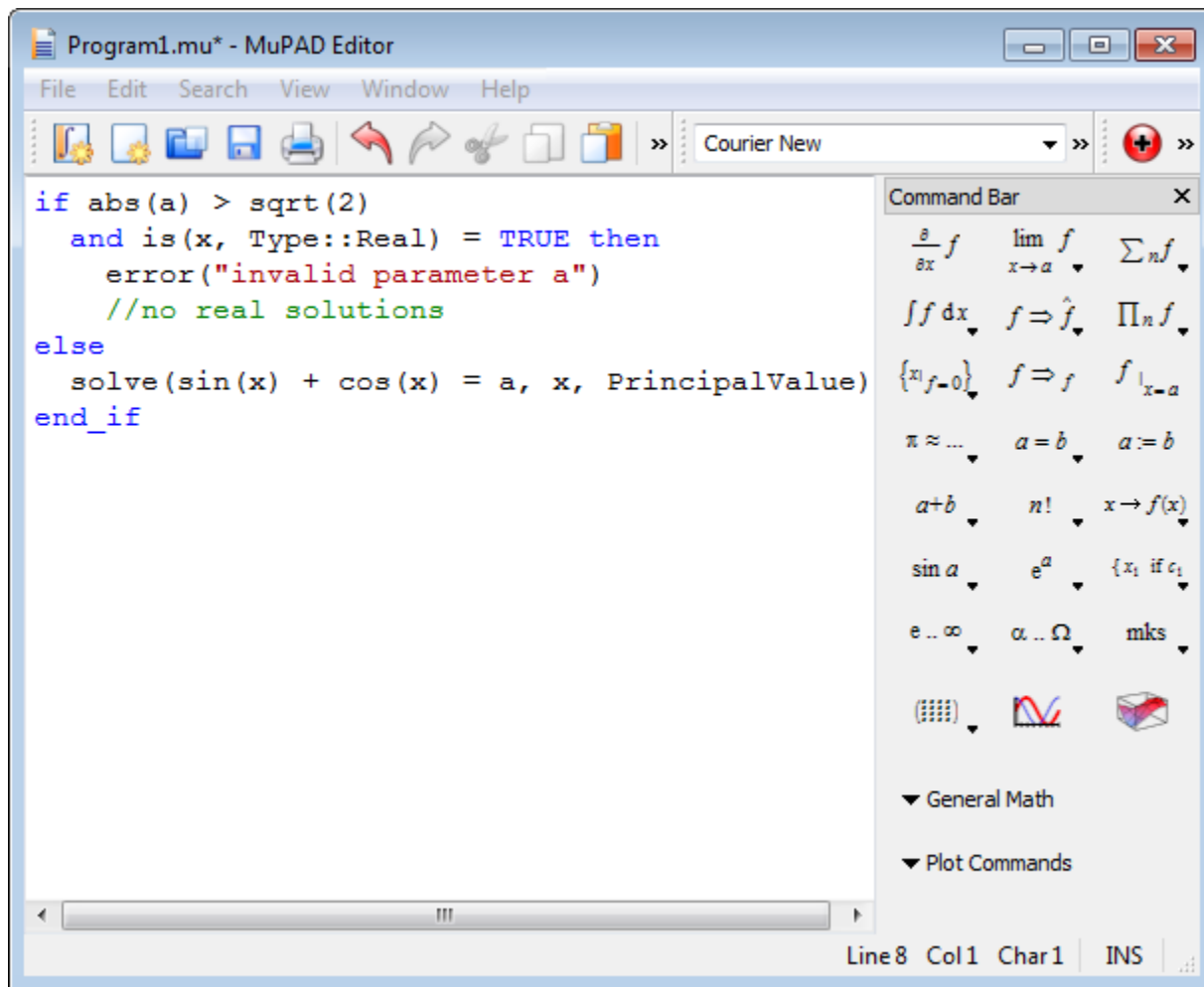
- Highlighting the MuPAD syntax elements, such as reserved words (for example, the names of the built-in functions), strings, comments, and so on
- Automatic indenting
- Using bookmarks to be able to find particular places in the code quickly

The Editor saves files in text format. By default, it uses the extension `.mu`. You can specify other extensions, for example, `.txt` or `.tst`. The extension `.mu` allows the Editor to recognize and open the files. Thus, if you want to open the files in the Editor later, save them using the extension `.mu`.

Alternatively, you can create and edit your code in any text editor or in the MATLAB Editor. Unlike the MATLAB Editor, the MuPAD Editor does not let you evaluate or debug code.

To open an existing MuPAD file with the extension `.mu` in the MuPAD Editor, double-click the file name. To start a new blank program file in the Editor, open the MuPAD welcome dialog box and click **New Editor**. For more ways to open the Editor window, see “Opening MuPAD Interfaces from MATLAB” on page 4-39.

After opening an existing file or creating a new file in the Editor, you can type mathematical expressions, commands and text as you would in a notebook. The Editor automatically highlights and indents your code.



For more information about the MuPAD Editor, see “Using the MuPAD Editor to Write New Functions” in the MuPAD documentation.

The MuPAD Debugger helps you find run-time errors in your code. This interface lets you:

- Execute your code step by step.



- Set breakpoints, including conditional breakpoints.
- Observe the values of the variables and expressions in each step.

To open the Debugger:

- 1** Open a new or existing MuPAD notebook. For instructions, see “Creating, Opening, and Saving MuPAD Notebooks” on page 4-11.
- 2** In the main menu of a notebook, select **Notebook > Debug**.
- 3** In the resulting dialog box, enter the procedure call that you want to debug.

Alternatively, use the `debug` function in the MuPAD notebook.

The screenshot shows the MuPAD Debugger interface for a file named 'factor.mu'. The main window displays the source code of a local procedure 'contains\_'. The code is as follows:

```

41 // Local procedure contains_
42
43 contains_ :=
44 proc(l, x)
45   local i, n;
46   begin
47     i := 1; n := nops(l);
48     while i <= n do
49       if specfunc::abs(l[i]-x) < 10^(-DIGITS) then
50         return(i)
51       end;
52       i := i + 1
53     end;
54     return(0)
55   end;
56
57 // local procedure round2zero
58 round2zero:=
59 proc(x, eps = 10^(-DIGITS))
60 begin

```

The Call Stack on the right shows the current procedure being executed is 'factor'. The Watch window at the bottom right displays the following data:

Expression	Value
args()	$x^2 - 1$
%	
p	$x^2 - 1$
_p_	NIL

The Output window at the bottom left is currently empty. The status bar at the bottom right indicates 'Mem 1 MB, T 0 s'.

The default layout of the Debugger window displays four panes:

- The main pane (top-left by default) displays the code that you debug. The Debugger only shows the code, but does not allow you to update it.
- The **Output** pane lets you type an expression and evaluate it anytime during the debugging process.
- The **Watch** pane shows values of the variables at each step during the debugging process.
- The **Call Stack** pane shows the names of the procedures that you debug.

You can close any pane, except for the main pane. If you close a pane, you can restore it again by selecting **View** and the name of the required pane. Using the **View** menu, you can also open the **Breakpoints** pane that shows the list of breakpoints in the code.

You cannot fix bugs directly in the Debugger window. If you work in the Debugger window and want to edit the code:

- 1 Open the file with the code in the Editor.

---

**Tip** If you did not yet save this code to a program file, display the code in a new Editor window by selecting **File > New Editor with Source**.

---

- 2 Close the Debugger if it is open.
- 3 Update the code in the Editor and save it.
- 4 Open a notebook.
- 5 In the notebook, select **Notebook > Read Commands** from the main menu and navigate to your updated file.
- 6 Open the Debugger from the notebook.

For details about the MuPAD Debugger, see “Tracing Errors with the MuPAD Debugger” in the MuPAD documentation.

## Notebook Files and Program Files

The two main types of files in MuPAD are:

- Notebook files, or notebooks
- Program files

A *notebook file* has the extension `.mn` and lets you store the result of the work performed in the notebook interface. A notebook file can contain text, graphics, and any MuPAD commands and their outputs. A notebook file can also contain procedures and functions.

By default, a notebook file opens in the notebook interface. Creating a new notebook or opening an existing one does not automatically start the MuPAD engine. This means that although you can see the results of computations as they were saved, MuPAD does not remember evaluating them. (The “MuPAD Workspace” is empty.) You can evaluate any or all commands after opening a notebook.

A *program file* is a text file that contains any code snippet that you want to store separately from other computations. Saving a code snippet as a program file can be very helpful when you want to use the code in several notebooks. Typically, a program file contains a single procedure, but it also can contain one or more procedures or functions, assignments, statements, tests, or any other valid MuPAD code.

---

**Tip** If you use a program file to store a procedure, MuPAD does not require the name of that program file to match the name of a procedure.

---

The most common approach is to write a procedure and save it as a program file with the extension `.mu`. This extension allows the MuPAD Editor to recognize and open the file later. Nevertheless, a program file is just a text file. You can save a program file with any extension that you use for regular text files.

To evaluate the commands from a program file, you must execute a program file in a notebook. For details about executing program files, see “Reading MuPAD Procedures” on page 4-49.

## Source Code of the MuPAD Library Functions

You can display the source code of the MuPAD built-in library functions. If you work in the MuPAD notebook interface, enter `expose (name)`, where `name` is the library function name. The notebook interface displays the code as plain text with the original line breaks and indentations.

You can also display the code of a MuPAD library function in the MATLAB Command Window. To do this, use the `evalin` or `feval` function to call the MuPAD `expose` function:

```
sprintf(char(feval(symengine, 'expose', 'numlib::tau')))  
  
ans =  
  
proc(a)  
  name numlib::tau;  
begin  
  if args(0) <> 1 then  
    error("wrong number of arguments")  
  else  
    if not testtype(a, Type::Numeric) then  
      return(procname(args()))  
    else  
      if domtype(a) <> DOM_INT then  
        error("argument must be an integer")  
      end_if  
    end_if  
  end_if;  
  numlib::numdivisors(a)  
end_proc
```

MuPAD also includes kernel functions written in C++. You cannot access the source the code of these functions.

## Integration of MuPAD and MATLAB

### In this section...

“Differences Between MATLAB and MuPAD Syntax” on page 4-28

“Copying Variables and Expressions Between the MATLAB Workspace and MuPAD Notebooks” on page 4-32

“Reserved Variable and Function Names” on page 4-35

“Opening MuPAD Interfaces from MATLAB” on page 4-39

“Calling Built-In MuPAD Functions from the MATLAB Command Window” on page 4-40

“Computing in the MATLAB Command Window vs. the MuPAD Notebook Interface” on page 4-43

“Using Your Own MuPAD Procedures” on page 4-48

“Clearing Assumptions and Resetting the Symbolic Engine” on page 4-51

### Differences Between MATLAB and MuPAD Syntax

There are several differences between MATLAB and MuPAD syntax. Be aware of which interface you are using in order to use the correct syntax:

- Use MATLAB syntax in the MATLAB workspace, *except* for the functions `evalin(symengine, ...)` and `feval(symengine, ...)`, which use MuPAD syntax.
- Use MuPAD syntax in MuPAD notebooks.

You must define MATLAB variables before using them. However, every expression entered in a MuPAD notebook is assumed to be a combination of symbolic variables unless otherwise defined. This means that you must be especially careful when working in MuPAD notebooks, since fewer of your typos cause syntax errors.

This table lists common tasks, meaning commands or functions, and how they differ in MATLAB and MuPAD syntax.

### Common Tasks in MATLAB and MuPAD Syntax

Task	MATLAB Syntax	MuPAD Syntax
Assignment	=	:=
List variables	whos	anames(All, User)
Numerical value of expression	double( <i>expression</i> )	float( <i>expression</i> )
Suppress output	;	:
Enter matrix	[x11,x12,x13; x21,x22,x23]	matrix([[x11,x12,x13], [x21,x22,x23]])
{a,b,c}	cell array	set
Linear algebra commands	Nothing extra needed	linalg:: prefix, or use(linalg)
Autocompletion	<b>Tab</b>	<b>Ctrl+space bar</b>
Equality, inequality comparison	==, ~=	=, <>

The next table lists differences between MATLAB expressions and MuPAD expressions.

### MATLAB vs. MuPAD Expressions

MATLAB Expression	MuPAD Expression
Inf	infinity
pi	PI
i	I
NaN	undefined
fix	trunc
log	ln
asin	arcsin

**MATLAB vs. MuPAD Expressions (Continued)**

<b>MATLAB Expression</b>	<b>MuPAD Expression</b>
acos	arccos
atan	arctan
asinh	arcsinh
acosh	arccosh
atanh	arctanh
acsc	arccsc
asec	arcsec
acot	arccot
acsch	arccsch
asech	arcsech
acoth	arccoth
besselj	besselJ
bessely	besselY
besseli	besselI
besselk	besselK
lambertw	lambertW
sinint	Si
cosint	Ci
eulergamma	EULER
conj	conjugate
catalan	CATALAN
laplace	transform::laplace
ilaplace	transform::invlaplace
ztrans	transform::ztrans
iztrans	transform::invztrans



The MuPAD definition of Fourier transform and inverse Fourier transform differ from their Symbolic Math Toolbox counterparts by the sign of the exponent:

	<b>Symbolic Math Toolbox Definition</b>	<b>MuPAD Definition</b>
Fourier transform	$F[f](w) = \int_{-\infty}^{\infty} f(x)e^{-iwx} dx.$ <p>Corresponding code is:</p> $F = \text{fourier}(f)$	$F[f](w) = \int_{-\infty}^{\infty} f(x)e^{iwx} dx.$ <p>Corresponding code is:</p> $F := \text{transform}::\text{fourier}(f,x,w)$
Inverse Fourier transform	$F^{-1}[f](x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(w)e^{iwx} dw.$ <p>Corresponding code is:</p> $\text{Finv} = \text{ifourier}(f)$	$F^{-1}[f](x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(w)e^{-iwx} dw.$ <p>Corresponding code is:</p> $\text{Finv} := \text{transform}::\text{invfourier}(f,w,x)$

The MuPAD definition of exponential integral differs from the Symbolic Math Toolbox counterpart.

	<b>Symbolic Math Toolbox Definition</b>	<b>MuPAD Definition</b>
Exponential integral	$\text{expint}(x) = -\text{Ei}(-x) =$ $\int_x^{\infty} \frac{\exp(-t)}{t} dt \text{ for } x > 0 =$ $\text{Ei}(1, x).$	$\text{Ei}(x) = \int_{-\infty}^x \frac{e^t}{t} dt \text{ for } x < 0.$ $\text{Ei}(n, x) = \int_1^{\infty} \frac{\exp(-xt)}{t^n} dt.$ <p>The definitions of Ei extend to the complex plane, with a branch cut along the negative real axis.</p>

## Copying Variables and Expressions Between the MATLAB Workspace and MuPAD Notebooks

You can copy a variable in a MuPAD notebook to a variable in the MATLAB workspace using a MATLAB command. Similarly, you can copy a variable or symbolic expression in the MATLAB workspace to a variable in a MuPAD notebook using a MATLAB command. To do either assignment, you need to know the handle to the MuPAD notebook you want to address.

The only way to assign variables between a MuPAD notebook and the MATLAB workspace is to open the notebook using the following syntax:

```
nb = mupad;
```

You can use any variable name for the handle `nb`. To open an existing notebook file, use the following syntax:

```
nb = mupad(file_name);
```

Here *file\_name* must be a full path unless the notebook is in the current folder. The handle `nb` is used only for communication between the MATLAB workspace and the MuPAD notebook.

- To copy a symbolic variable in the MATLAB workspace to a variable in the MuPAD notebook engine with the same name, enter this command in the MATLAB Command Window:

```
setVar(notebook_handle,variable)
```

For example, if `nb` is the handle to the notebook and `z` is the variable, enter:

```
setVar(nb,z)
```

There is no indication in the MuPAD notebook that variable `z` exists. Check that it exists by entering `z` in an input area of the notebook, or by entering the command `anames(All, User)` in the notebook.

- To assign a symbolic expression to a variable in a MuPAD notebook, enter:

```
setVar(notebook_handle, 'variable', expression)
```

at the MATLAB command line. For example, if `nb` is the handle to the notebook, `exp(x) - sin(x)` is the expression, and `z` is the variable, enter:

```
syms x
setVar(nb, 'z', exp(x) - sin(x))
```

For this type of assignment, `x` must be a symbolic variable in the MATLAB workspace.

Again, there is no indication in the MuPAD notebook that variable `z` exists. Check that it exists by entering `z` in an input area of the notebook, or by entering the command `anames(All, User)` in the notebook.

- To copy a symbolic variable in a MuPAD notebook to a variable in the MATLAB workspace, enter in the MATLAB Command Window:

```
MATLABvar = getVar(notebook_handle, 'variable');
```

For example, if `nb` is the handle to the notebook, `z` is the variable in the MuPAD notebook, and `u` is the variable in the MATLAB workspace, enter:

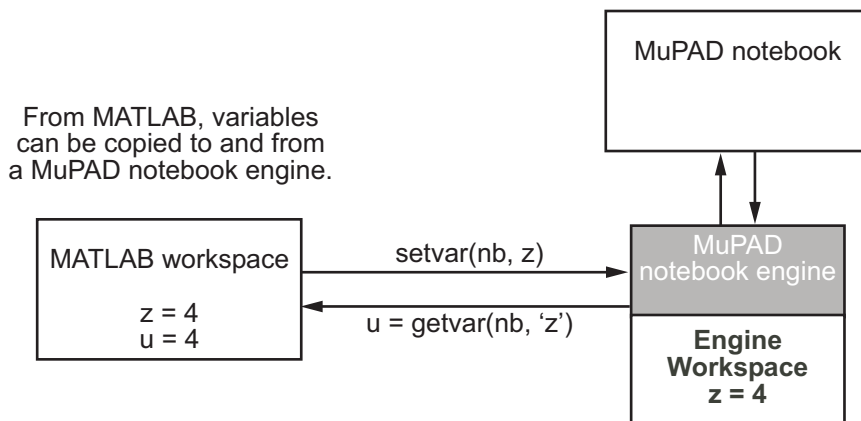
```
u = getVar(nb, 'z')
```

Communication between the MATLAB workspace and the MuPAD notebook occurs in the notebook's engine. Therefore, variable `z` must be synchronized into the notebook's MuPAD engine before using `getVar`, and not merely displayed in the notebook. If you try to use `getVar` to copy an undefined variable `z` in the MuPAD engine, the resulting MATLAB variable `u` is empty. For details, see “Synchronizing a Notebook and its Engine” on page 4-19.

---

**Tip** Do all copying and assignments from the MATLAB workspace, not from a MuPAD notebook.

---



### Copying and Pasting Using the System Clipboard

You can also copy and paste between notebooks and the MATLAB workspace using standard editing commands. If you copy a result in a MuPAD notebook to the system clipboard, you might get the text associated with the expression, or a picture, depending on your operating system and application support.

For example, consider this MuPAD expression:

$$\left[ \begin{array}{l} y := \exp(x)/(1 + x^2) \\ \frac{e^x}{x^2 + 1} \end{array} \right]$$

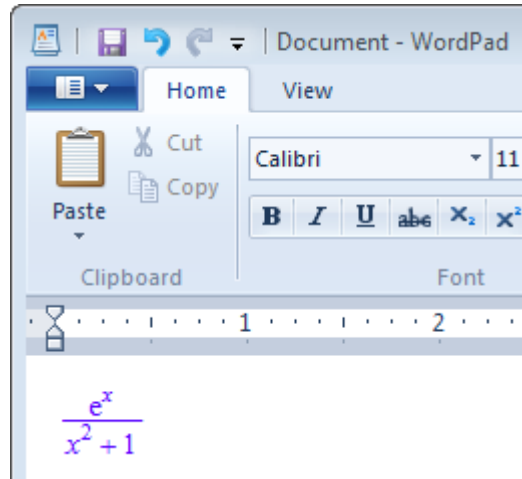
Select the output with the mouse and copy it to the clipboard:

$$\left[ \begin{array}{l} y := \exp(x)/(1 + x^2) \\ \frac{e^x}{x^2 + 1} \end{array} \right]$$

Paste this into the MATLAB workspace. The result is text:

$$\exp(x)/(x^2 + 1)$$

If you paste it into Microsoft® WordPad on a Windows® system, the result is a picture.



## Reserved Variable and Function Names

Both MATLAB and MuPAD have their own reserved keywords, such as function names, special values, and names of mathematical constants. Using reserved keywords as variable or function names can result in errors. If a variable name or a function name is a reserved keyword in one or both interfaces, you can get errors or incorrect results. If you work in one interface and a name is a reserved keyword in another interface, the error and warning messages are produced by the interface you work in. These messages can specify the cause of the problem incorrectly.

---

**Tip** The best approach is to avoid using reserved keywords as variable or function names, especially if you use both interfaces.

---

## Conflicts Caused by MuPAD Function Names

In MuPAD, function names are protected. Normally, the system does not let you redefine a standard function or use its name as a variable. (To be able to modify a standard MuPAD function you must first remove its protection.)

Even when you work in the MATLAB Command Window, the MuPAD engine handles symbolic computations. Therefore, MuPAD function names are reserved keywords in this case. Using a MuPAD function name while performing symbolic computations in the MATLAB Command Window can lead to incorrect results:

```
solve('D - 10')
```

The warning message does not indicate the real cause of the problem:

```
Warning: 1 equations in 0 variables.  
Warning: Explicit solution could not be found.  
> In solve at 81  
  
ans =  
[ empty sym ]
```

To fix this issue, use a variable name that is not a reserved keyword:

```
solve('x - 10')
```

```
ans =  
10
```

Alternatively, use the `syms` function to declare `D` as a symbolic variable. Then call the symbolic solver without using quotes:

```
syms D;  
solve(D - 10)
```

In this case, the toolbox replaces `D` with some other variable name before passing the expression to the MuPAD engine:

```
ans =  
10
```

To list all MuPAD function names, enter this command in the MATLAB Command Window:

```
evalin(symengine, 'anames()')
```

If you work in a MuPAD notebook, enter:

```
anames()
```

## Conflicts Caused by Syntax Conversions

Many mathematical functions, constants, and special values use different syntaxes in MATLAB and MuPAD. See the table MATLAB® vs. MuPAD® Expressions on page 4-29 for these expressions. When you use such functions, constants, or special values in the MATLAB Command Window, the toolbox internally converts the original MATLAB expression to the corresponding MuPAD expression and passes the converted expression to the MuPAD engine. When the toolbox gets the results of computations, it converts the MuPAD expressions in these results to the MATLAB expressions.

Now suppose you write a procedure in MuPAD, and then call it from the MATLAB Command Window. For example, write this MuPAD procedure that uses the local variable `i`:

```
myProc := proc(n)
  local x;
  local i;
begin
  x := 0;
  for i from 0 to n do
    x := x + ((-1)^i)/(2*i + 1);
  end_for;
  x := x*4;
end_proc;
```

Save the procedure as `myProc.mu` in the folder `C:/MuPAD`. Execute it in the MATLAB Command Window:

```
eng=symengine;
eng.feval('read', 'C:/MuPAD/myProc.mu');
```

This command results in the following error:

```
?? Error using ==> mupadengine.mupadengine>mupadengine.feval at 141
Error: Identifier expected (check aliases) [proc];

during evaluation of 'read';
```

```
while reading file 'C:/MuPAD/myProc.mu'
```

In this example, using lowercase `i` as a variable causes the problem. Executing procedure `myProc` in a MuPAD notebook does not cause an error because MuPAD uses uppercase `I` to represent an imaginary unit. MATLAB uses `i` to represent an imaginary unit, but MATLAB lets you redefine this variable. (The name `i` is not protected.)

The problem appears when you try to execute procedure `myProc` in the MATLAB Command Window. As with all other symbolic expressions, the toolbox converts lowercase `i` to uppercase `I`, and then passes the converted expression to the MuPAD engine. This causes an error because `I` is a protected function in MuPAD, and the procedure is trying to assign new values to the protected function. To fix this problem, use another variable name instead of `i`. For example, rewrite the procedure using the variable name `k`:

```
myProc := proc(n)
  local x;
  local k;
begin
  x := 0;
  for k from 0 to n do
    x := x + ((-1)^k)/(2*k + 1);
  end_for;
  x := x^4;
end_proc;
```

Alternatively, you can use the `Plain` option as follows:

```
eng=symengine;
eng.feval('read', 'C:/MuPAD/myProc.mu', 'Plain');
```

---

**Tip** If your MuPAD procedure uses the `log` function to compute the logarithm to an arbitrary base, use the `Plain` option. Without this option, the toolbox treats `log` as the natural logarithm, and therefore, converts it to `ln` before passing the procedure to the MuPAD engine.

---



## Opening MuPAD Interfaces from MATLAB

You can open an existing MuPAD notebook, a program file with the extension `.mu`, a help file with the extension `.muphlp`, or a graphic file (`.xvc` or `.xvz`) by double-clicking the file name. The system opens the file in the appropriate interface. Alternatively, use the `mupad` function in the MATLAB Command Window and specify the path to the file:

```
mupad('H:\Documents\Notes\myProc.mu')
```

If you perform computations in both interfaces, do not forget to use handles to notebooks. The toolbox uses this handle for communication between the MATLAB workspace and the MuPAD notebook. If you use the MATLAB Command Window only to open a notebook, and then perform all your computations in that notebook, you can skip using a handle. Also, you can skip using a handle when opening program files, help files, and graphic files.

Symbolic Math Toolbox also provides these functions for opening MuPAD files in the interfaces with which these files are associated:

- `openmn` opens a notebook in the notebook interface.
- `openmu` opens a program file with the extension `.mu` in the MuPAD Editor.
- `openmuphlp` opens a help file in the MuPAD Help Browser.
- `openxvc` opens an `XVC` graphics file in the MuPAD Graphics window.
- `openxvz` opens an `XVZ` graphics file in the MuPAD Graphics window.

These functions accomplish the same task as the `mupad` function. The system calls these functions when you double-click the file name.

You also can use the MuPAD welcome dialog box to access MuPAD interfaces. This dialog box lets you open existing files as well as create new empty notebooks and program files. To open this dialog box, type `mupadwelcome` in the MATLAB Command Window. For details, see “Creating, Opening, and Saving MuPAD Notebooks” on page 4-11.

Once you open any MuPAD interface, you can use the main menu or the toolbar in that interface to open other interfaces or additional files. See “Getting Started” in the MuPAD documentation.

---

**Note** You cannot access the MuPAD Debugger interface from the MATLAB Command Window.

---

For information about the Debugger, see “Other MuPAD Interfaces: Editor and Debugger” on page 4-21.

## Calling Built-In MuPAD Functions from the MATLAB Command Window

To access MuPAD functions and procedures at the MATLAB command line, use `evalin(symengine, ...)` or `feval(symengine, ...)`. These functions are designed to work like the existing MATLAB `evalin` and `feval` functions.

---

**Note** You cannot use `evalin` and `feval` to access the MuPAD function `log` that represents the logarithm to an arbitrary base. Instead, both commands evaluate the natural logarithm.

---

### **evalin**

For `evalin`, the syntax is

```
y = evalin(symengine, 'MuPAD_Expression');
```

Use `evalin` when you want to perform computations in the MuPAD language, while working in the MATLAB workspace. For example, to make a three-element symbolic vector of the `sin(kx)` function, `k = 1 to 3`, enter:

```
y = evalin(symengine, 'sin(k*x) $ k = 1..3')
```

The result is:

```
y =  
sin(x), sin(2*x), sin(3*x)
```

**feval**

For evaluating a MuPAD function, you can also use the `feval` function. `feval` has a different syntax than `evalin`, so it can be simpler to use. The syntax is:

```
y = feval(symengine, 'MuPAD_Function', x1, ..., xn);
```

*MuPAD\_Function* represents the name of a MuPAD function. The arguments  $x_1, \dots, x_n$  must be symbolic variables, numbers, or strings. For example, to find the tenth element in the Fibonacci sequence, enter:

```
z = feval(symengine, 'numlib::fibonacci', 10)
```

The result is:

```
z =
55
```

The next example compares the use of a symbolic solution of an equation to the solution returned by the MuPAD numeric `fsolve` function near the point  $x = 3$ . For information on this function, enter `doc(symengine, 'numeric::fsolve')` at the MATLAB command line. The symbolic solver

```
syms x
f = sin(x^2);
solve(f)
```

returns

```
ans =
0
0
```

The numeric solver `fsolve`

```
feval(symengine, 'numeric::fsolve', f, 'x=3')
```

returns

```
ans =
[x = 3.0699801238394654654386548746677946]
```

As you might expect, the answer is the numerical value of  $\sqrt{3\pi}$ . The setting of MATLAB format does not affect the display; it is the full returned value from the MuPAD 'numeric::fsolve' function.

### Using evalin vs. feval

The `evalin(symengine, ...)` function causes the MuPAD engine to evaluate a string. Since the MuPAD engine workspace is generally empty, expressions returned by `evalin(symengine, ...)` are not simplified or evaluated according to their definitions in the MATLAB workspace. For example:

```
syms x
y = x^2;
evalin(symengine, 'cos(y)')

ans =
cos(y)
```

Variable `y` is not expressed in terms of `x` because `y` is unknown to the MuPAD engine workspace.

In contrast, `feval(symengine, ...)` can pass symbolic variables that exist in the MATLAB workspace, and these variables are evaluated before being processed in the MuPAD engine. For example:

```
syms x
y = x^2;
feval(symengine, 'cos', y)

ans =
cos(x^2)
```

### Floating-Point Arguments of evalin and feval

By default, MuPAD performs all computations in an exact form. When you call the `evalin` or `feval` function with floating-point numbers as arguments, the toolbox converts these arguments to rational numbers before passing them to MuPAD. For example, when you calculate the incomplete gamma function, the result is the following symbolic expression:

```
y = feval(symengine, 'igamma', 0.1, 2.5)
```

```
y =
igamma(1/10, 5/2)
```

To approximate the result numerically with double precision, use the `double` function:

```
format long;
double(y)

ans =
0.028005841168289
```

Alternatively, use quotes to prevent the conversion of floating-point arguments to rational numbers. (The toolbox treats arguments enclosed in quotes as strings.) When MuPAD performs arithmetic operations on numbers involving at least one floating-point number, it automatically switches to numeric computations and returns a floating-point result:

```
feval(symengine,'igamma', '0.1', 2.5)

ans =
0.028005841168289177028337498391181
```

For further computations, set the format for displaying outputs back to short:

```
format short;
```

## Computing in the MATLAB Command Window vs. the MuPAD Notebook Interface

When computing with Symbolic Math Toolbox, you can choose to work in the MATLAB Command Window or in the MuPAD notebook interface. The MuPAD engine that performs all symbolic computations is the same for both interfaces. The choice of the interface mostly depends on your preferences.

Working in the MATLAB Command Window lets you perform all symbolic computations using the familiar MATLAB language. The toolbox contains hundreds of MATLAB symbolic functions for common tasks, such as differentiation, integration, simplification, transforms, and equation solving. If your task requires a few specialized symbolic functions not available directly from this interface, you can use `evalin` or `feval` to call MuPAD

functions. See “Calling Built-In MuPAD Functions from the MATLAB Command Window” on page 4-40.

Working in the MATLAB Command Window is recommended if you use other toolboxes or MATLAB as a primary tool for your current task and only want to embed a few symbolic computations in your code.

Working in the MuPAD notebook interface requires you to use the MuPAD language, which is optimized for symbolic computations. In addition to solving common mathematical problems, MuPAD functions cover specialized areas, such as number theory and combinatorics. Also, for some computations the performance is better in the MuPAD notebook interface than in the MATLAB Command Window. The reason is that the engine returns the results in the MuPAD language. To display them in the MATLAB Command Window, the toolbox translates the results to the MATLAB language.

Working in the MuPAD notebook interface is recommended when your task mainly consists of symbolic computations. It is also recommended if you want to document your work and results, for example, embed graphics, animations, and descriptive text with your calculations. Symbolic results computed in the MuPAD notebook interface can be accessed from the MATLAB Command Window, which helps you integrate symbolic results into larger MATLAB applications.

Learning the MuPAD language and using the MuPAD notebook interface for your symbolic computations provides the following benefits.

### **Results Displayed in Typeset Math**

By default, the MuPAD notebook interface displays results in typeset math making them look very similar to what you see in mathematical books. In addition, the notebook interface

- Uses standard mathematical notations in output expressions.
- Uses abbreviations to make a long output expression with common subexpressions shorter and easier to read. You can disable abbreviations.
- Wraps long output expressions, including long numbers, fractions and matrices, to make them fit the page. If you resize the notebook window,

MuPAD automatically adjusts outputs. You can disable wrapping of output expressions.

Alternatively, you can display pretty-printed outputs similar to those that you get in the MATLAB Command Window when you use `pretty`. You can also display outputs as plain text. For details, see “Using Different Output Modes” in the MuPAD documentation.

In a MuPAD notebook, you can copy or move output expressions, including expressions in typeset math, to any input or text region within the notebook, or to another notebook. If you copy or move an output expression to an input region, the expression appears as valid MuPAD input.

## Graphics and Animations

The MuPAD notebook interface provides very extensive graphic capabilities to help you visualize your problem and display results. Here you can create a wide variety of plots, including:

- 2-D and 3-D plots in Cartesian, polar, and spherical coordinates
- Plots of continuous and piecewise functions and functions with singularities
- Plots of discrete data sets
- Surfaces and volumes by using predefined functions
- Turtle graphics and Lindenmayer systems
- Animated 2-D and 3-D plots

Graphics in the MuPAD notebook interface is interactive. You can explore and edit plots, for example:

- Change colors, fonts, legends, axes appearance, grid lines, tick marks, line, and marker styles.
- Zoom and rotate plots without reevaluating them.
- Display coordinates of any point on the plot.

After you create and customize a plot, you can export it to various vector and bitmap file formats, including EPS, SVG, PDF, PNG, GIF, BMP, TIFF, and

JPEG. The set of the file formats available for exporting graphics from a MuPAD notebook can be limited by your operating system.

You can export animations as AVI files or as sequences of static images.

### **More Functionality in Specialized Mathematical Areas**

While both MATLAB and MuPAD interfaces provide functions for performing common mathematical tasks, the notebook interface also provides functions that cover many specialized areas. For example, MuPAD libraries support computations in the following areas:

- Combinatorics
- Graph theory
- Gröbner bases
- Linear optimization
- Polynomial algebra
- Number theory
- Statistics

MuPAD libraries also provide large collections of functions for working with ordinary differential equations, integral and discrete transforms, linear algebra, and more.

### **More Options for Common Symbolic Functions**

Most functions for performing common mathematical computations are available in both MATLAB and MuPAD interfaces. For example, you can solve equations and systems of equations using `solve`, simplify expressions using `simplify`, compute integrals using `int`, and compute limits using `limit`. Note that although the function names are the same, the syntax of the function calls depends on the interface that you use.

Results of symbolic computations can be very long and complicated, especially because the toolbox assumes all values to be complex by default. For many symbolic functions you can use additional parameters and options to help you limit the number and complexity and also to control the form of returned



results. For example, `solve` accepts the `Real` option that lets you restrict all symbolic parameters of an equation to real numbers. It also accepts the `VectorFormat` option that you can use to get solutions of a system as a set of vectors.

Typically, the functions available in the notebook interface accept more options than the analogous functions in the MATLAB Command Window. For example, in the notebook interface you can use the `VectorFormat` option. This option is not directly available for the `solve` function called in the MATLAB Command Window.

### **Possibility to Expand Existing Functionality**

The MuPAD programming language supports multiple programming styles, including imperative, functional, and object-oriented programming. The system includes a few basic functions written in C++, but the majority of the MuPAD built-in functionality is implemented as library functions written in the MuPAD language. You can extend the built-in functionality by writing custom symbolic functions and libraries, defining new function environments, data types, and operations on them in the MuPAD language. MuPAD implements data types as domains (classes). Domains with similar mathematical structure typically belong to a category. Domains and categories allow you to use the concepts of inheritance, overloading methods and operators. The language also uses axioms to state properties of domains and categories.

“Programming Fundamentals” in the MuPAD documentation contains the basic information to get you started with object-oriented programming in MuPAD. For more information, see “Language Extensibility” in the MuPAD documentation.

MuPAD also lets you load dynamic modules anytime during a MuPAD session. Dynamic modules consist of machine code compiled from C/C++ code, and can contain libraries compiled from other programming languages. See “Dynamic Modules” in the MuPAD documentation.

## Using Your Own MuPAD Procedures

### Writing MuPAD Procedures

A MuPAD procedure is a text file that you can write in any text editor or in the MATLAB Editor. The recommended practice is to use the MuPAD Editor.

To define a procedure, use the `proc` function. Enclose the code in the `begin` and `end_proc` functions:

```
myProcedure:= proc(n)
begin
  if n = 1 or n = 0 then
    1
  else
    n * myProc(n - 1)
  end_if;
end_proc:
```

By default, a MuPAD procedure returns the result of the last executed command. You can force a procedure to return another result by using `return`. In both cases, a procedure returns only one result. To get multiple results from a procedure, use the `print` function or data structures inside the procedure.

- If you just want to display the results, and do not need to use them in further computations, use the `print` function. With `print`, your procedure still returns one result, but prints intermediate results on screen. For example, this procedure prints the value of its argument in each call:

```
myProcPrint:= proc(n)
begin
  print(n);
  if n = 0 or n = 1 then
    return(1);
  end_if;
  n * myProcPrint(n - 1);
end_proc:
```

- If you want to use multiple results of a procedure, use ordered data structures, such as lists or matrices as return values. In this case, the result of the last executed command is technically one object, but it can

contain more than one value. For example, this procedure returns the list of two entries:

```
myProcSort:= proc(a, b)
begin
  if a < b then
    [a, b]
  else
    [b, a]
  end_if;
end_proc;
```

Avoid using unordered data structures, such as sequences and sets, to return multiple results of a procedure. The order of the entries in these structures can change unpredictably.

When you save the procedure, it is recommended to use the extension `.mu`. For details, see “Notebook Files and Program Files” on page 4-26. The name of the file can differ from the name of the procedure. Also, you can save multiple procedures in one file.

## Before Calling a Procedure

To be able to call a procedure, you must first execute it in a notebook. If you write a procedure in the same notebook, simply evaluate the input region that contains the procedure. If you write a procedure in a separate file, you must *read* the procedure into a notebook. *Reading* a procedure means finding and executing the procedure.

**Reading MuPAD Procedures.** If you work in the MuPAD notebook interface and create a separate program file that contains a procedure, use one of the following methods to execute the procedure in a notebook. The first approach is to select **Notebook > Read Commands** from the main menu.

Alternatively, you can use the `read` function. The function call `read(filename)` searches for the program file in this order:

- 1 Folders specified by the environment variable `READPATH`
- 2 `filename` regarded as an absolute path

3 Current folder (depends on the operating system)

4 Folders specified by the environment variable LIBPATH

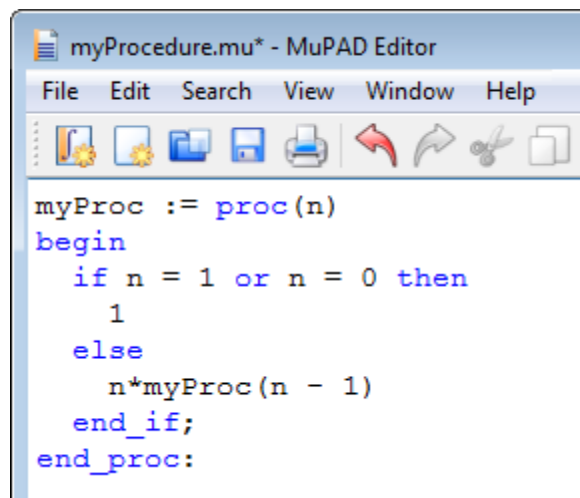
If you want to call the procedure from the MATLAB Command Window, you still need to execute that procedure before calling it. See “Calling Your Own MuPAD Procedures” on page 4-50.

**Using Startup Commands and Scripts.** Alternatively, you can add a MuPAD procedure to startup commands of a particular notebook. This method lets you execute the procedure every time you start a notebook engine. Startup commands are executed silently, without any visible outputs in the notebook. You can copy the procedure to the dialog box that specifies startup commands or attach the procedure as a startup script. For information, see “Hiding Code Lines” in the MuPAD documentation.

## Calling Your Own MuPAD Procedures

You can extend the functionality available in the toolbox by writing your own procedures in the MuPAD language. This section explains how to call such procedures at the MATLAB Command Window.

Suppose you wrote the `myProc` procedure that computes the factorial of a nonnegative integer.



```
myProcedure.mu* - MuPAD Editor
File Edit Search View Window Help
[Icons: Open, Save, Print, Undo, Redo, Cut, Paste]
myProc := proc (n)
begin
  if n = 1 or n = 0 then
    1
  else
    n*myProc(n - 1)
  end_if;
end_proc;
```

Save the procedure as a file with the extension `.mu`. For example, save the procedure as `myProcedure.mu` in the folder `C:/MuPAD`.

Now, switch to the MATLAB Command Window. Before calling the procedure at the MATLAB command line, enter:

```
eng=symengine;  
eng.feval('read','C:/MuPAD/myProcedure.mu');
```

The `read` command reads and executes the `myProcedure.mu` file in MuPAD. After that, you can call the `myProc` procedure with any valid parameter. For example, compute the factorial of 15:

```
eng.feval('myProc', 15)
```

```
ans =  
1307674368000
```

## Clearing Assumptions and Resetting the Symbolic Engine

The symbolic engine workspace associated with the MATLAB workspace is usually empty. The MATLAB workspace tracks the values of symbolic variables, and passes them to the symbolic engine for evaluation as necessary. However, the symbolic engine workspace contains all assumptions you make about symbolic variables, such as whether a variable is real or positive. These assumptions can affect solutions to equations, simplifications, and transformations, as explained in “Examples of the Effect of Assumptions” on page 4-54.

---

**Note** These commands

```
syms x  
x = sym('x')  
clear x
```

clear any existing value of `x` in the MATLAB workspace, but do not clear assumptions about `x` in the symbolic engine workspace.

---

If you make an assumption about the nature of a variable, for example, using the commands

```
syms x real
```

or

```
syms x positive
```

then clearing the variable `x` from the MATLAB workspace does not clear the assumption from the symbolic engine workspace. To clear the assumption, enter the command

```
syms x clear
```

For details, see “Checking a Variable’s Assumptions” on page 4-53 and “Examples of the Effect of Assumptions” on page 4-54.

If you reset the symbolic engine by entering the command

```
reset(symengine)
```

or if you change symbolic engines with the `symengine` command, MATLAB no longer recognizes any symbolic variables that exist in the MATLAB workspace. Clear the variables with the `clear` command, or renew them with the `syms` or `sym` command.

This example shows how the MATLAB workspace and the symbolic engine workspace respond to a sequence of commands.

Step	Command	MATLAB Workspace	MuPAD Engine Workspace
1	<code>syms x positive</code>	<code>x</code>	<code>x is positive</code>
2	<code>clear x</code>	empty	<code>x is positive</code>
3	<code>syms x</code>	<code>x</code>	<code>x is positive</code>
4	<code>syms x clear</code>	<code>x</code>	empty

## Checking a Variable's Assumptions

To check whether a variable, say  $x$ , has any assumptions in the symbolic engine workspace associated with the MATLAB workspace, enter the following command in the MATLAB Command Window:

```
evalin(symengine, 'getprop(x)')
```

- If the returned answer is  $\mathbb{C}_-$ , there are no assumptions about the variable. ( $\mathbb{C}_-$  means it can be any complex number.)
- If the returned value is anything else, there are assumptions about the variable.

For example:

```
syms x real
evalin(symengine, 'getprop(x)')

ans =
R_

syms x clear

syms z
evalin(symengine, 'assume(z <> 0)')
evalin(symengine, 'getprop(z)')

ans =
C_ minus {0}

syms z clear
evalin(symengine, 'getprop(z)')

ans =
C_
```

For details about basic sets that can be returned as assumptions, enter:

```
doc(symengine, 'solvelib::BasicSet')
```

## Examples of the Effect of Assumptions

Assumptions can affect the answers returned by the `solve` function. They also can affect the results of simplifications. The only assumptions you can make using MATLAB commands are `real` or `positive`.

For example, consider what transpires when solving the equation  $x^3 = 1$ :

```
syms x
solve('x^3 = 1')

ans =
      1
 - (3^(1/2)*i)/2 - 1/2
  (3^(1/2)*i)/2 - 1/2

syms x real
solve('x^3 = 1')

ans =
1
```

However, clearing `x` does not change the underlying assumption that `x` is real:

```
clear x
syms x
solve('x^3 = 1')

ans =
1
```

Clearing `x` with `syms x clear` clears the assumption:

```
syms x clear
solve('x^3 = 1')

ans =
      1
 - (3^(1/2)*i)/2 - 1/2
  (3^(1/2)*i)/2 - 1/2
```

Using `evalin` or `feval`, you can make a variety of assumptions about an expression; see “Calling Built-In MuPAD Functions from the MATLAB



Command Window” on page 4-40. To clear all such assumptions, use the command `syms x clear`, as in this example:

```

evalin(symengine,'assume(a <> 0)');
evalin(symengine,'solve(a*x^2 + b*x + c = 0,x)')

ans =
{-(b - (b^2 - 4*a*c)^(1/2))/(2*a),...
 -(b + (b^2 - 4*a*c)^(1/2))/(2*a)}

syms a clear
evalin(symengine,'solve(a*x^2 + b*x + c = 0,x)')

ans =
piecewise([a <> 0, {-(b - (b^2 - 4*a*c)^(1/2))/(2*a),...
 -(b + (b^2 - 4*a*c)^(1/2))/(2*a)}],...
 [a = 0 and b <> 0, {-c/b}], [a = 0 and b = 0 and c = 0, C_],...
 [a = 0 and b = 0 and c <> 0, {}])

```

## Integrating Symbolic Computations in Other Toolboxes and Simulink

### In this section...

“Creating MATLAB Functions from MuPAD Expressions” on page 4-56

“Creating MATLAB Function Blocks from MuPAD Expressions” on page 4-59

“Creating Simscape Equations from MuPAD Expressions” on page 4-61

### Creating MATLAB Functions from MuPAD Expressions

Symbolic Math Toolbox lets you create a MATLAB function from a symbolic expression. A MATLAB function created from a symbolic expression accepts numeric arguments and evaluates the expression applied to the arguments. You can generate a function handle or a file that contains a MATLAB function. The generated file is available for use in any MATLAB calculation, independent of a license for Symbolic Math Toolbox functions.

If you work in the MATLAB Command Window, see “Generating MATLAB Functions” on page 3-135.

When you use the MuPAD notebook interface, all your symbolic expressions are written in the MuPAD language. To be able to create a MATLAB function from such expressions, you must convert it to the MATLAB language. There are two approaches for converting a MuPAD expression to the MATLAB language:

- Assign the MuPAD expression to a variable, and copy that variable from a notebook to the MATLAB workspace. This approach lets you create a function handle or a file that contains a MATLAB function. It also requires using a handle to the notebook.
- Generate MATLAB code from the MuPAD expression in a notebook. This approach limits your options to creating a file. You can skip creating a handle to the notebook.

The generated MATLAB function can depend on the approach that you chose. For example, code can be optimized differently or not optimized at all.

Suppose you want to create a MATLAB function from a symbolic matrix that converts spherical coordinates of any point to its Cartesian coordinates. First, open a MuPAD notebook with the handle `notebook_handle`:

```
notebook_handle = mupad;
```

In this notebook, create the symbolic matrix `S` that converts spherical coordinates to Cartesian coordinates:

```
x := r*sin(a)*cos(b):
y := r*sin(a)*sin(b):
z := r*cos(b):
S := matrix([x, y, z]):
```

Now convert matrix `S` to the MATLAB language. Choose the best approach for your task.

### Copying MuPAD Variables to the MATLAB Workspace

If your notebook has a handle, like `notebook_handle` in this example, you can copy variables from that notebook to the MATLAB workspace with the `getVar` function, and then create a MATLAB function. For example, to convert the symbolic matrix `S` to a MATLAB function:

- 1 Copy variable `S` to the MATLAB workspace:

```
S = getVar(notebook_handle, 'S')
```

Variable `S` and its value (the symbolic matrix) appear in the MATLAB workspace and in the MATLAB Command Window:

```
S =
  r*cos(b)*sin(a)
  r*sin(a)*sin(b)
  r*cos(b)
```

- 2 Use `matlabFunction` to create a MATLAB function from the symbolic matrix. To generate a MATLAB function handle, use `matlabFunction` without additional parameters:

```
h = matlabFunction(S)
```

```
h =
    @(a,b,r)[r.*cos(b).*sin(a);r.*sin(a).*sin(b);r.*cos(b)]
```

To generate a file containing the MATLAB function, use the parameter file and specify the path to the file and its name. For example, save the MATLAB function to the file `cartesian.m` in the current folder:

```
S = matlabFunction(S,'file', 'cartesian.m');
```

You can open and edit `cartesian.m` in the MATLAB Editor.

```
1 function S = cartesian(a,b,r)
2 %CARTESIAN
3 % S = CARTESIAN(A,B,R)
4
5 t2 = sin(a);
6 t3 = cos(b);
7 S = [r.*t2.*t3;r.*t2.*sin(b);r.*t3];
```

### Generating MATLAB Code in a MuPAD Notebook

To generate the MATLAB code from a MuPAD expression within the MuPAD notebook, use the `generate::MATLAB` function. Then, you can create a new file that contains an empty MATLAB function, copy the code, and paste it there. Alternatively, you can create a file with a MATLAB formatted string representing a MuPAD expression, and then add appropriate syntax to create a valid MATLAB function.

- 1 In the MuPAD notebook interface, use the `generate::MATLAB` function to generate MATLAB code from the MuPAD expression. Instead of printing the result on screen, use the `fprint` function to create a file and write the generated code to that file:

```
fprint(Unquoted, Text, "cartesian.m", generate::MATLAB(S)):
```

---

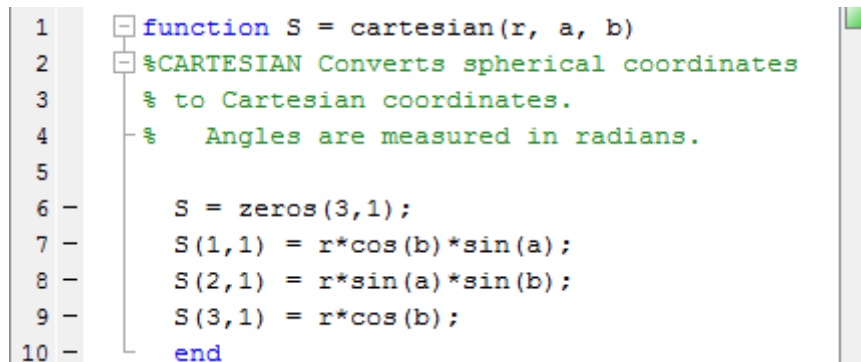
**Note** If the file with this name already exists, `fprint` replaces the contents of this file with the converted expression.

---

- 2** Open `cartesian.m`. It contains a MATLAB formatted string representing matrix `S`:

```
S = zeros(3,1);
S(1,1) = r*cos(b)*sin(a);
S(2,1) = r*sin(a)*sin(b);
S(3,1) = r*cos(b);
```

- 3** To convert this file to a valid MATLAB function, add the keywords `function` and `end`, the function name (must match the file name), input and output arguments, and comments:



```
1 function S = cartesian(r, a, b)
2 %CARTESIAN Converts spherical coordinates
3 % to Cartesian coordinates.
4 % Angles are measured in radians.
5
6 S = zeros(3,1);
7 S(1,1) = r*cos(b)*sin(a);
8 S(2,1) = r*sin(a)*sin(b);
9 S(3,1) = r*cos(b);
10 end
```

## Creating MATLAB Function Blocks from MuPAD Expressions

Symbolic Math Toolbox lets you create a MATLAB function block from a symbolic expression. The generated block is available for use in Simulink models, whether or not the computer that runs the simulations has a license for Symbolic Math Toolbox.

If you work in the MATLAB Command Window, see “Generating MATLAB Function Blocks” on page 3-140.

The MuPAD notebook interface does not provide a function for generating a block. Therefore, to be able to create a block from the MuPAD expression:

- 1** In a MuPAD notebook, assign that expression to a variable.

- 2 Use the `getVar` function to copy that variable from a notebook to the MATLAB workspace.

For details about these steps, see “Copying MuPAD Variables to the MATLAB Workspace” on page 4-57.

When the expression that you want to use for creating a MATLAB function block appears in the MATLAB workspace, use the `emlBlock` function to create a block from that expression.

For example, open a MuPAD notebook with the handle `notebook_handle`:

```
notebook_handle = mupad;
```

In this notebook, create the following symbolic expression:

```
r := sqrt(x^2 + y^2)
```

Use `getVar` to copy variable `r` to the MATLAB workspace:

```
r = getVar(notebook_handle, 'r')
```

Variable `r` and its value appear in the MATLAB workspace and in the MATLAB Command Window:

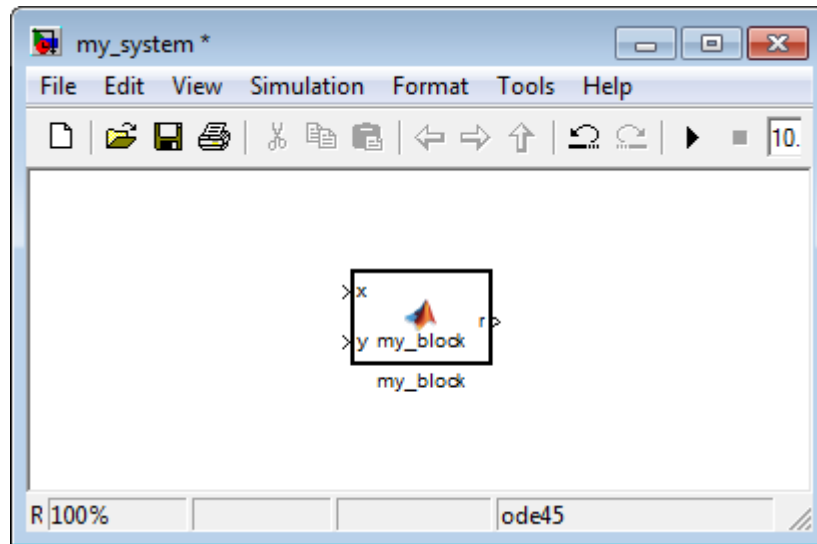
```
r =  
(x^2 + y^2)^(1/2)
```

Before generating a MATLAB Function block from the expression, create an empty model or open an existing one. For example, create and open the new model `my_system`:

```
new_system('my_system');  
open_system('my_system')
```

Since the variable and its value are in the MATLAB workspace, you can use `emlBlock` to generate the block `my_block`:

```
emlBlock('my_system/my_block', r)
```



You can open and edit the block in the MATLAB Editor. To open the block, select **Edit > Open Block** or use the context menu.

```
function r = my_block(x,y)
    %#codegen

    r = sqrt(x.^2+y.^2);
```

## Creating Simscape Equations from MuPAD Expressions

Symbolic Math Toolbox lets you integrate symbolic computations into the Simscape modeling workflow by using the results of these computations in the Simscape equation section.

If you work in the MATLAB Command Window, see “Generating Simscape Equations” on page 3-144.

If you work in the MuPAD notebook interface, you can:

- Assign the MuPAD expression to a variable, copy that variable from a notebook to the MATLAB workspace, and use `simscapeEquation` to generate the Simscape equation in the MATLAB Command Window.
- Generate the Simscape equation from the MuPAD expression in a notebook.

In both cases, to use the generated equation, you must manually copy the equation and paste it to the equation section of the Simscape component file.

For example, follow these steps to generate a Simscape equation from the solution of the ordinary differential equation computed in the notebook interface:

- 1 Open a MuPAD notebook with the handle `notebook_handle`:

```
notebook_handle = mupad;
```

- 2 In this notebook, define the following equation:

```
s:= ode(y'(t) = y(t)^2, y(t)):
```

- 3 Decide whether you want to generate the Simscape equation in the MuPAD notebook interface or in the MATLAB Command Window.

### Generating Simscape Equations in the MuPAD Notebook Interface

To generate the Simscape equation in the same notebook, use `generate::Simscape`. To display generated Simscape code on screen, use the `print` function. To remove quotes and expand special characters like line breaks and tabs, use the printing option `Unquoted`:

```
print(Unquoted, generate::Simscape(s))
```

This command returns the Simscape equation that you can copy and paste to the Simscape equation section:

```
-y^2+y.der == 0.0;
```



## Generating Simscape Equations in the MATLAB Command Window

To generate the Simscape equation in the MATLAB Command Window, follow these steps:

- 1 Use `getVar` to copy variable `s` to the MATLAB workspace:

```
s = getVar(notebook_handle, 's')
```

Variable `s` and its value appear in the MATLAB workspace and in the MATLAB Command Window:

```
s =  
ode(D(y)(t) - y(t)^2, y(t))
```

- 2 Use `simscapeEquation` to generate the Simscape equation from `s`:

```
simscapeEquation(s)
```

You can copy and paste the generated equation to the Simscape equation section. Do not copy the automatically generated variable `ans` and the equal sign that follows it.

```
ans =  
s == (-y^2+y.der == 0.0);
```



# Function Reference

---

Calculus (p. 5-2)	Perform calculus operations on symbolic expressions
Linear Algebra (p. 5-2)	Symbolic matrix manipulation
Simplification (p. 5-3)	Modify or simplify symbolic data
Solution of Equations (p. 5-3)	Solve symbolic expression
Variable-Precision Arithmetic (p. 5-4)	Computing that requires exact control over numeric accuracy
Arithmetic Operations (p. 5-4)	Perform arithmetic on symbolic expressions
Special Functions (p. 5-5)	Specific mathematical applications
MuPAD (p. 5-5)	Access MuPAD
Pedagogical and Graphical Applications (p. 5-6)	Provide more information with plots and calculations
Conversions (p. 5-7)	Convert symbolic data from one data type to another
Basic Operations (p. 5-8)	Basic operations of symbolic data
Integral and Z-Transforms (p. 5-9)	Perform integral transforms and z-transforms

## Calculus

<code>diff</code>	Differentiate symbolic expression
<code>int</code>	Integrate symbolic expression
<code>jacobian</code>	Jacobian matrix
<code>limit</code>	Compute limit of symbolic expression
<code>symsum</code>	Evaluate symbolic sum of series
<code>taylor</code>	Taylor series expansion

## Linear Algebra

<code>colspace</code>	Column space of matrix
<code>det</code>	Compute determinant of symbolic matrix
<code>diag</code>	Create or extract diagonals of symbolic matrices
<code>eig</code>	Compute symbolic eigenvalues and eigenvectors
<code>expm</code>	Compute symbolic matrix exponential
<code>inv</code>	Compute symbolic matrix inverse
<code>jordan</code>	Jordan form of matrix
<code>null</code>	Form basis for null space of matrix
<code>poly</code>	Characteristic polynomial of matrix
<code>rank</code>	Compute rank of symbolic matrix
<code>rref</code>	Compute reduced row echelon form of matrix
<code>svd</code>	Compute singular value decomposition of symbolic matrix

<code>tril</code>	Return lower triangular part of symbolic matrix
<code>triu</code>	Return upper triangular part of symbolic matrix

## Simplification

<code>coeffs</code>	List coefficients of multivariate polynomial
<code>collect</code>	Collect coefficients
<code>expand</code>	Symbolic expansion of polynomials and elementary functions
<code>factor</code>	Factorization
<code>horner</code>	Horner nested polynomial representation
<code>numden</code>	Numerator and denominator
<code>simple</code>	Search for simplest form of symbolic expression
<code>simplify</code>	Algebraic simplification
<code>subexpr</code>	Rewrite symbolic expression in terms of common subexpressions
<code>subs</code>	Symbolic substitution in symbolic expression or matrix

## Solution of Equations

<code>compose</code>	Functional composition
<code>dsolve</code>	Symbolic solution of ordinary differential equations

<code>finverse</code>	Functional inverse
<code>solve</code>	Solve equations and systems

## Variable-Precision Arithmetic

<code>digits</code>	Variable-precision accuracy
<code>vpa</code>	Variable-precision arithmetic

## Arithmetic Operations

<code>+</code>	Addition
<code>-</code>	Subtraction
<code>*</code>	Multiplication
<code>.*</code>	Array multiplication
<code>\</code>	Left division
<code>.\</code>	Array left division
<code>/</code>	Right division
<code>./</code>	Array right division
<code>^</code>	Matrix or scalar raised to a power
<code>.^</code>	Array raised to a power
<code>'</code>	Complex conjugate transpose
<code>.'</code>	Real transpose

## Special Functions

<code>cosint</code>	Cosine integral
<code>dirac</code>	Dirac delta
<code>gamma</code>	Gamma function
<code>heaviside</code>	Compute Heaviside step function
<code>hypergeom</code>	Generalized hypergeometric
<code>lambertw</code>	Lambert W function
<code>mfun</code>	Numeric evaluation of special mathematical function
<code>mfunlist</code>	List special functions for use with <code>mfun</code>
<code>sinint</code>	Sine integral
<code>zeta</code>	Compute Riemann zeta function

## MuPAD

<code>clear all</code>	Remove items from MATLAB workspace and reset MuPAD engine
<code>doc</code>	Get help for MuPAD functions
<code>evalin</code>	Evaluate MuPAD expressions
<code>feval</code>	Evaluate MuPAD expressions
<code>getVar</code>	Get variable from MuPAD notebook
<code>mupad</code>	Start MuPAD notebook
<code>mupadwelcome</code>	Start MuPAD interfaces
<code>openmn</code>	Open MuPAD notebook
<code>openmu</code>	Open MuPAD program file
<code>openmuphp</code>	Open MuPAD help file

<code>openxvc</code>	Open MuPAD XVC graphics file
<code>openxvz</code>	Open MuPAD XVZ graphics file
<code>reset</code>	Close MuPAD engine
<code>setVar</code>	Assign variable in MuPAD notebook
<code>symengine</code>	Return symbolic engine
<code>trace</code>	Enable and disable tracing of MuPAD commands

## **Pedagogical and Graphical Applications**

<code>ezcontour</code>	Contour plotter
<code>ezcontourf</code>	Filled contour plotter
<code>ezmesh</code>	3-D mesh plotter
<code>ezmeshc</code>	Combined mesh and contour plotter
<code>ezplot</code>	Function plotter
<code>ezplot3</code>	3-D parametric curve plotter
<code>ezpolar</code>	Polar coordinate plotter
<code>ezsurf</code>	3-D colored surface plotter
<code>ezsurfz</code>	Combined surface and contour plotter
<code>funtool</code>	Function calculator
<code>rsums</code>	Interactive evaluation of Riemann sums
<code>taylortool</code>	Taylor series calculator



## Conversions

<code>ccode</code>	C code representation of symbolic expression
<code>char</code>	Convert symbolic objects to strings
<code>double</code>	Convert symbolic matrix to MATLAB numeric form
<code>emlBlock</code>	Convert symbolic expression to MATLAB Function block
<code>fortran</code>	Fortran representation of symbolic expression
<code>int16</code>	Convert symbolic matrix to signed integers
<code>int32</code>	Convert symbolic matrix to signed integers
<code>int64</code>	Convert symbolic matrix to signed integers
<code>int8</code>	Convert symbolic matrix to signed integers
<code>latex</code>	LaTeX representation of symbolic expression
<code>matlabFunction</code>	Convert symbolic expression to function handle or file
<code>poly2sym</code>	Polynomial coefficient vector to symbolic polynomial
<code>simscapeEquation</code>	Convert symbolic expressions to Simscape language equations
<code>single</code>	Convert symbolic matrix to single precision
<code>sym2poly</code>	Symbolic-to-numeric polynomial conversion

<code>uint16</code>	Convert symbolic matrix to unsigned integers
<code>uint32</code>	Convert symbolic matrix to unsigned integers
<code>uint64</code>	Convert symbolic matrix to unsigned integers
<code>uint8</code>	Convert symbolic matrix to unsigned integers

## Basic Operations

<code>ceil</code>	Round symbolic matrix toward positive infinity
<code>conj</code>	Symbolic complex conjugate
<code>eq</code>	Perform symbolic equality test
<code>fix</code>	Round toward zero
<code>floor</code>	Round symbolic matrix toward negative infinity
<code>frac</code>	Symbolic matrix element-wise fractional parts
<code>imag</code>	Imaginary part of complex number
<code>log10</code>	Logarithm base 10 of entries of symbolic matrix
<code>log2</code>	Logarithm base 2 of entries of symbolic matrix
<code>mod</code>	Symbolic matrix element-wise modulus
<code>pretty</code>	Prettyprint symbolic expressions
<code>quorem</code>	Symbolic matrix element-wise quotient and remainder

<code>real</code>	Real part of complex symbolic number
<code>round</code>	Symbolic matrix element-wise round
<code>size</code>	Symbolic matrix dimensions
<code>sort</code>	Sort symbolic vectors, matrices, or polynomials
<code>sym</code>	Define symbolic objects
<code>syms</code>	Shortcut for constructing symbolic objects
<code>symvar</code>	Find symbolic variables in symbolic expression or matrix

## Integral and Z-Transforms

<code>fourier</code>	Fourier integral transform
<code>ifourier</code>	Inverse Fourier integral transform
<code>ilaplace</code>	Inverse Laplace transform
<code>iztrans</code>	Inverse z-transform
<code>laplace</code>	Laplace transform
<code>ztrans</code>	z-transform



# Functions — Alphabetical List

---

# Arithmetic Operations

---

<b>Purpose</b>	Perform arithmetic operations on symbols
<b>Syntax</b>	A+B A-B A*B A.*B A\B A.\B B/A A./B A^B A.^B A' A.'
<b>Description</b>	<p><b>+</b> Matrix addition. A+B adds A and B. A and B must have the same dimensions, unless one is scalar.</p> <p><b>-</b> Matrix subtraction. A-B subtracts B from A. A and B must have the same dimensions, unless one is scalar.</p> <p><b>*</b> Matrix multiplication. A*B is the linear algebraic product of A and B. The number of columns of A must equal the number of rows of B, unless one is a scalar.</p> <p><b>.*</b> Array multiplication. A.*B is the entry-by-entry product of A and B. A and B must have the same dimensions, unless one is scalar.</p> <p><b>\</b> Matrix left division. A\B solves the symbolic linear equations A*X=B for X. Note that A\B is roughly equivalent to inv(A)*B. Warning messages are produced if X does not exist or is not unique. Rectangular matrices A are allowed, but the equations must be consistent; a least squares solution is <i>not</i> computed.</p>

- .\      Array left division.  $A.\backslash B$  is the matrix with entries  $B(i,j)/A(i,j)$ .  $A$  and  $B$  must have the same dimensions, unless one is scalar.
- /      Matrix right division.  $B/A$  solves the symbolic linear equation  $X*A=B$  for  $X$ . Note that  $B/A$  is the same as  $(A.\backslash B.')$ '. Warning messages are produced if  $X$  does not exist or is not unique. Rectangular matrices  $A$  are allowed, but the equations must be consistent; a least squares solution is not computed.
- ./     Array right division.  $A./B$  is the matrix with entries  $A(i,j)/B(i,j)$ .  $A$  and  $B$  must have the same dimensions, unless one is scalar.
- ^      Matrix power.  $A^B$  raises the square matrix  $A$  to the integer power  $B$ . If  $A$  is a scalar and  $B$  is a square matrix,  $A^B$  raises  $A$  to the matrix power  $B$ , using eigenvalues and eigenvectors.  $A^B$ , where  $A$  and  $B$  are both matrices, is an error.
- .^     Array power.  $A.^B$  is the matrix with entries  $A(i,j)^B(i,j)$ .  $A$  and  $B$  must have the same dimensions, unless one is scalar.
- '      Matrix Hermitian transpose. If  $A$  is complex,  $A'$  is the complex conjugate transpose.
- .'     Array transpose.  $A.'$  is the real transpose of  $A$ .  $A.'$  does not conjugate complex entries.

## Examples

The following statements

```
syms a b c d;  
A = [a b; c d];  
A*A/A  
A*A-A^2  
  
return
```

# Arithmetic Operations

---

```
[ a, b]
[ c, d]
```

```
[ 0, 0]
[ 0, 0]
```

The following statements

```
syms a11 a12 a21 a22 b1 b2;
A = [a11 a12; a21 a22];
B = [b1 b2];
X = B/A;
x1 = X(1)
x2 = X(2)
```

return

```
x1 =
-(a21*b2 - a22*b1)/(a11*a22 - a12*a21)

x2 =
(a11*b2 - a12*b1)/(a11*a22 - a12*a21)
```

## See Also

`null` | `solve`



<b>Purpose</b>	C code representation of symbolic expression
<b>Syntax</b>	<code>ccode(s)</code> <code>ccode(s, 'file', fileName)</code>
<b>Description</b>	<p><code>ccode(s)</code> returns a fragment of C that evaluates the symbolic expression <code>s</code>.</p> <p><code>ccode(s, 'file', fileName)</code> writes an “optimized” C code fragment that evaluates the symbolic expression <code>s</code> to the file named <code>fileName</code>. “Optimized” means intermediate variables are automatically generated in order to simplify the code. MATLAB generates intermediate variables as a lowercase letter <code>t</code> followed by an automatically generated number, for example <code>t32</code>.</p>
<b>Examples</b>	<p>The statements</p> <pre>syms x f = taylor(log(1+x)); ccode(f)</pre> <p>return</p> <pre>t0 = x - (x*x)*(1.0/2.0) + (x*x*x)*(1.0/3.0) - (x*x*x*x)*(1.0/4.0) + ... (x*x*x*x*x)*(1.0/5.0);</pre> <p>The statements</p> <pre>H = sym(hilb(3)); ccode(H)</pre> <p>return</p> <pre>H[0][0] = 1.0; H[0][1] = 1.0/2.0; H[0][2] = 1.0/3.0; H[1][0] = 1.0/2.0;</pre>

```
H[1][1] = 1.0/3.0;  
H[1][2] = 1.0/4.0;  
H[2][0] = 1.0/3.0;  
H[2][1] = 1.0/4.0;  
H[2][2] = 1.0/5.0;
```

The statements

```
syms x  
z = exp(-exp(-x));  
ccode(diff(z,3), 'file', 'ccodetest');
```

return a file named `ccodetest` containing the following:

```
t2 = exp(-x);  
t3 = exp(-t2);  
t0 = t3*exp(x*(-2.0))*(-3.0)+t3*exp(x*(-3.0))+t2*t3;
```

## See Also

[fortran](#) | [latex](#) | [matlabFunction](#) | [pretty](#)

## How To

- “Generating Code from Symbolic Expressions” on page 3-134

---

<b>Purpose</b>	Round symbolic matrix toward positive infinity
<b>Syntax</b>	$Y = \text{ceil}(x)$
<b>Description</b>	$Y = \text{ceil}(x)$ is the matrix of the smallest integers greater than or equal to $x$ .
<b>Examples</b>	<pre>x = sym(-5/2); [fix(x) floor(x) round(x) ceil(x) frac(x)]  ans = [ -2, -3, -3, -2, -1/2]</pre>
<b>See Also</b>	<a href="#">round</a>   <a href="#">floor</a>   <a href="#">fix</a>   <a href="#">frac</a>

# char

---

**Purpose** Convert symbolic objects to strings

**Syntax** `char(A)`  
`char(A, d)`

**Description** `char(A)` converts a symbolic scalar or a symbolic array to a string.  
`char(A, d)` converts a symbolic scalar or array to a string. For symbolic arrays, the second parameter specifies the form of the resulting string. For symbolic scalars, this parameter does not affect the result.

**Input Arguments**

**A**  
A symbolic scalar or a symbolic array

**d**  
A number that specifies the format of the resulting string. For symbolic arrays:

`char(A, 1)` results in `matrix([...])`  
`char(A, 2)` results in `matrix([[...],[...]])`  
`char(A, d)` for all other values of the parameter `d` results in `array([1..m, 1..n, 1..p], [(1,1,1) = xxx, ..., (m,n,p) = xxx])`

**Examples** Convert symbolic expressions to strings, and then concatenate the strings:

```
syms x;  
y = char(x^3 + x^2 + 2*x - 1);  
name = [y, ' presents a polynomial expression']
```

The result is:

```
name =  
x^3 + x^2 + 2*x - 1 presents a polynomial expression
```

---

Convert a symbolic matrix to a string:

```
A = sym(hilb(3))
char(A)
```

The result is:

```
A =
[ 1, 1/2, 1/3]
[ 1/2, 1/3, 1/4]
[ 1/3, 1/4, 1/5]

ans =
matrix([[1,1/2,1/3],[1/2,1/3,1/4],[1/3,1/4,1/5]])
```

**See Also**

[sym](#) | [double](#) | [pretty](#)

# clear all

---

**Purpose** Remove items from MATLAB workspace and reset MuPAD engine

**Syntax** `clear all`

**Description** `clear all` clears all objects in the MATLAB workspace and closes the MuPAD engine associated with the MATLAB workspace resetting all its assumptions.

**See Also** `reset`

**Purpose** List coefficients of multivariate polynomial

**Syntax**

```
C = coeffs(p)
C = coeffs(p, x)
[C, T] = coeffs(p, x)
```

**Description**

`C = coeffs(p)` returns the coefficients of the polynomial `p` with respect to all the indeterminates of `p`.

`C = coeffs(p, x)` returns the coefficients of the polynomial `p` with respect to `x`.

`[C, T] = coeffs(p, x)` returns a list of the coefficients and a list of the terms of `p`. There is a one-to-one correspondence between the coefficients and the terms of `p`.

**Examples** List the coefficients of the following single-variable polynomial:

```
syms x
t = 16*log(x)^2 + 19*log(x) + 11;
coeffs(t)
```

The result is:

```
ans =
[ 11, 19, 16]
```

List the coefficients of the following polynomial with respect to the indeterminate `sin(x)`:

```
syms a b c x
y = a + b*sin(x) + c*sin(2*x);
coeffs(y, sin(x))
```

The result is:

```
ans =
```

# coeffs

---

```
[ a + c*sin(2*x), b]
```

---

List the coefficients of the following multivariable polynomial with respect to all the indeterminates and with respect to the variable  $x$  only:

```
syms x y
z = 3*x^2*y^2 + 5*x*y^3;
coeffs(z)
coeffs(z,x)
```

The results are:

```
ans =
[ 5, 3]

ans =
[ 5*y^3, 3*y^2]
```

---

Display the list of the coefficients and the list of the terms of the following polynomial expression:

```
syms x y
z = 3*x^2*y^2 + 5*x*y^3;
[c,t] = coeffs(z,y)
```

The results are:

```
c =
[ 5*x, 3*x^2]

t =
[ y^3, y^2]
```

## See Also

`sym2poly`



<b>Purpose</b>	Collect coefficients
<b>Syntax</b>	$R = \text{collect}(S)$ $R = \text{collect}(S, v)$
<b>Description</b>	$R = \text{collect}(S)$ returns an array of collected polynomials for each polynomial in the array $S$ of polynomials. $R = \text{collect}(S, v)$ collects terms containing the variable $v$ .
<b>Examples</b>	<p>The following statements</p> <pre>syms x y; R1 = collect((exp(x)+x)*(x+2)) R2 = collect((x+y)*(x^2+y^2+1), y) R3 = collect([(x+1)*(y+1),x+y])</pre> <p>return</p> <pre>R1 = x^2 + (exp(x) + 2)*x + 2*exp(x)</pre> <pre>R2 = y^3 + x*y^2 + (x^2 + 1)*y + x*(x^2 + 1)</pre> <pre>R3 = [ y + x*(y + 1) + 1, x + y]</pre>
<b>See Also</b>	<code>expand</code>   <code>factor</code>   <code>simple</code>   <code>simplify</code>   <code>syms</code>

# colspace

---

	column space
<b>Purpose</b>	Column space of matrix
<b>Syntax</b>	<code>B = colspace(A)</code>
<b>Description</b>	<code>B = colspace(A)</code> returns a matrix whose columns form a basis for the column space of A. The matrix A can be symbolic or numeric.
<b>Examples</b>	<p>Find the basis for the column space of this matrix:</p> <pre>A = sym([2,0;3,4;0,5]) B = colspace(A)</pre> <p>The result is:</p> <pre>A = [ 2, 0] [ 3, 4] [ 0, 5]</pre> <pre>B = [ 1, 0] [ 0, 1] [-15/8, 5/4]</pre>
<b>See Also</b>	<code>null</code>   <code>size</code>

**Purpose**

Functional composition

**Syntax**

```
compose(f,g)
compose(f,g,z)
compose(f,g,x,z)
compose(f,g,x,y,z)
```

**Description**

`compose(f,g)` returns  $f(g(y))$  where  $f = f(x)$  and  $g = g(y)$ . Here  $x$  is the symbolic variable of  $f$  as defined by `symvar` and  $y$  is the symbolic variable of  $g$  as defined by `symvar`.

`compose(f,g,z)` returns  $f(g(z))$  where  $f = f(x)$ ,  $g = g(y)$ , and  $x$  and  $y$  are the symbolic variables of  $f$  and  $g$  as defined by `symvar`.

`compose(f,g,x,z)` returns  $f(g(z))$  and makes  $x$  the independent variable for  $f$ . That is, if  $f = \cos(x/t)$ , then `compose(f,g,x,z)` returns  $\cos(g(z)/t)$  whereas `compose(f,g,t,z)` returns  $\cos(x/g(z))$ .

`compose(f,g,x,y,z)` returns  $f(g(z))$  and makes  $x$  the independent variable for  $f$  and  $y$  the independent variable for  $g$ . For  $f = \cos(x/t)$  and  $g = \sin(y/u)$ , `compose(f,g,x,y,z)` returns  $\cos(\sin(z/u)/t)$  whereas `compose(f,g,x,u,z)` returns  $\cos(\sin(y/z)/t)$ .

**Examples**

Suppose

```
syms x y z t u;
f = 1/(1 + x^2); g = sin(y); h = x^t; p = exp(-y/u);
```

Then

```
a = compose(f,g)
b = compose(f,g,t)
c = compose(h,g,x,z)
d = compose(h,g,t,z)
e = compose(h,p,x,y,z)
f = compose(h,p,t,u,z)
```

returns:

## compose

---

```
a =  
1/(sin(y)^2 + 1)
```

```
b =  
1/(sin(t)^2 + 1)
```

```
c =  
sin(z)^t
```

```
d =  
x^sin(z)
```

```
e =  
(1/exp(z/u))^t
```

```
f =  
x^(1/exp(y/z))
```

### See Also

`finverse` | `subs` | `syms`

<b>Purpose</b>	Symbolic complex conjugate
<b>Syntax</b>	<code>conj(X)</code>
<b>Description</b>	<code>conj(X)</code> is the complex conjugate of $X$ . For a complex $X$ , $\text{conj}(X) = \text{real}(X) - i*\text{imag}(X)$ .
<b>See Also</b>	<code>real</code>   <code>imag</code>

# cosint

---

**Purpose** Cosine integral

**Syntax** `Y = cosint(X)`

**Description** `Y = cosint(X)` evaluates the cosine integral function at the elements of `X`, a numeric matrix, or a symbolic matrix. The cosine integral function is defined by

$$Ci(x) = \gamma + \ln(x) + \int_0^x \frac{\cos t - 1}{t} dt,$$

where  $\gamma$  is Euler's constant 0.577215664...

**Examples** Compute cosine integral for a numerical value:

```
cosint(7.2)
```

The result is:

```
0.0960
```

Compute the cosine integral for `[0:0.1:1]` :

```
cosint([0:0.1:1])
```

The result is:

```
Columns 1 through 6
```

```
-Inf -1.7279 -1.0422 -0.6492 -0.3788 -0.1778
```

```
Columns 7 through 11
```

```
-0.0223 0.1005 0.1983 0.2761 0.3374
```

The statements

```
syms x;
```

```
f = cosint(x);  
diff(f)
```

```
return
```

```
cos(x)/x
```

**See Also**

```
sinint
```

# det

---

**Purpose** Compute determinant of symbolic matrix

**Syntax** `r = det(A)`

**Description** `r = det(A)` computes the determinant of A, where A is a symbolic or numeric matrix. `det(A)` returns a symbolic expression for a symbolic A and a numeric value for a numeric A.

**Examples** Compute the determinant of the following symbolic matrix:

```
syms a b c d;  
det([a, b; c, d])
```

The result is:

```
ans =  
a*d - b*c
```

---

Compute the determinant of the following matrix containing the symbolic numbers:

```
A = sym([2/3 1/3; 1 1])  
r = det(A)
```

The result is:

```
A =  
[ 2/3, 1/3]  
[ 1, 1]  
  
r =  
1/3
```

**See Also** `rank` | `eig`



**Purpose**

Create or extract diagonals of symbolic matrices

**Syntax**

```
diag(A, k)
diag(A)
```

**Description**

`diag(A, k)` returns a square symbolic matrix of order  $n + \text{abs}(k)$ , with the elements of  $A$  on the  $k$ -th diagonal.  $A$  must present a row or column vector with  $n$  components. The value  $k = 0$  signifies the main diagonal. The value  $k > 0$  signifies the  $k$ -th diagonal above the main diagonal. The value  $k < 0$  signifies the  $k$ -th diagonal below the main diagonal. If  $A$  is a square symbolic matrix, `diag(A, k)` returns a column vector formed from the elements of the  $k$ -th diagonal of  $A$ .

`diag(A)`, where  $A$  is a vector with  $n$  components, returns an  $n$ -by- $n$  diagonal matrix having  $A$  as its main diagonal. If  $A$  is a square symbolic matrix, `diag(A)` returns the main diagonal of  $A$ .

**Examples**

Create a symbolic matrix with the main diagonal presented by the elements of the vector  $v$ :

```
syms a b c;
v = [a b c];
diag(v)
```

The result is:

```
ans =
[ a, 0, 0]
[ 0, b, 0]
[ 0, 0, c]
```

---

Create a symbolic matrix with the second diagonal below the main one presented by the elements of the vector  $v$ :

```
syms a b c;
v = [a b c];
```

# diag

---

```
diag(v, -2)
```

The result is:

```
ans =  
[ 0, 0, 0, 0, 0]  
[ 0, 0, 0, 0, 0]  
[ a, 0, 0, 0, 0]  
[ 0, b, 0, 0, 0]  
[ 0, 0, c, 0, 0]
```

---

Extract the main diagonal from a square matrix:

```
syms a b c x y z;  
A = [a, b, c; 1, 2, 3; x, y, z];  
diag(A)
```

The result is

```
ans =  
a  
2  
z
```

---

Extract the first diagonal above the main one:

```
syms a b c x y z;  
A = [a, b, c; 1, 2, 3; x, y, z];  
diag(A, 1)
```

The result is:

```
ans =  
b  
3
```

**See Also**      `tril | triu`

# diff

---

**Purpose** Differentiate symbolic expression

**Syntax**

```
diff(expr)
diff(expr, v)
diff(expr, sym('v'))
diff(expr, n)
diff(expr, v, n)
diff(expr, n, v)
```

**Description** `diff(expr)` differentiates a symbolic expression `expr` with respect to its free variable as determined by `symvar`.

`diff(expr, v)` and `diff(expr, sym('v'))` differentiate `expr` with respect to `v`.

`diff(expr, n)` differentiates `expr`  $n$  times.  $n$  is a positive integer.

`diff(expr, v, n)` and `diff(expr, n, v)` differentiate `expr` with respect to `v`  $n$  times.

**Examples** Differentiate the following single-variable expression one time:

```
syms x;
diff(sin(x^2))
```

The result is

```
ans =
2*x*cos(x^2)
```

---

Differentiate the following single-variable expression six times:

```
syms t;
diff(t^6,6)
```

The result is

```
ans =
```

---

720

---

Differentiate the following expression with respect to t:

```
syms x t;  
diff(sin(x*t^2), t)
```

The result is

```
ans =  
2*t*x*cos(t^2*x)
```

**See Also**

[int](#) | [jacobian](#) | [symvar](#)

# digits

---

**Purpose** Variable-precision accuracy

**Syntax** `digits`  
`digits(d)`  
`d = digits`

**Description** `digits` specifies the minimum number of significant (nonzero) decimal digits that MuPAD software uses to do variable-precision arithmetic (VPA). The default value is 32 digits.

`digits(d)` sets the current VPA accuracy to at least  $d$  significant (nonzero) decimal digits. The value  $d$  must be a positive integer larger than 1 and smaller than  $2^{29} + 1$ .

`d = digits` returns the current VPA accuracy.

**Examples** The `digits` function specifies the number of significant (nonzero) digits. For example, use 4 significant digits to compute the ratio  $1/3$  and the ratio  $1/3000$ :

```
old = digits;  
digits(4);  
vpa(1/3)  
vpa(1/3000)  
digits(old);
```

```
ans =  
0.3333
```

```
ans =  
0.0003333
```

---

To change the VPA accuracy for one operation without changing the current `digits` setting, use the `vpa` function. For example, compute the ratio  $1/3$  with the default 32 digits, 10 digits, and 40 digits:

```
vpa(1/3)
vpa(1/3, 10)
vpa(1/3, 40)

ans =
  0.33333333333333333333333333333333

ans =
  0.3333333333

ans =
  0.33333333333333333333333333333333
```

---

The number of digits that you specify by the `vpa` function or the `digits` function is the minimal number of digits. Internally, the toolbox can use more digits than you specify. These additional digits are called guard digits. For example, set the number of digits to 4, and then display the floating-point approximation of  $1/3$  using 4 digits:

```
old = digits;
digits(4);
a = vpa(1/3)

a =
  0.3333
```

Now, display `a` using 20 digits. The result shows that the toolbox internally used more than 4 digits when computing `a`. The last digits in the following result are incorrect because of the round-off error:

```
digits(20);
vpa(a)
digits(old);

ans =
  0.333333333333333303016843
```





---

Suppose, you convert a number to a symbolic object, and then perform VPA operations on that object. The results can depend on the conversion technique that you used to convert a floating-point number to a symbolic object. The `sym` function lets you choose the conversion technique by specifying the optional second argument, which can be 'r', 'f', 'd', or 'e'. The default is 'r'. For example, convert the constant  $\pi=3.141592653589793\dots$  to a symbolic object:

```
r = sym(pi)
f = sym(pi, 'f')
d = sym(pi, 'd')
e = sym(pi, 'e')

r =
pi

f =
884279719003555/281474976710656

d =
3.1415926535897931159979634685442

e =
pi - (198*eps)/359
```

Set the number of digits to 4. Three of the four numeric approximations give the same result:

```
digits(4);
vpa(r)
vpa(f)
vpa(d)
vpa(e)

ans =
3.142
```

# digits

---

```
ans =  
3.142
```

```
ans =  
3.142
```

```
ans =  
3.142 - 0.5515*eps
```

Now, set the number of digits to 40. The numeric approximation of 1/10 depends on the technique that you used to convert 1/10 to the symbolic object:

```
digits(40);  
vpa(r)  
vpa(f)  
vpa(d)  
vpa(e)
```

```
ans =  
3.141592653589793238462643383279502884197
```

```
ans =  
3.141592653589793115997963468544185161591
```

```
ans =  
3.1415926535897931159979634685442
```

```
ans =  
3.141592653589793238462643383279502884197 - ...  
0.5515320334261838440111420612813370473538*eps
```

## See Also

[double](#) | [vpa](#)

**Purpose** Dirac delta

**Syntax** `dirac(x)`

**Description** `dirac(x)` returns the Dirac delta function of  $x$ .

The Dirac delta function, `dirac`, has the value 0 for all  $x$  not equal to 0 and the value `Inf` for  $x = 0$ . Several Symbolic Math Toolbox functions return answers in terms of `dirac`.

**Examples** `dirac` has the property that

$$\int_{-\infty}^{\infty} \text{dirac}(x-a) * f(x) = f(a)$$

for any function  $f$  and real number  $a$ . For example:

```
syms x a
a = 5;
int(dirac(x-a)*sin(x),-inf, inf)

ans =
sin(5)
```

`dirac` also has the following relationship to the function `heaviside`:

```
syms x;
diff(heaviside(x),x)

ans =
dirac(x)
```

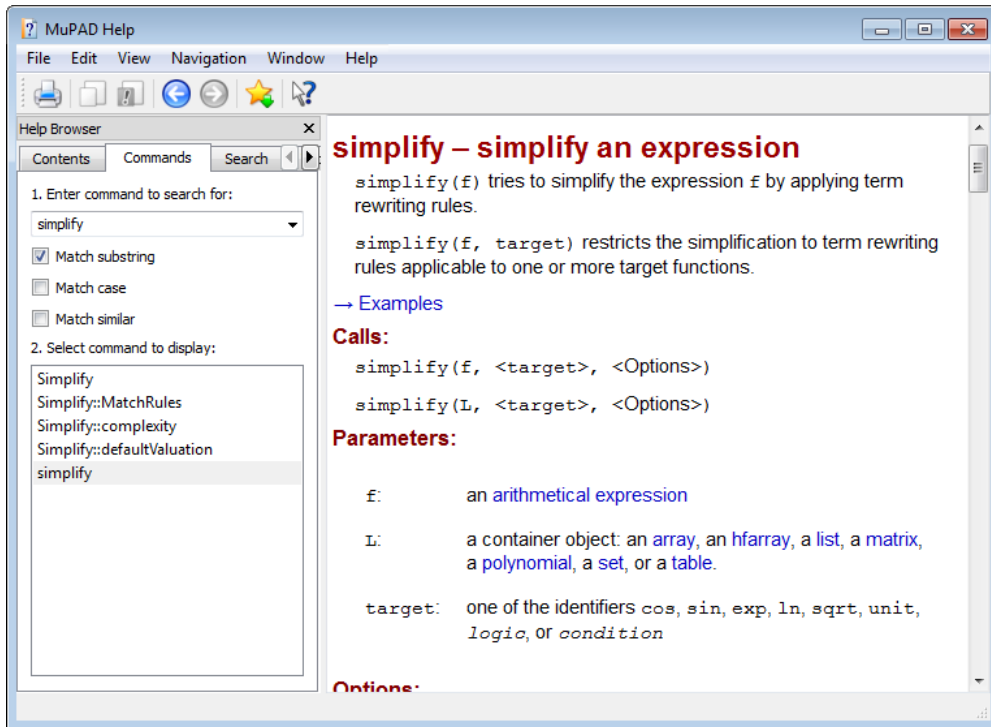
**See Also** `heaviside`

**Purpose** Get help for MuPAD functions

**Syntax** `doc(symengine)`  
`doc(symengine, 'MuPAD_function_name')`

**Description** `doc(symengine)` opens the MuPAD help browser.  
`doc(symengine, 'MuPAD_function_name')` opens the MuPAD help browser at the definition of `MuPAD_function_name`.

**Examples** `doc(symengine, 'simplify')` opens the following window.



**How To** • “Getting Help for MuPAD” on page 4-10

**Purpose** Convert symbolic matrix to MATLAB numeric form

**Syntax** `r = double(S)`

**Description** `r = double(S)` converts the symbolic object `S` to a numeric object. If `S` is a symbolic constant or constant expression, `double` returns a double-precision floating-point number representing the value of `S`. If `S` is a symbolic matrix whose entries are constants or constant expressions, `double` returns a matrix of double precision floating-point numbers representing the values of `S`'s entries.

**Examples** Find the numeric value for the expression  $\frac{1+\sqrt{5}}{2}$ :

```
double(sym('(1+sqrt(5))/2'))
```

The result is:

```
1.6180
```

The following statements

```
a = sym(2*sqrt(2));  
b = sym((1-sqrt(3))^2);  
T = [a, b];  
double(T)
```

```
return
```

```
ans =  
2.8284    0.5359
```

**See Also** `sym` | `vpa`

# dsolve

---

**Purpose** Symbolic solution of ordinary differential equations

**Syntax** `dsolve('eq1','eq2',...,'cond1','cond2',...,'v')`  
`dsolve(...,'IgnoreAnalyticConstraints',value)`

**Description** `dsolve('eq1','eq2',...,'cond1','cond2',...,'v')` symbolically solves the ordinary differential equations `eq1`, `eq2`, ... using `v` as the independent variable. Here `cond1`, `cond2`, ... specify boundary or initial conditions or both. You also can use the following syntax: `dsolve('eq1','eq2',...,'cond1','cond2',...,'v')`. The default independent variable is `t`.

The letter `D` denotes differentiation with respect to the independent variable. The primary default is  $d/dx$ . The letter `D` followed by a digit denotes repeated differentiation. For example, `D2` is  $d^2/dx^2$ . Any character immediately following a differentiation operator is a dependent variable. For example, `D3y` denotes the third derivative of  $y(x)$  or  $y(t)$ .

You can specify initial and boundary conditions by equations like  $y(a) = b$  or  $Dy(a) = b$ , where  $y$  is a dependent variable and  $a$  and  $b$  are constants. If the number of the specified initial conditions is less than the number of dependent variables, the resulting solutions contain the arbitrary constants `C1`, `C2`, ...

You can input each equation or a condition as a separate symbolic equation. The `dsolve` command accepts up to 12 input arguments.

`dsolve` can produce the following three types of outputs:

- For one equation and one output, `dsolve` returns the resulting solution with multiple solutions to a nonlinear equation in a symbolic vector.
- For several equations and an equal number of outputs, `dsolve` sorts the results alphabetically and assigns them to the outputs.
- For several equations and a single output, `dsolve` returns a structure containing the solutions.

If `dsolve` cannot find a closed-form (explicit) solution, it attempts to find an implicit solution. When `dsolve` returns an implicit solution, it issues a warning. If `dsolve` cannot find either an explicit or an implicit solution, then it issues a warning and returns the empty `sym`. In such a case, you can find a numeric solution, using the MATLAB `ode23` or `ode45` functions. In some cases involving nonlinear equations, the output is an equivalent lower order differential equation or an integral.

`dsolve(..., 'IgnoreAnalyticConstraints', value)` accepts the following values:

- `value = 'all'` applies the purely algebraic simplifications to the expressions on both sides of equations. These simplifications might not be generally valid. The default value of this option is `all`.
- `value = 'none'` solves ordinary differential equations without additional assumptions. The results obtained with this option are correct for all values of the arguments.

---

**Note** By default, the solver does not guarantee general correctness and completeness of the results. If you do not set the option `IgnoreAnalyticConstraints` to `none`, always verify results returned by the `dsolve` command.

---

If you do not set the value of the option `IgnoreAnalyticConstraints` to `none`, the solver applies the following rules to the expressions on both sides of an equation:

- The solutions of polynomial equations must be complete.
- $\ln(a) + \ln(b) = \ln(a \cdot b)$  for all values of  $a$  and  $b$ . In particular, the following equality is valid for all values of  $a$ ,  $b$ , and  $c$ :

$$(a \cdot b)^c = a^c \cdot b^c$$

- $\ln(a^b) = b \cdot \ln a$  for all values of  $a$  and  $b$ . In particular, the following equality is valid for all values of  $a$ ,  $b$ , and  $c$ :

$$(a^b)^c = a^{b \cdot c}$$

- The following equality is valid for all values of  $x$ :
  - $\ln(e^x) = x$
  - $\arcsin(\sin(x)) = x$ ,  $\arccos(\cos(x)) = x$ ,  $\arctan(\tan(x)) = x$
  - $\operatorname{arcsinh}(\sinh(x)) = x$ ,  $\operatorname{arccosh}(\cosh(x)) = x$ ,  $\operatorname{arctanh}(\tanh(x)) = x$
  - $W_k(x \cdot e^x) = x$  for all values of  $k$
- The solver can multiply both sides of an equation by any expression except 0.

## Examples

### Solving Ordinary Differential Equations Symbolically

```
dsolve('Dx = -a*x')
```

```
ans =  
C2/exp(a*t)
```

### Specifying the Dependent Variable

The following differential equation presents  $f$  as a dependent variable:

```
dsolve('Df = f + sin(t)')
```

```
ans =  
C4*exp(t) - sin(t)/2 - cos(t)/2
```



**Specifying the Independent Variable**

```
dsolve('(Dy)^2 + y^2 = 1', 's')
```

```
ans =
```

```

      1
     -1
cosh(C11 + s*i)
cosh(C7 - s*i)
```

**Setting Initial and Boundary Conditions**

```
dsolve('Dy = a*y', 'y(0) = b')
```

```
ans =
```

```
b*exp(a*t)
```

```
dsolve('D2y = -a^2*y', 'y(0) = 1', 'Dy(pi/a) = 0')
```

```
ans =
```

```
1/(2*exp(a*t*i)) + exp(a*t*i)/2
```

**Solving a System of Differential Equations**

```
z = dsolve('Dx = y', 'Dy = -x')
```

```
z =
```

```

y: [1x1 sym]
x: [1x1 sym]
```

Enter z.x and z.y to see the results:

```
z.x
```

```
ans =
```

```
C19*cos(t) + C20*sin(t)
```

```
z.y
```

```
ans =
C20*cos(t) - C19*sin(t)
```

## Using the IgnoreAnalyticConstraints Option

By default, the solver applies the set of purely algebraic simplifications that are not correct in general, but that can result in simple and practical solutions:

```
y = dsolve('Dy = 1/sqrt(y)', 'y(0) = 1')

y =
((3*t)/2 + 1)^(2/3)
```

To obtain complete and generally correct solutions, set the value of the option `IgnoreAnalyticConstraints` to `none`:

```
y = dsolve('Dy = 1/sqrt(y)', 'y(0) = 1',...
'IgnoreAnalyticConstraints', 'none')

y =
piecewise([C29 in Z_, Dom::ImageSet(exp(pi*1*(-4*i)/3))*((3*t)/2 +...
exp(C29*pi*(-3*i))*exp(pi*1*((4*i)/3))^(3/2))^(2/3), 1, Z_ intersect..
solve(- (2*pi)/3 + 2*C29*pi < arg(exp(pi*X504*((4*i)/3))), X504) inters
solve(arg(exp(pi*X504*((4*i)/3))) <= (2*pi)/3 + 2*C29*pi, X504) inters
Dom::Interval([- (3*(pi/2 - arg(C27 + t)/3))/(2*pi)],...
(3*(pi/2 + arg(C27 + t)/3))/(2*pi))], [not C29 in Z_, {}])
```

If you apply algebraic simplifications, you can get explicit solutions for some equations for which the solver cannot compute them using strict mathematical rules:

```
dsolve('Dy = 19.6/sqrt(y) + 0.00196*y', 'y(0) = 1')

ans =
(exp((147*t)/50000 + log(10001)) - 10000)^(2/3)
```

versus

```

dsolve('Dy = 19.6/sqrt(y) + 0.00196*y', 'y(0) = 1',
'IgnoreAnalyticConstraints','none')

ans =
piecewise([(3*Im(C36))/2 in Dom::Interval(-pi, [pi]) and...
C38 in Z_ and C39 in Z_,...
Dom::ImageSet(exp(pi*1*(-(4*i)/3))*(exp((147*t)/50000 +...
log((-1)^(3*C38)*exp(pi*1*((4*i)/3))^(3/2) + 10000) +...
C39*pi*(2*i)) - 10000)^(2/3), 1, Z_ intersect...
solve(-(2*pi)/3 + 2*C38*pi < arg(exp(pi*X2803*((4*i)/3))), X2803)
solve(arg(exp(pi*X2803*((4*i)/3))) <= (2*pi)/3 + 2*C38*pi, X2803) i
Dom::Interval([-3*(pi/2 - arg(exp((3*C36)/2 + (147*t)/50000) - 100
(3*(pi/2 + arg(exp((3*C36)/2 + (147*t)/50000) - 10000)/3))/(2*pi))
[not (3*Im(C36))/2 in Dom::Interval(-pi, [pi])...
or not C38 in Z_ or not C39 in Z_, {}])

```

## Diagnostics

If dsolve cannot find an analytic solution for an equation, it prints the warning:

```
Warning: Explicit solution could not be found.
```

and returns an empty sym object.

## See Also

syms

**Purpose** Compute symbolic eigenvalues and eigenvectors

**Syntax**

```
lambda = eig(A)
[V,D] = eig(A)
[V,D,P] = eig(A)
lambda = eig(vpa(A))
[V,D] = eig(vpa(A))
```

**Description** `lambda = eig(A)` returns a symbolic vector containing the eigenvalues of the square symbolic matrix `A`.

`[V,D] = eig(A)` returns matrices `V` and `D`. The columns of `V` present eigenvectors of `A`. The diagonal matrix `D` contains eigenvalues. If the resulting `V` has the same size as `A`, the matrix `A` has a full set of linearly independent eigenvectors that satisfy  $A*V = V*D$ .

`[V,D,P] = eig(A)` returns a vector of indices `P`. The length of `P` equals to the total number of linearly independent eigenvectors, so that  $A*V = V*D(P,P)$ .

`lambda = eig(vpa(A))` returns numeric eigenvalues using variable-precision arithmetic.

`[V,D] = eig(vpa(A))` returns numeric eigenvectors using variable-precision arithmetic. If `A` does not have a full set of eigenvectors, the columns of `V` are not linearly independent.

**Examples** Compute the eigenvalues for the magic square of order 5:

```
M = sym(magic(5));
eig(M)
```

The result is:

```
ans =
                                     65
(625/2 - (5*3145^(1/2))/2)^(1/2)
((5*3145^(1/2))/2 + 625/2)^(1/2)
-(625/2 - (5*3145^(1/2))/2)^(1/2)
```

$$-\left(\left(5 \cdot 3145^{1/2}\right)/2 + 625/2\right)^{1/2}$$


---

Compute the eigenvalues for the magic square of order 5 using variable-precision arithmetic:

```
M = sym(magic(5));
eig(vpa(M))
```

The result is:

```
ans =
                                     65.0
21.27676547147379553062642669797423
13.12628093070921880252564308594914
-13.126280930709218802525643085949
-21.276765471473795530626426697974
```

---

Compute the eigenvalues and eigenvectors for one of the MATLAB test matrices:

```
A = sym(gallery(5))
[v, lambda] = eig(A)
```

The results are:

```
A =
[ -9, 11, -21, 63, -252]
[ 70, -69, 141, -421, 1684]
[ -575, 575, -1149, 3451, -13801]
[ 3891, -3891, 7782, -23345, 93365]
[ 1024, -1024, 2048, -6144, 24572]

v =
0
21/256
-71/128
```

```
973/256  
1
```

```
lambda =  
[ 0, 0, 0, 0, 0]  
[ 0, 0, 0, 0, 0]  
[ 0, 0, 0, 0, 0]  
[ 0, 0, 0, 0, 0]  
[ 0, 0, 0, 0, 0]
```

## See Also

[jordan](#) | [poly](#) | [svd](#) | [vpa](#)

---

<b>Purpose</b>	Convert symbolic expression to MATLAB Function block
<b>Syntax</b>	<pre>emlBlock(block, f) emlBlock(block, f1, f2, ...) emlBlock(block, f1, f2, ..., param1, value1,...)</pre>
<b>Description</b>	<p><code>emlBlock(block, f)</code> converts the symbolic expression <code>f</code> to a MATLAB Function block that you can use in Simulink models. The parameter <code>block</code> specifies the name of the block you create or modify. The <code>block</code> should be a string.</p> <p><code>emlBlock(block, f1, f2, ...)</code> converts a list of the symbolic expressions <code>f1, f2, ...</code> to a MATLAB Function block with multiple outputs.</p> <p><code>emlBlock(block, f1, f2, ..., param1, value1,...)</code> converts a list of the symbolic expressions <code>f1, f2, ...</code> to a MATLAB Function block with multiple outputs, with the following options for parameter/value pairs:</p> <ul style="list-style-type: none"><li>• <code>Parameter = 'functionName'</code> allows you to set the name of the function. <code>value</code> should be a string. By default, <code>value</code> coincides with the name of the block.</li><li>• <code>Parameter = 'outputs'</code> allows you to set the names of the output ports. <code>value</code> should be a cell array of strings. The number of <code>value</code> entries should equal or exceed the number of free variables in the symbolic expression <code>f</code>. The default name of an output port consists of the word <code>out</code> followed by the output port number, for example, <code>out3</code>.</li><li>• <code>Parameter = 'vars'</code> allows you to set the order of the variables and the corresponding input ports of a block. The default order is alphabetical. <code>value</code> should be either a cell array of strings or symbolic arrays, or a vector of symbolic variables. The number of <code>value</code> entries should equal or exceed the number of free variables in the symbolic expression <code>f</code>.</li></ul>

---

**Tip** To convert a MuPAD expression or function to a MATLAB Function block, use `f = evalin(symengine, 'MuPAD_Expression')` or `f = feval(symengine, 'MuPAD_Function', x1, ..., xn)`. `emlBlock` cannot correctly convert some MuPAD expressions to a block. These expressions do not trigger an error message. When converting a MuPAD expression or function that is not on the MATLAB vs. MuPAD Expressions list, always check the results of conversion. To verify the results, you can:

- Run the simulation containing the resulting block.
  - Open the block and verify that all the functions are defined in MATLAB Function Library Reference.
- 

## Examples

Before you can convert a symbolic expression to a MATLAB Function block, create an empty model or open an existing one:

```
new_system('my_system');
open_system('my_system');
```

Use `emlBlock` to create the block `my_block` containing the symbolic expression:

```
syms x y z
f = x^2 + y^2 + z^2;
emlBlock('my_system/my_block', f);
```

If you use the name of an existing block, the `emlBlock` command replaces the definition of an existing block with the converted symbolic expression.

You can open and edit the resulting block. To open a block, select **Edit > Open Block** or use the context menu:

```
function f = my_block(x,y,z)
%#codegen
```



```
f = x.^2 + y.^2 + z.^2;
```

The following example generates a block and sets the function name to *my\_function*:

```
emlBlock('my_system/my_block', x, y, z,...
        'functionName', 'my_function')
```

You can change the order of the input ports:

```
emlBlock('my_system/my_block', x, y, z,...
        'vars', [y z x])
```

Also, you can rename the output variables and the corresponding ports:

```
emlBlock('my_system/my_block', x, y, z,...
        'outputs', {'name1', 'name2', 'name3'})
```

`emlBlock` accepts several options simultaneously:

```
emlBlock('my_system/my_block', x, y, z,...
        'functionName', 'my_function', 'vars', [y z x],...
        'outputs', {'name1', 'name2', 'name3'})
```

You also can convert MuPAD expressions:

```
syms x y;
f = evalin(symengine, 'arcsin(x) + arccos(y)');
emlBlock('my_system/my_block', f);
```

The resulting block contains the same expressions written in the MATLAB language:

```
function f = my_block(x,y)
    %#codegen

    f = asin(x) + acos(y);
```

## See Also

`ccode` | `fortran` | `matlabFunction` | `subs` | `sym2poly`

## How To

- “Generating Code from Symbolic Expressions” on page 3-134

**Purpose** Perform symbolic equality test

**Syntax** `eq(A, B)`  
`A == B`

**Description** `eq(A, B)` compares each element of `A` for equality with the corresponding element of `B`. If the elements are not equal or if either element is undefined, the test fails. `eq` does not expand or simplify expressions before making the comparison.

`A == B` is the alternate syntax for `eq(A, B)`.

**Examples** Check equality of two symbolic matrices:

```
A = sym(hilb(10));
B = sym([1/11 1/12 1/13 1/14 1/15 1/16]);
eq(A(9, 3:8), B)
```

The result is:

```
ans =
     1     1     1     1     1     1
```

---

Check the trigonometric identity:

```
syms x;
sin(x)^2 + cos(x)^2 == 1
```

The symbolic equality test might fail to recognize mathematical equivalence of polynomial or trigonometric expressions because it does not simplify or expand them. The result is:

```
ans =
     0
```

When testing mathematical equivalence of such expressions, simplify the difference between the expressions, and then compare the result with 0:

```
syms x;  
simplify(sin(x)^2 + cos(x)^2 - 1) == 0
```

The result is:

```
ans =  
    1
```

**See Also**

`simplify`

**Purpose** Evaluate MuPAD expressions

**Syntax**

```
result = evalin(symengine, 'MuPAD_expression')
[result,status] = evalin(symengine, 'MuPAD_expression')
```

**Description** `result = evalin(symengine, 'MuPAD_expression')` evaluates the MuPAD expression *MuPAD\_expression*, and returns *result* as a symbolic object.

`[result,status] = evalin(symengine, 'MuPAD_expression')` returns the error status in *status* and the error message in *result* if *status* is nonzero. If *status* is 0, *result* is a symbolic object; otherwise, it is a string.

**Examples** Compute the discriminant of the following polynomial:

```
evalin(symengine, 'polylib::discrim(a*x^2+b*x+c,x)')
```

The result is:

```
ans =
  b^2 - 4*a*c
```

Do not use `evalin` to access the MuPAD `log` function that represents the logarithm to an arbitrary base. The `evalin` command evaluates `log` as the natural logarithm (the appropriate MuPAD function is `ln`):

```
evalin(symengine, 'log(E)')
```

```
ans =
  1
```

Evaluating `log` with two parameters results in the following error:

```
evalin(symengine, 'log(10, 10)')
```

```
??? Error using ==> mupadengine.mupadengine>mupadengine.evalin at 124
Error: wrong no of args [ln]
```

# evalin

---

## See Also

doc | feval

## How To

- “Calling Built-In MuPAD Functions from the MATLAB Command Window” on page 4-40

**Purpose** Compute symbolic matrix exponential

**Syntax** `expm(A)`

**Description** `expm(A)` computes the matrix exponential of the symbolic matrix `A`.

**Examples** Compute the matrix exponential for the following matrix and simplify the result:

```
syms t;
A = [0 1; -1 0];
simplify(expm(t*A))
```

The result is:

```
ans =
[ cos(t), sin(t)]
[ -sin(t), cos(t)]
```

**See Also** `eig`

# expand

---

**Purpose** Symbolic expansion of polynomials and elementary functions

**Syntax** `expand(S)`

**Description** `expand(S)` writes each element of a symbolic expression `S` as a product of its factors. `expand` is often used with polynomials. It also expands trigonometric, exponential, and logarithmic functions.

**Examples** Expand the expression:

```
syms x;  
expand((x-2)*(x-4))
```

The result is:

```
ans =  
x^2 - 6*x + 8
```

Expand the trigonometric expression:

```
syms x y;  
expand(cos(x+y))
```

The result is:

```
ans =  
cos(x)*cos(y) - sin(x)*sin(y)
```

Expand the exponent:

```
syms a b;  
expand(exp((a+b)^2))
```

The result is:

```
ans =  
exp(2*a*b)*exp(a^2)*exp(b^2)
```

Expand the expressions that form a vector:



```
syms t;  
expand([sin(2*t), cos(2*t)])
```

The result is:

```
ans =  
[ 2*cos(t)*sin(t), cos(t)^2 - sin(t)^2]
```

## See Also

[collect](#) | [factor](#) | [horner](#) | [simple](#) | [simplify](#) | [syms](#)

**Purpose** Contour plotter

**Syntax**  
`ezcontour(f)`  
`ezcontour(f, domain)`  
`ezcontour(..., n)`

**Description** `ezcontour(f)` plots the contour lines of  $f(x,y)$ , where  $f$  is a symbolic expression that represents a mathematical function of two variables, such as  $x$  and  $y$ .

The function  $f$  is plotted over the default domain  $-2\pi < x < 2\pi$ ,  $-2\pi < y < 2\pi$ . MATLAB software chooses the computational grid according to the amount of variation that occurs; if the function  $f$  is not defined (singular) for points on the grid, then these points are not plotted.

`ezcontour(f, domain)` plots  $f(x,y)$  over the specified domain. `domain` can be either a 4-by-1 vector  $[xmin, xmax, ymin, ymax]$  or a 2-by-1 vector  $[min, max]$  (where,  $min < x < max$ ,  $min < y < max$ ).

If  $f$  is a function of the variables  $u$  and  $v$  (rather than  $x$  and  $y$ ), then the domain endpoints  $umin$ ,  $umax$ ,  $vmin$ , and  $vmax$  are sorted alphabetically. Thus, `ezcontour(u^2 - v^3, [0, 1], [3, 6])` plots the contour lines for  $u^2 - v^3$  over  $0 < u < 1$ ,  $3 < v < 6$ .

`ezcontour(..., n)` plots  $f$  over the default domain using an  $n$ -by- $n$  grid. The default value for  $n$  is 60.

`ezcontour` automatically adds a title and axis labels.

**Examples** The following mathematical expression defines a function of two variables,  $x$  and  $y$ .

$$f(x,y) = 3(1-x)^2 e^{-x^2-(y+1)^2} - 10\left(\frac{x}{5} - x^3 - y^5\right) e^{-x^2-y^2} - \frac{1}{3} e^{-(x+1)^2-y^2}.$$

`ezcontour` requires a `sym` argument that expresses this function using MATLAB syntax to represent exponents, natural logs, etc. This function is represented by the symbolic expression

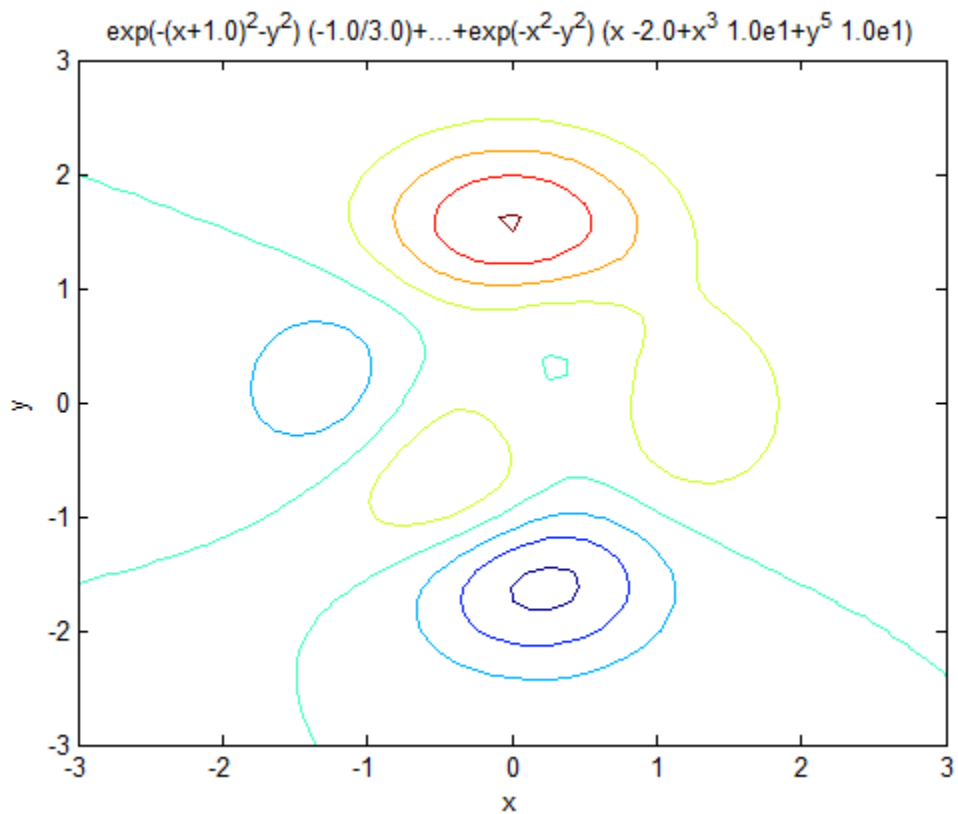
```
syms x y
f = 3*(1-x)^2*exp(-(x^2)-(y+1)^2)...
    - 10*(x/5 - x^3 - y^5)*exp(-x^2-y^2)...
    - 1/3*exp(-(x+1)^2 - y^2);
```

For convenience, this expression is written on three lines.

Pass the sym `f` to `ezcontour` along with a domain ranging from -3 to 3 and specify a computational grid of 49-by-49.

```
ezcontour(f, [-3,3], 49)
```

## ezcontour



In this particular case, the title is too long to fit at the top of the graph so MATLAB abbreviates the string.

### See Also

[contour](#) | [ezcontourf](#) | [ezmesh](#) | [ezmeshc](#) | [ezplot](#) | [ezplot3](#) | [ezpolar](#) | [ezsurf](#) | [ezsurf](#)

**Purpose**

Filled contour plotter

**Syntax**

```
ezcontour(f)
ezcontour(f, domain)
ezcontourf(..., n)
```

**Description**

`ezcontour(f)` plots the contour lines of  $f(x,y)$ , where  $f$  is a `sym` that represents a mathematical function of two variables, such as  $x$  and  $y$ .

The function  $f$  is plotted over the default domain  $-2\pi < x < 2\pi$ ,  $-2\pi < y < 2\pi$ . MATLAB software chooses the computational grid according to the amount of variation that occurs; if the function  $f$  is not defined (singular) for points on the grid, then these points are not plotted.

`ezcontour(f, domain)` plots  $f(x,y)$  over the specified domain. `domain` can be either a 4-by-1 vector  $[x_{min}, x_{max}, y_{min}, y_{max}]$  or a 2-by-1 vector  $[min, max]$  (where,  $min < x < max$ ,  $min < y < max$ ).

If  $f$  is a function of the variables  $u$  and  $v$  (rather than  $x$  and  $y$ ), then the domain endpoints  $u_{min}$ ,  $u_{max}$ ,  $v_{min}$ , and  $v_{max}$  are sorted alphabetically. Thus, `ezcontourf(u^2 - v^3, [0, 1], [3, 6])` plots the contour lines for  $u^2 - v^3$  over  $0 < u < 1$ ,  $3 < v < 6$ .

`ezcontourf(..., n)` plots  $f$  over the default domain using an  $n$ -by- $n$  grid. The default value for  $n$  is 60.

`ezcontourf` automatically adds a title and axis labels.

**Examples**

The following mathematical expression defines a function of two variables,  $x$  and  $y$ .

$$f(x,y) = 3(1-x)^2 e^{-x^2-(y+1)^2} - 10\left(\frac{x}{5} - x^3 - y^5\right) e^{-x^2-y^2} - \frac{1}{3} e^{-(x+1)^2-y^2}.$$

`ezcontourf` requires a `sym` argument that expresses this function using MATLAB syntax to represent exponents, natural logs, etc. This function is represented by the symbolic expression

## ezcontourf

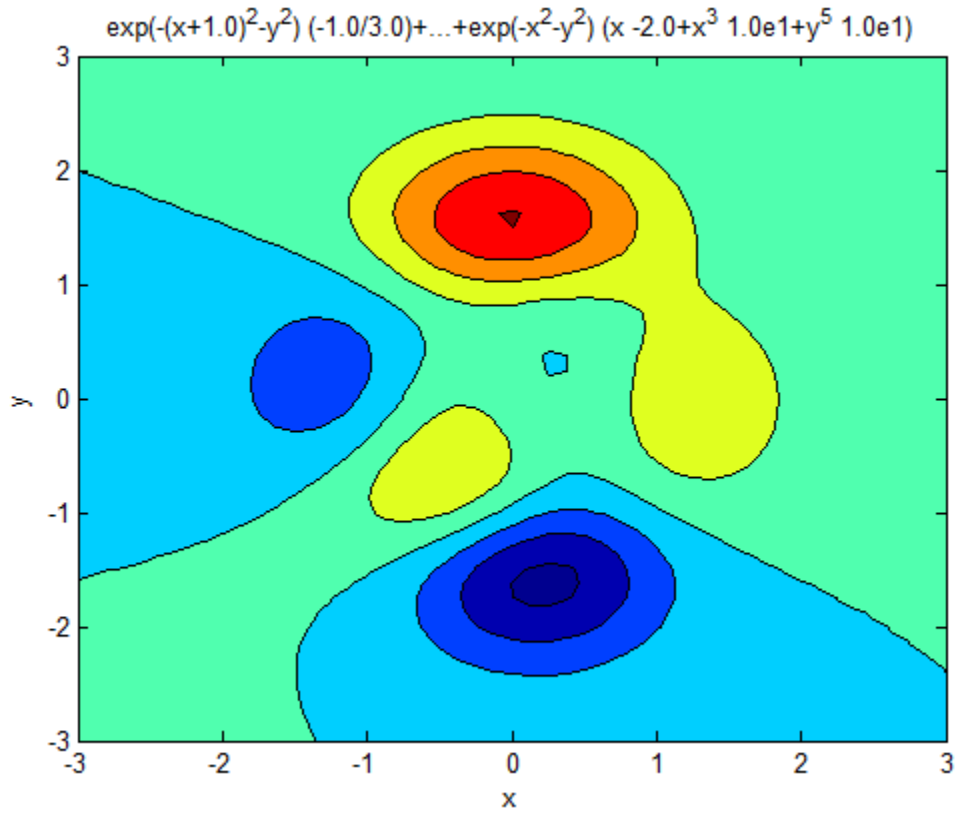
---

```
syms x y
f = 3*(1-x)^2*exp(-(x^2)-(y+1)^2)...
    - 10*(x/5 - x^3 - y^5)*exp(-x^2-y^2)...
    - 1/3*exp(-(x+1)^2 - y^2);
```

For convenience, this expression is written on three lines.

Pass the sym `f` to `ezcontourf` along with a domain ranging from -3 to 3 and specify a grid of 49-by-49.

```
ezcontourf(f, [-3,3], 49)
```



In this particular case, the title is too long to fit at the top of the graph so MATLAB abbreviates the string.

### See Also

[contourf](#) | [ezcontour](#) | [ezmesh](#) | [ezmeshc](#) | [ezplot](#) | [ezplot3](#) | [ezpolar](#) | [ezsurf](#) | [ezsurfz](#)

# ezmesh

---

**Purpose** 3-D mesh plotter

**Syntax**

```
ezmesh(f)
ezmesh(f, domain)
ezmesh(x,y,z)
ezmesh(x,y,z,[smin,smax,tmin,tmax])
ezmesh(x,y,z,[min,max])
ezmesh(...,n)
ezmesh(...,'circ')
```

**Description** `ezmesh(f)` creates a graph of  $f(x,y)$ , where  $f$  is a symbolic expression that represents a mathematical function of two variables, such as  $x$  and  $y$ .

The function  $f$  is plotted over the default domain  $-2\pi < x < 2\pi$ ,  $-2\pi < y < 2\pi$ . MATLAB software chooses the computational grid according to the amount of variation that occurs; if the function  $f$  is not defined (singular) for points on the grid, then these points are not plotted.

`ezmesh(f, domain)` plots  $f$  over the specified domain. `domain` can be either a 4-by-1 vector  $[xmin, xmax, ymin, ymax]$  or a 2-by-1 vector  $[min, max]$  (where,  $min < x < max$ ,  $min < y < max$ ).

If  $f$  is a function of the variables  $u$  and  $v$  (rather than  $x$  and  $y$ ), then the domain endpoints  $umin$ ,  $umax$ ,  $vmin$ , and  $vmax$  are sorted alphabetically. Thus, `ezmesh(u^2 - v^3, [0, 1], [3, 6])` plots  $u^2 - v^3$  over  $0 < u < 1$ ,  $3 < v < 6$ .

`ezmesh(x, y, z)` plots the parametric surface  $x = x(s,t)$ ,  $y = y(s,t)$ , and  $z = z(s,t)$  over the square  $-2\pi < s < 2\pi$ ,  $-2\pi < t < 2\pi$ .

`ezmesh(x, y, z, [smin, smax, tmin, tmax])` or `ezmesh(x, y, z, [min, max])` plots the parametric surface using the specified domain.

`ezmesh(..., n)` plots  $f$  over the default domain using an  $n$ -by- $n$  grid. The default value for  $n$  is 60.

`ezmesh(..., 'circ')` plots  $f$  over a disk centered on the domain.



**Examples**

This example visualizes the function,

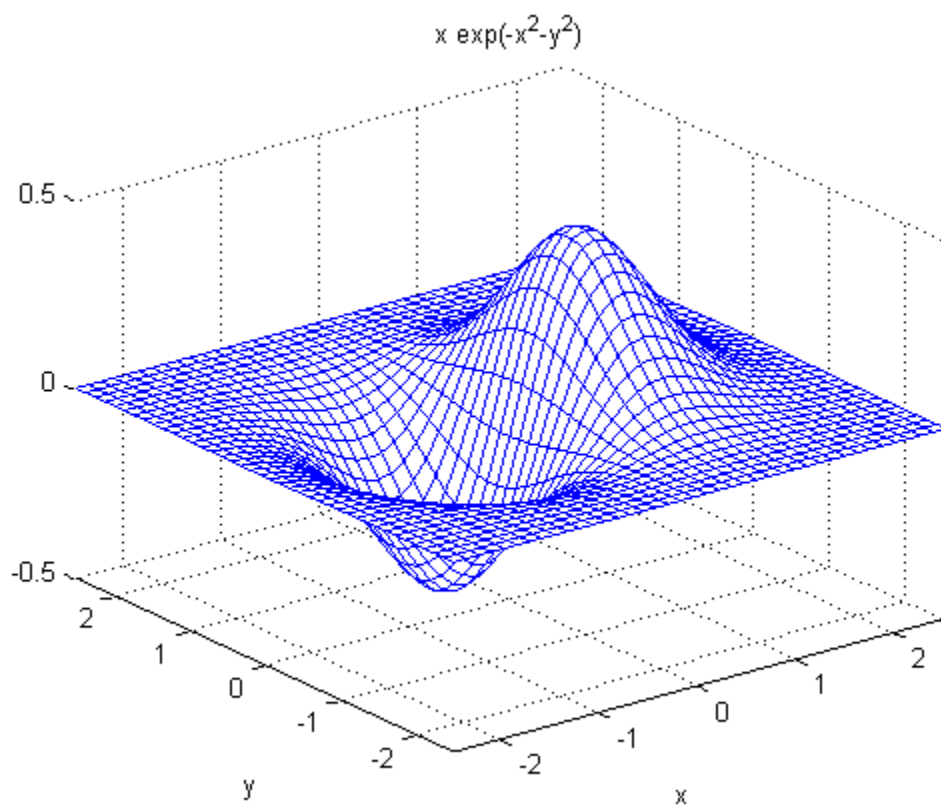
$$f(x, y) = xe^{-x^2-y^2},$$

with a mesh plot drawn on a 40-by-40 grid. The mesh lines are set to a uniform blue color by setting the colormap to a single color.

```
syms x y
ezmesh(x*exp(-x^2-y^2), [-2.5, 2.5], 40)
colormap([0 0 1])
```

## ezmesh

---



### See Also

[ezcontour](#) | [ezcontourf](#) | [ezmeshc](#) | [ezplot](#) | [ezplot3](#) | [ezpolar](#) | [ezsurf](#) | [ezsurfz](#) | [ezsurfz](#) | [mesh](#)

**Purpose**

Combined mesh and contour plotter

**Syntax**

```
ezmeshc(f)
ezmeshc(f, domain)
ezmeshc(x, y, z)
ezmeshc(x, y, z, [smin, smax, tmin, tmax])
ezmeshc(x, y, z, [min, max])
ezmeshc(..., n)
ezmeshc(..., 'circ')
```

**Description**

`ezmeshc(f)` creates a graph of  $f(x,y)$ , where  $f$  is a symbolic expression that represents a mathematical function of two variables, such as  $x$  and  $y$ .

The function  $f$  is plotted over the default domain  $-2\pi < x < 2\pi$ ,  $-2\pi < y < 2\pi$ . MATLAB software chooses the computational grid according to the amount of variation that occurs; if the function  $f$  is not defined (singular) for points on the grid, then these points are not plotted.

`ezmeshc(f, domain)` plots  $f$  over the specified domain. `domain` can be either a 4-by-1 vector  $[xmin, xmax, ymin, ymax]$  or a 2-by-1 vector  $[min, max]$  (where,  $min < x < max$ ,  $min < y < max$ ).

If  $f$  is a function of the variables  $u$  and  $v$  (rather than  $x$  and  $y$ ), then the domain endpoints  $umin$ ,  $umax$ ,  $vmin$ , and  $vmax$  are sorted alphabetically. Thus, `ezmeshc(u^2 - v^3, [0, 1], [3, 6])` plots  $u^2 - v^3$  over  $0 < u < 1$ ,  $3 < v < 6$ .

`ezmeshc(x, y, z)` plots the parametric surface  $x = x(s,t)$ ,  $y = y(s,t)$ , and  $z = z(s,t)$  over the square  $-2\pi < s < 2\pi$ ,  $-2\pi < t < 2\pi$ .

`ezmeshc(x, y, z, [smin, smax, tmin, tmax])` or `ezmeshc(x, y, z, [min, max])` plots the parametric surface using the specified domain.

`ezmeshc(..., n)` plots  $f$  over the default domain using an  $n$ -by- $n$  grid. The default value for  $n$  is 60.

`ezmeshc(..., 'circ')` plots  $f$  over a disk centered on the domain.

## Examples

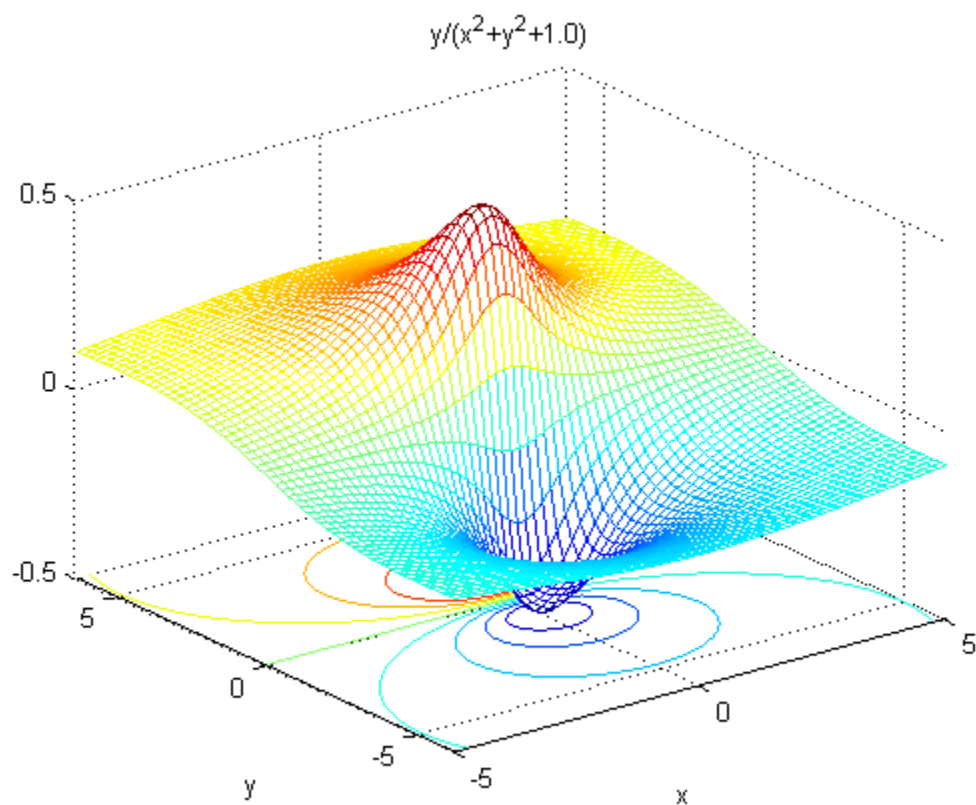
Create a mesh/contour graph of the expression,

$$f(x, y) = \frac{y}{1 + x^2 + y^2},$$

over the domain  $-5 < x < 5$ ,  $-2\pi < y < 2\pi$ .

```
syms x y
ezmeshc(y/(1 + x^2 + y^2), [-5,5, -2*pi, 2*pi])
```

Use the mouse to rotate the axes to better observe the contour lines (this picture uses a view of azimuth = -65 and elevation = 26).

**See Also**

[ezcontour](#) | [ezcontourf](#) | [ezmesh](#) | [ezplot](#) | [ezplot3](#) | [ezpolar](#) | [ezsurf](#) | [ezsurfz](#) | [ezsurfz](#) | [meshc](#)

# ezplot

---

## Purpose

Function plotter

## Syntax

```
ezplot(f)
ezplot(f,[xmin xmax])
ezplot(f,[xmin xmax], fign)
ezplot(f,[xmin, xmax, ymin, ymax])
ezplot(x,y)
ezplot(x,y,[tmin,tmax])
ezplot(...,figure)
```

## Description

`ezplot(f)` plots the expression  $f = f(x)$  over the default domain  $-2\pi < x < 2\pi$ .

`ezplot(f,[xmin xmax])` plots  $f = f(x)$  over the specified domain. It opens and displays the result in a window labeled **Figure No. 1**. If any plot windows are already open, `ezplot` displays the result in the highest numbered window.

`ezplot(f,[xmin xmax], fign)` opens (if necessary) and displays the plot in the window labeled `fign`.

For implicitly defined functions,  $f = f(x,y)$ .

`ezplot(f)` plots  $f(x,y) = 0$  over the default domain  $-2\pi < x < 2\pi$ ,  $-2\pi < y < 2\pi$ .

`ezplot(f,[xmin, xmax, ymin, ymax])` plots  $f(x,y) = 0$  over  $x_{\min} < x < x_{\max}$  and  $y_{\min} < y < y_{\max}$ .

`ezplot(f,[min,max])` plots  $f(x,y) = 0$  over  $\min < x < \max$  and  $\min < y < \max$ .

If  $f$  is a function of the variables  $u$  and  $v$  (rather than  $x$  and  $y$ ), then the domain endpoints  $u_{\min}$ ,  $u_{\max}$ ,  $v_{\min}$ , and  $v_{\max}$  are sorted alphabetically. Thus, `ezplot(u^2 - v^2 - 1, [-3,2, -2,3])` plots  $u^2 - v^2 - 1 = 0$  over  $-3 < u < 2$ ,  $-2 < v < 3$ .

`ezplot(x,y)` plots the parametrically defined planar curve  $x = x(t)$  and  $y = y(t)$  over the default domain  $0 < t < 2\pi$ .

`ezplot(x,y,[tmin,tmax])` plots  $x = x(t)$  and  $y = y(t)$  over  $t_{\min} < t < t_{\max}$ .

`ezplot(...,figure)` plots the given function over the specified domain in the figure window identified by the handle `figure`.

## Algorithms

If you do not specify a plot range, `ezplot` samples the function between  $-2\pi$  and  $2\pi$  and selects a subinterval where the variation is significant as the plot domain. For the range, `ezplot` omits extreme values associated with singularities.

## Examples

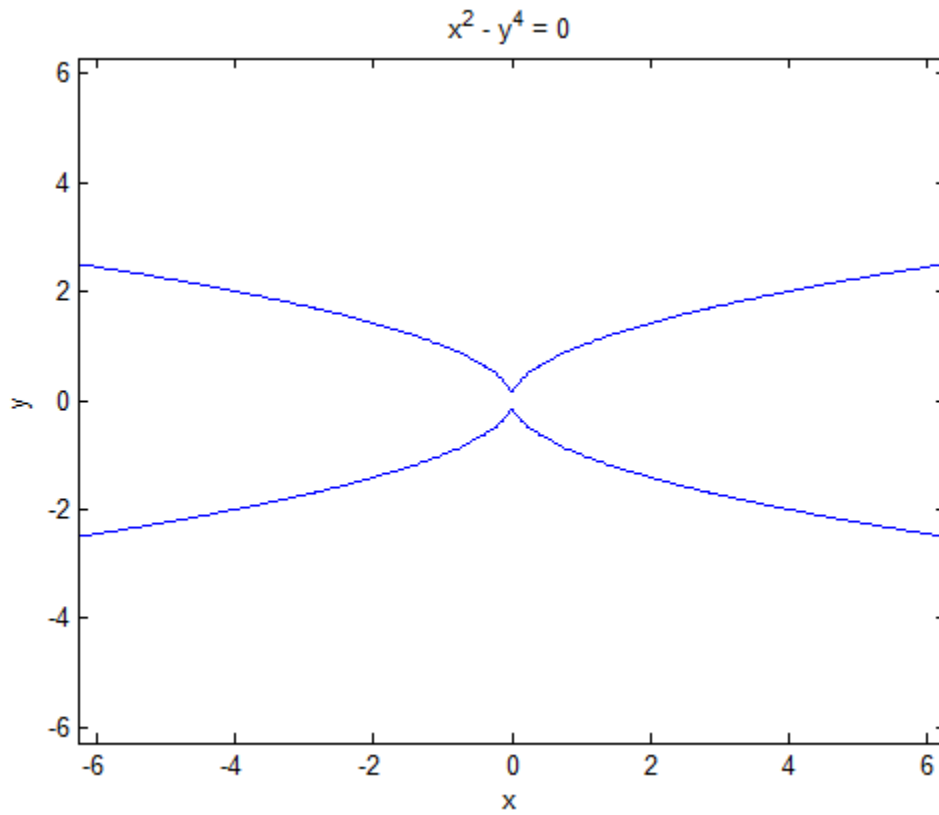
This example plots the implicitly defined function,

$$x^2 - y^4 = 0$$

over the domain  $[-2\pi, 2\pi]$ .

```
syms x y
ezplot(x^2-y^4)
```

# ezplot

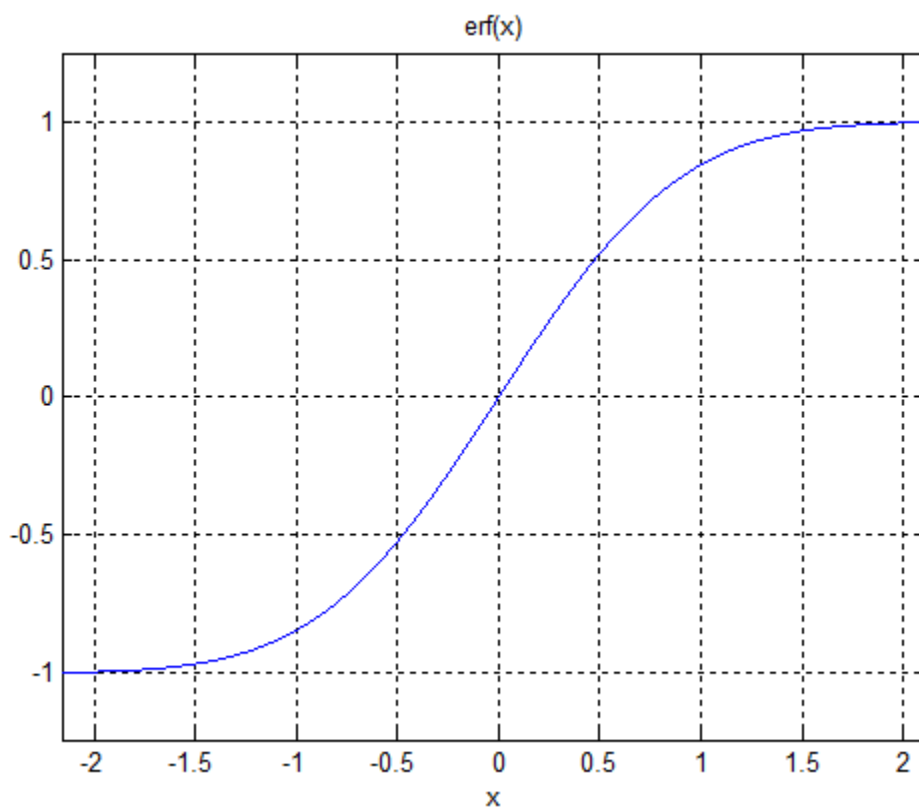


The following statements

```
syms x
ezplot(erf(x))
grid
```

plot a graph of the error function.



**See Also**

[ezcontour](#) | [ezcontourf](#) | [ezmesh](#) | [ezmeshc](#) | [ezplot3](#) | [ezpolar](#) | [ezsurf](#) | [ezsurfz](#) | [ezsurfz](#) | [plot](#)

# ezplot3

---

**Purpose** 3-D parametric curve plotter

**Syntax**  
`ezplot3(x,y,z)`  
`ezplot3(x,y,z,[tmin,tmax])`  
`ezplot3(...,'animate')`

**Description** `ezplot3(x,y,z)` plots the spatial curve  $x = x(t)$ ,  $y = y(t)$ , and  $z = z(t)$  over the default domain  $0 < t < 2\pi$ .  
`ezplot3(x,y,z,[tmin,tmax])` plots the curve  $x = x(t)$ ,  $y = y(t)$ , and  $z = z(t)$  over the domain  $tmin < t < tmax$ .  
`ezplot3(...,'animate')` produces an animated trace of the spatial curve.

**Examples** This example plots the parametric curve,  $x = \sin(t)$ ,  $y = \cos(t)$ ,  $z = t$  over the domain  $[0, 6\pi]$ .

```
syms t;  
ezplot3(sin(t), cos(t), t,[0,6*pi])
```



# ezpolar

---

**Purpose** Polar coordinate plotter

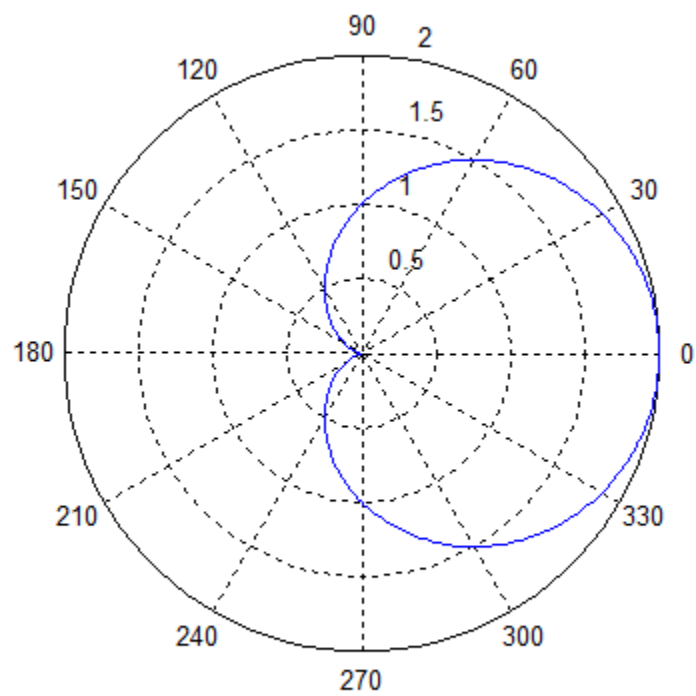
**Syntax** `ezpolar(f)`  
`ezpolar(f, [a, b])`

**Description** `ezpolar(f)` plots the polar curve  $r = f(\theta)$  over the default domain  $0 < \theta < 2\pi$ .  
`ezpolar(f, [a, b])` plots  $f$  for  $a < \theta < b$ .

**Examples** This example creates a polar plot of the function,

```
1 + cos(t)
over the domain [0, 2π].

syms t
ezpolar(1 + cos(t))
```



$$r = \cos(t) + 1$$

## Purpose

3-D colored surface plotter

## Syntax

```
ezsurf(f)
ezsurf(f, domain)
ezsurf(x,y,z)
ezsurf(x,y,z,[smin,smax,tmin,tmax])
ezsurf(x,y,z,[min,max])
ezsurf(...,n)
ezsurf(...,'circ')
```

`ezsurf(f)` plots over the default domain  $-2\pi < x < 2\pi$ ,  $-2\pi < y < 2\pi$ . MATLAB software chooses the computational grid according to the amount of variation that occurs; if the function  $f$  is not defined (singular) for points on the grid, then these points are not plotted.

`ezsurf(f, domain)` plots  $f$  over the specified domain. `domain` can be either a 4-by-1 vector  $[xmin, xmax, ymin, ymax]$  or a 2-by-1 vector  $[min, max]$  (where,  $min < x < max$ ,  $min < y < max$ ).

If  $f$  is a function of the variables  $u$  and  $v$  (rather than  $x$  and  $y$ ), then the domain endpoints  $umin$ ,  $umax$ ,  $vmin$ , and  $vmax$  are sorted alphabetically. Thus, `ezsurf(u^2 - v^3, [0, 1], [3, 6])` plots  $u^2 - v^3$  over  $0 < u < 1$ ,  $3 < v < 6$ .

`ezsurf(x,y,z)` plots the parametric surface  $x = x(s,t)$ ,  $y = y(s,t)$ , and  $z = z(s,t)$  over the square  $-2\pi < s < 2\pi$ ,  $-2\pi < t < 2\pi$ .

`ezsurf(x,y,z,[smin,smax,tmin,tmax])` or `ezsurf(x,y,z,[min,max])` plots the parametric surface using the specified domain.

`ezsurf(...,n)` plots  $f$  over the default domain using an  $n$ -by- $n$  grid. The default value for  $n$  is 60.

`ezsurf(...,'circ')` plots  $f$  over a disk centered on the domain.

## Examples

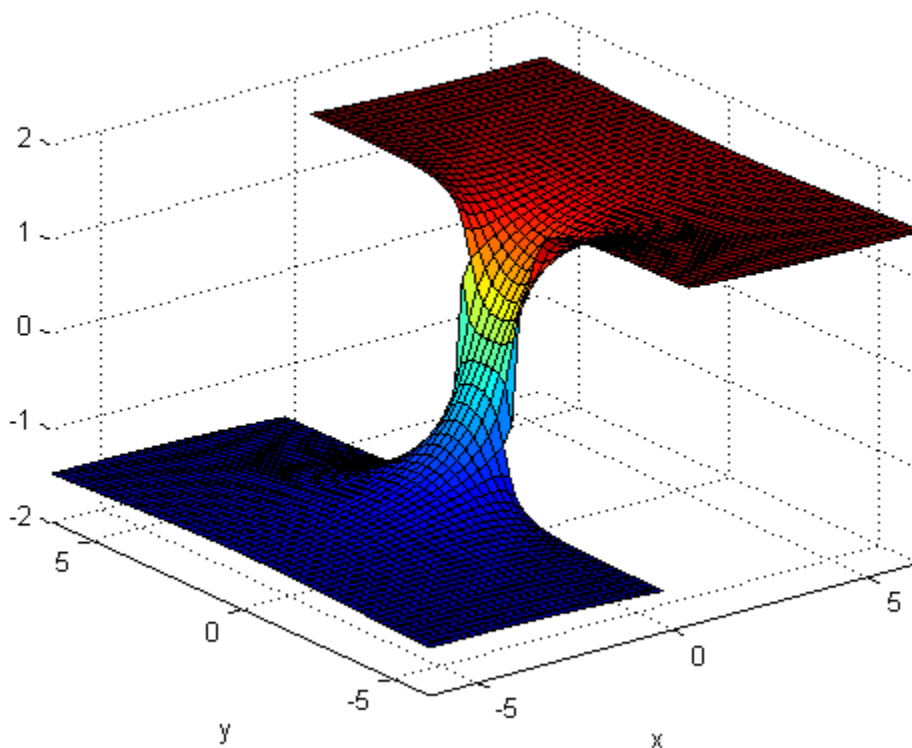
`ezsurf` does not graph points where the mathematical function is not defined (these data points are set to NaNs, which MATLAB does not plot). This example illustrates this filtering of singularities/discontinuous points by graphing the function,

$$f(x,y) = \text{real}(\text{atan}(x + iy))$$

over the default domain  $-2\pi < x < 2\pi$ ,  $-2\pi < y < 2\pi$ .

```
syms x y
ezsurf(real(atan(x+i*y)))
```

$$\text{atan}(x + y i)/2 + \text{conj}(\text{atan}(x + y i))/2$$



Note also that ezsurf creates graphs that have axis labels, a title, and extend to the axis limits.

# ezsurf

---

## See Also

ezcontour | ezcontourf | ezmesh | ezmeshc | ezplot | ezpolar |  
ezsurf | surf



**Purpose**

Combined surface and contour plotter

**Syntax**

```
ezsurf(f)
ezsurf(f,domain)
ezsurf(x,y,z)
ezsurf(x,y,z,[smin,smax,tmin,tmax])
ezsurf(x,y,z,[min,max])
ezsurf(...,n)
ezsurf(...,'circ')
```

**Description**

`ezsurf(f)` creates a graph of  $f(x,y)$ , where  $f$  is a symbolic expression that represents a mathematical function of two variables, such as  $x$  and  $y$ .

The function  $f$  is plotted over the default domain  $-2\pi < x < 2\pi$ ,  $-2\pi < y < 2\pi$ . MATLAB software chooses the computational grid according to the amount of variation that occurs; if the function  $f$  is not defined (singular) for points on the grid, then these points are not plotted.

`ezsurf(f,domain)` plots  $f$  over the specified domain. `domain` can be either a 4-by-1 vector  $[xmin, xmax, ymin, ymax]$  or a 2-by-1 vector  $[min, max]$  (where,  $min < x < max$ ,  $min < y < max$ ).

If  $f$  is a function of the variables  $u$  and  $v$  (rather than  $x$  and  $y$ ), then the domain endpoints  $umin$ ,  $umax$ ,  $vmin$ , and  $vmax$  are sorted alphabetically. Thus, `ezsurf(u^2 - v^3, [0,1],[3,6])` plots  $u^2 - v^3$  over  $0 < u < 1$ ,  $3 < v < 6$ .

`ezsurf(x,y,z)` plots the parametric surface  $x = x(s,t)$ ,  $y = y(s,t)$ , and  $z = z(s,t)$  over the square  $-2\pi < s < 2\pi$ ,  $-2\pi < t < 2\pi$ .

`ezsurf(x,y,z,[smin,smax,tmin,tmax])` or `ezsurf(x,y,z,[min,max])` plots the parametric surface using the specified domain.

`ezsurf(...,n)` plots  $f$  over the default domain using an  $n$ -by- $n$  grid. The default value for  $n$  is 60.

`ezsurf(...,'circ')` plots  $f$  over a disk centered on the domain.

## Examples

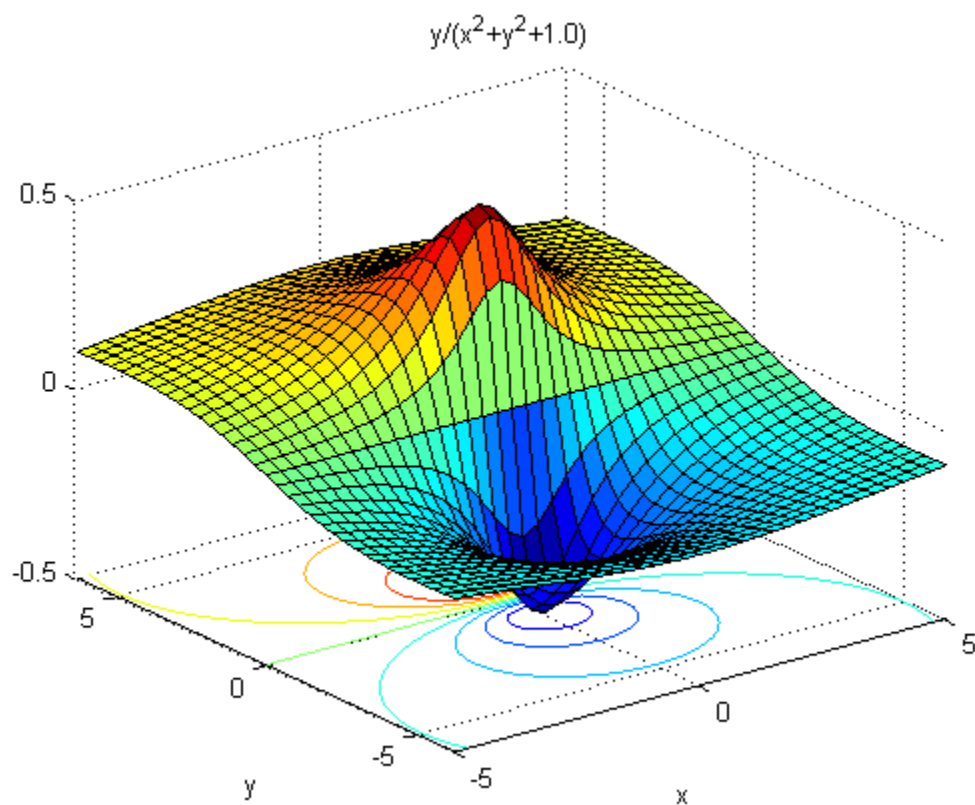
Create a surface/contour plot of the expression,

$$f(x, y) = \frac{y}{1 + x^2 + y^2},$$

over the domain  $-5 < x < 5$ ,  $-2\pi < y < 2\pi$ , with a computational grid of size 35-by-35

```
syms x y
ezsurf(y/(1 + x^2 + y^2), [-5,5, -2*pi,2*pi], 35)
```

Use the mouse to rotate the axes to better observe the contour lines (this picture uses a view of azimuth = -65 and elevation = 26).

**See Also**

[ezcontour](#) | [ezcontourf](#) | [ezmesh](#) | [ezmeshc](#) | [ezplot](#) | [ezpolar](#) | [ezsurf](#) | [surf](#)

# factor

---

**Purpose** Factorization

**Syntax** `factor(X)`

**Description** `factor(X)` can take a positive integer, an array of symbolic expressions, or an array of symbolic integers as an argument. If  $N$  is a positive integer, `factor(N)` returns the prime factorization of  $N$ .

If  $S$  is a matrix of polynomials or integers, `factor(S)` factors each element. If any element of an integer array has more than 16 digits, you must use `sym` to create that element, for example, `sym('N')`.

**Examples** Factorize the two-variable expression:

```
syms x y;  
factor(x^3-y^3)
```

The result is:

```
ans =  
(x - y)*(x^2 + x*y + y^2)
```

Factorize the expressions that form a vector:

```
syms a b;  
factor([a^2 - b^2, a^3 + b^3])
```

The result is:

```
ans =  
[ (a - b)*(a + b), (a + b)*(a^2 - a*b + b^2)]
```

Factorize the symbolic number:

```
factor(sym('12345678901234567890'))
```

The result is:

```
ans =
```

$$2 \cdot 3^2 \cdot 5 \cdot 101 \cdot 3541 \cdot 3607 \cdot 3803 \cdot 27961$$
**See Also**`collect | expand | horner | simplify | simple`

# feval

---

**Purpose** Evaluate MuPAD expressions

**Syntax** `result = feval(symengine,F,x1,...,xn)`  
`[result,status] = feval(symengine,F,x1,...,xn)`

**Description** `result = feval(symengine,F,x1,...,xn)` evaluates `F`, which is either a MuPAD function name or a symbolic object, with arguments `x1,...,xn`, with `result` a symbolic object.

`[result,status] = feval(symengine,F,x1,...,xn)` returns the error status in `status`, and the error message in `result` if `status` is nonzero. If `status` is 0, `result` is a symbolic object. Otherwise, `result` is a string.

**Examples**

```
syms a b c x
p = a*x^2+b*x+c;
feval(symengine,'polylib::discrim', p, x)

ans =
b^2 - 4*a*c
```

Alternatively, the same calculation based on variables not defined in the MATLAB workspace is:

```
feval(symengine,'polylib::discrim', 'a*x^2
+ b*x + c', 'x')

ans =
b^2 - 4*a*c
```

Do not use `feval` to access the MuPAD `log` function that represents the logarithm to an arbitrary base. The `feval` command evaluates `log` as the natural logarithm (the appropriate MuPAD function is `ln`):

```
feval(symengine,'log', 'E')

ans =
1
```

Evaluating `log` with two parameters results in the following error:

```
feval(symengine, 'log', '10', '10')
```

```
??? Error using ==> mupadengine.mupadengine>mupadengine.feval at 163  
Error: wrong no of args [1n]
```

## See Also

`doc | evalin`

## How To

- “Calling Built-In MuPAD Functions from the MATLAB Command Window” on page 4-40

# findsym

---

**Purpose** Determine variables in symbolic expression or matrix

---

**Note** `findsym` is not recommended. Use `symvar` instead.

---

**Syntax** `findsym(S)`  
`findsym(S, n)`

**Description** `findsym(S)` for a symbolic expression or matrix `S`, returns all symbolic variables in `S` in lexicographical order, separated by commas. If `S` does not contain any variables, `findsym` returns an empty string.

`findsym(S, n)` returns the `n` variables alphabetically closest to `x`:

- 1** The variables are sorted by the first letters in their names. The ordering is `x y w z v u ... a X Y W Z V U ... A`. The name of a symbolic variable cannot begin with a number.
- 2** For all subsequent letters the ordering is alphabetical, with all uppercase letters having precedence over lowercase:  
`0 1 ... 9 A B ... Z a b ...z`.

`findsym(S)` can return variables in different order than `findsym(S, n)`.

**See Also** `symvar`



## Purpose

Functional inverse

## Syntax

```
g = finverse(f)
g = finverse(f,v)
```

## Description

`g = finverse(f)` returns the functional inverse of `f`. `f` is a scalar sym representing a function of one symbolic variable, say `x`. Then `g` is a scalar sym that satisfies  $f(g(x)) = x$ . That is, `finverse(f)` returns  $f^{-1}$ , provided  $f^{-1}$  exists.

`g = finverse(f,v)` uses the symbolic variable `v`, where `v` is a sym, as the independent variable. Then `g` is a scalar sym that satisfies  $f(g(v)) = v$ . Use this form when `f` contains more than one symbolic variable.

## Examples

Compute functional inverse for the trigonometric function:

```
syms x u v;
finverse(1/tan(x))
```

The result is:

```
ans =
atan(1/x)
```

Compute functional inverse for the exponent function:

```
finverse(exp(u - 2*v), u)
```

The result is:

```
ans =
2*v + log(u)
```

## See Also

`compose` | `syms`

# fix

---

**Purpose**            Round toward zero

**Syntax**            `fix(X)`

**Description**        `fix(X)` is the matrix of the integer parts of  $X$ .  
`fix(X) = floor(X)` if  $X$  is positive and `ceil(X)` if  $X$  is negative.

**See Also**            `round` | `ceil` | `floor` | `frac`

**Purpose** Round symbolic matrix toward negative infinity

**Syntax** `floor(X)`

**Description** `floor(X)` is the matrix of the greatest integers less than or equal to  $X$ .

**Examples**

```
x = sym(-5/2);  
[fix(x) floor(x) round(x) ceil(x) frac(x)]  
  
ans =  
[ -2, -3, -3, -2, -1/2]
```

**See Also** `round` | `ceil` | `fix` | `frac`

**Purpose** Fortran representation of symbolic expression

**Syntax** `fortran(S)`  
`fortran(S, 'file', fileName)`

**Description** `fortran(S)` returns the Fortran code equivalent to the expression `S`.  
`fortran(S, 'file', fileName)` writes an “optimized” Fortran code fragment that evaluates the symbolic expression `S` to the file named `fileName`. “Optimized” means intermediate variables are automatically generated in order to simplify the code. MATLAB generates intermediate variables as a lowercase letter `t` followed by an automatically generated number, for example `t32`.

**Examples** The statements

```
syms x
f = taylor(log(1+x));
fortran(f)
```

return

```
ans =
      t0 = x-x**2*(1.000/2.000)+x**3*(1.000/3.000)-x**4*(1.000/4.000)+x*
      +*5*(1.000/5.000)
```

The statements

```
H = sym(hilb(3));
fortran(H)
```

return

```
ans =
      H(1,1) = 1.000
      H(1,2) = 1.000/2.000
      H(1,3) = 1.000/3.000
      H(2,1) = 1.000/2.000
```

```
H(2,2) = 1.0D0/3.0D0
H(2,3) = 1.0D0/4.0D0
H(3,1) = 1.0D0/3.0D0
H(3,2) = 1.0D0/4.0D0
H(3,3) = 1.0D0/5.0D0
```

The statements

```
syms x
z = exp(-exp(-x));
fortran(diff(z,3), 'file', 'fortrantest');
```

return a file named `fortrantest` containing the following:

```
t7 = exp(-x)
t8 = exp(-t7)
t0 = t8*exp(x*(-2))*(-3)+t8*exp(x*(-3))+t7*t8
```

## See Also

[ccode](#) | [latex](#) | [matlabFunction](#) | [pretty](#)

## How To

- “Generating Code from Symbolic Expressions” on page 3-134

# fourier

---

**Purpose**            Fourier integral transform

**Syntax**             $F = \text{fourier}(f)$   
                       $F = \text{fourier}(f, v)$   
                       $F = \text{fourier}(f, u, v)$

**Description**       $F = \text{fourier}(f)$  is the Fourier transform of the symbolic scalar  $f$  with default independent variable  $x$ . The default return is a function of  $w$ . The Fourier transform is applied to a function of  $x$  and returns a function of  $w$ .

$$f = f(x) \Rightarrow F = F(w)$$

If  $f = f(w)$ , `fourier` returns a function of  $t$ .

$$F = F(t)$$

By definition,

$$F(w) = \int_{-\infty}^{\infty} f(x)e^{-iwx} dx$$

where  $x$  is the symbolic variable in  $f$  as determined by `symvar`.

$F = \text{fourier}(f, v)$  makes  $F$  a function of the symbol  $v$  instead of the default  $w$ .

$$F(v) = \int_{-\infty}^{\infty} f(x)e^{-ivx} dx$$

$F = \text{fourier}(f, u, v)$  makes  $f$  a function of  $u$  and  $F$  a function of  $v$  instead of the default variables  $x$  and  $w$ , respectively.

$$F(v) = \int_{-\infty}^{\infty} f(u)e^{-ivu} du$$

Examples

Fourier Transform	MATLAB Commands
$f(x) = e^{-x^2}$ $F[f](w) = \int_{-\infty}^{\infty} f(x)e^{-ixw} dx$ $= \sqrt{\pi} e^{-w^2/4}$	<pre>syms x; f = exp(-x^2); fourier(f)</pre> <p>returns</p> <pre>ans = pi^(1/2)/exp(w^2/4)</pre>
$g(w) = e^{- w }$ $F[g](t) = \int_{-\infty}^{\infty} g(w)e^{-itw} dw$ $= \frac{2}{1+t^2}$	<pre>syms w; g = exp(-abs(w)); fourier(g)</pre> <p>returns</p> <pre>ans = 2/(v^2 + 1)</pre>
$f(x) = xe^{- x }$ $F[f](u) = \int_{-\infty}^{\infty} f(x)e^{-ixu} dx$ $= -\frac{4iu}{(1+u^2)^2}$	<pre>syms x u; f = x*exp(-abs(x)); fourier(f,u)</pre> <p>returns</p> <pre>ans = -(u*4*i)/(u^2 + 1)^2</pre>
$f(x,v) = e^{-x^2 \frac{ v  \sin v}{v}}, x \text{ real}$ $F[f(v)](u) = \int_{-\infty}^{\infty} f(x,v)e^{-ivu} dv$	<pre>syms v u; syms x real; f = exp(-x^2*abs(v))*sin(v)/v; fourier(f,v,u)</pre> <p>returns</p>

# fourier

---

Fourier Transform	MATLAB Commands
$= -\arctan \frac{u-1}{x^2} + \arctan \frac{u+1}{x^2}$	<pre>ans = piecewise([x &lt;&gt; 0,... atan((u + 1)/x^2)... - atan((u - 1)/x^2)])</pre>

## See Also

ifourier | laplace | ztrans



---

<b>Purpose</b>	Symbolic matrix element-wise fractional parts
<b>Syntax</b>	<code>frac(X)</code>
<b>Description</b>	<code>frac(X)</code> is the matrix of the fractional parts of the elements: <code>frac(X) = X - fix(X)</code> .
<b>Examples</b>	<pre>x = sym(-5/2); [fix(x) floor(x) round(x) ceil(x) frac(x)]  ans = [ -2, -3, -3, -2, -1/2]</pre>
<b>See Also</b>	<code>round</code>   <code>ceil</code>   <code>floor</code>   <code>fix</code>

# funtool

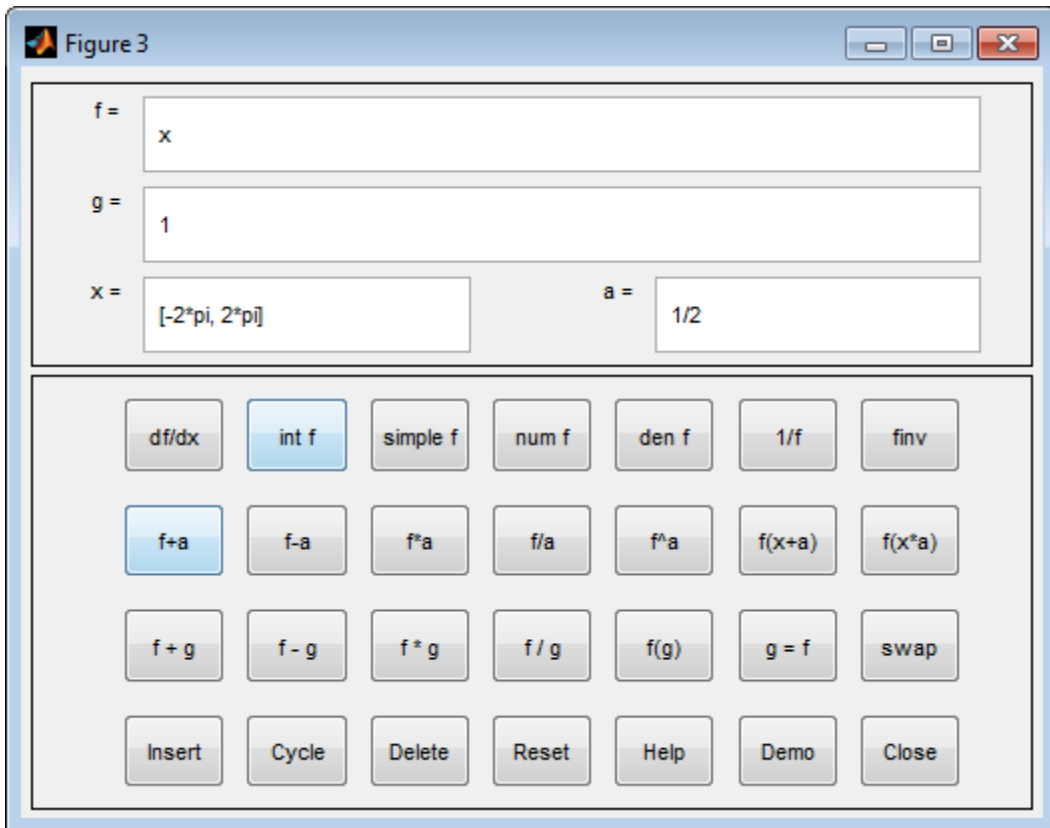
---

**Purpose**            Function calculator

**Syntax**            funtool

**Description**        funtool is a visual function calculator that manipulates and displays functions of one variable. At the click of a button, for example, funtool draws a graph representing the sum, product, difference, or ratio of two functions that you specify. funtool includes a function memory that allows you to store functions for later retrieval.

At startup, funtool displays graphs of a pair of functions,  $f(x) = x$  and  $g(x) = 1$ . The graphs plot the functions over the domain  $[-2\pi, 2\pi]$ . funtool also displays a control panel that lets you save, retrieve, redefine, combine, and transform  $f$  and  $g$ .



### Text Fields

The top of the control panel contains a group of editable text fields.

- f=** Displays a symbolic expression representing  $f$ . Edit this field to redefine  $f$ .
- g=** Displays a symbolic expression representing  $g$ . Edit this field to redefine  $g$ .

- x=** Displays the domain used to plot  $f$  and  $g$ . Edit this field to specify a different domain.
- a=** Displays a constant factor used to modify  $f$  (see button descriptions in the next section). Edit this field to change the value of the constant factor.

funtool redraws  $f$  and  $g$  to reflect any changes you make to the contents of the control panel's text fields.

## Control Buttons

The bottom part of the control panel contains an array of buttons that transform  $f$  and perform other operations.

The first row of control buttons replaces  $f$  with various transformations of  $f$ .

- df/dx** Derivative of  $f$
- int f** Integral of  $f$
- simple f** Simplified form of  $f$ , if possible
- num f** Numerator of  $f$
- den f** Denominator of  $f$
- 1/f** Reciprocal of  $f$
- finv** Inverse of  $f$

The operators **intf** and **finv** may fail if the corresponding symbolic expressions do not exist in closed form.

The second row of buttons translates and scales  $f$  and the domain of  $f$  by a constant factor. To specify the factor, enter its value in the field labeled **a=** on the calculator control panel. The operations are

- f+a** Replaces  $f(x)$  by  $f(x) + a$ .
- f-a** Replaces  $f(x)$  by  $f(x) - a$ .

<b>f*a</b>	Replaces $f(x)$ by $f(x) * a$ .
<b>f/a</b>	Replaces $f(x)$ by $f(x) / a$ .
<b>f^a</b>	Replaces $f(x)$ by $f(x) ^ a$ .
<b>f(x+a)</b>	Replaces $f(x)$ by $f(x + a)$ .
<b>f(x*a)</b>	Replaces $f(x)$ by $f(x * a)$ .

The first four buttons of the third row replace  $f$  with a combination of  $f$  and  $g$ .

<b>f+g</b>	Replaces $f(x)$ by $f(x) + g(x)$ .
<b>f-g</b>	Replaces $f(x)$ by $f(x) - g(x)$ .
<b>f*g</b>	Replaces $f(x)$ by $f(x) * g(x)$ .
<b>f/g</b>	Replaces $f(x)$ by $f(x) / g(x)$ .

The remaining buttons on the third row interchange  $f$  and  $g$ .

<b>g=f</b>	Replaces $g$ with $f$ .
<b>swap</b>	Replaces $f$ with $g$ and $g$ with $f$ .

The first three buttons in the fourth row allow you to store and retrieve functions from the calculator's function memory.

<b>Insert</b>	Adds $f$ to the end of the list of stored functions.
<b>Cycle</b>	Replaces $f$ with the next item on the function list.
<b>Delete</b>	Deletes $f$ from the list of stored functions.

The other four buttons on the fourth row perform miscellaneous functions:

<b>Reset</b>	Resets the calculator to its initial state.
<b>Help</b>	Displays the online help for the calculator.

# funtool

---

**Demo**            Runs a short demo of the calculator.

**Close**           Closes the calculator's windows.

**See Also**        `ezplot` | `syms`

<b>Purpose</b>	Gamma function
<b>Syntax</b>	gamma(x) gamma(A)
<b>Description</b>	gamma(x) returns the gamma function of a symbolic variable or symbolic expression x.  gamma(A) returns the gamma function of the elements of a symbolic vector or a symbolic matrix A.
<b>Input Arguments</b>	x  A symbolic variable or a symbolic expression  A  A vector or a matrix of symbolic variables or expressions

## Definitions **gamma Function**

The following integral defines the gamma function:

$$\Gamma(z) = \int_0^{\infty} t^{z-1} e^{-t} dt.$$

## Examples

Differentiate the gamma function, and then substitute the variable  $t$  with the value 1:

```
syms t
u = diff(gamma(t^3 + 1))
subs(u, 1)

u =
3*t^2*gamma(t^3 + 1)*psi(t^3 + 1)

ans =
1.2684
```

# gamma

---

---

Compute the limit of the following expression that involves the gamma function:

```
syms x;  
limit(x/gamma(x), x, inf)
```

```
ans =  
0
```

---

Simplify the following expression:

```
syms x;  
simplify(gamma(x)*gamma(1 - x))
```

```
ans =  
pi/sin(pi*x)
```

## See Also

[mfun](#) | [mfunlist](#)



**Purpose** Get variable from MuPAD notebook

**Syntax** `y = getVar(nb, 'z')`

**Description** `y = getVar(nb, 'z')` assigns the symbolic variable `z` in the MuPAD notebook `nb` to a symbolic variable `y` in the MATLAB workspace.

**Examples** Start a new MuPAD notebook and define a handle `mpnb` to that notebook:

```
mpnb = mupad;
```

In the MuPAD notebook, enter the command `f:=x^2`. This command creates the variable `f` and assigns the value `x^2` to this variable. At this point, the variable and its value exist only in MuPAD. Now, return to the MATLAB Command Window and use the `getVar` function:

```
f = getVar(mpnb, 'f')
```

After you use `getVar`, the variable `f` appears in the MATLAB workspace. The value of the variable `f` is `x^2`.

**See Also** `mupad` | `setVar`

# heaviside

---

**Purpose** Compute Heaviside step function

**Syntax** `heaviside(x)`

**Description** `heaviside(x)` has the value 0 for  $x < 0$ , 1 for  $x > 0$ , and 0.5 for  $x = 0$ .

**Examples** For  $x < 0$  the function `heaviside(x)` returns 0:

```
heaviside(sym(-3))
```

```
ans =  
0
```

For  $x > 0$  the function, `heaviside(x)` returns 1:

```
heaviside(sym(3))
```

```
ans =  
1
```

For  $x = 0$  the function, `heaviside(x)` returns 1/2:

```
heaviside(sym(0))
```

```
ans =  
1/2
```

For numeric  $x = 0$  the function, `heaviside(x)` returns the numeric result:

```
heaviside(0)
```

```
ans =  
0.5000
```

**See Also** `dirac`

**Purpose** Horner nested polynomial representation

**Syntax** horner(P)

**Description** Suppose P is a matrix of symbolic polynomials. horner(P) transforms each element of P into its Horner, or nested, representation.

**Examples** Find nested polynomial representation of the polynomial:

```
syms x
horner(x^3-6*x^2+11*x-6)
```

The result is

```
ans =
x*(x*(x - 6) + 11) - 6
```

Find nested polynomial representation for the polynomials that form a vector:

```
syms x y
horner([x^2+x;y^3-2*y])
```

The result is:

```
ans =
x*(x + 1)
y*(y^2 - 2)
```

**See Also** expand | factor | simple | simplify | syms

# hypergeom

---

**Purpose** Generalized hypergeometric

**Syntax** hypergeom(n, d, z)

**Description** hypergeom(n, d, z) is the generalized hypergeometric function  $F(n, d, z)$ , also known as the Barnes extended hypergeometric function and denoted by  ${}_jF_k$  where  $j = \text{length}(n)$  and  $k = \text{length}(d)$ . For scalar a, b, and c, hypergeom([a, b], c, z) is the Gauss hypergeometric function  ${}_2F_1(a, b; c; z)$ .

The definition by a formal power series is

$$F(n, d, z) = \sum_{k=0}^{\infty} \frac{C_{n,k}}{C_{d,k}} \cdot \frac{z^k}{k!},$$

where

$$C_{v,k} = \prod_{j=1}^{|v|} \frac{\Gamma(v_j + k)}{\Gamma(v_j)}.$$

Either of the first two arguments may be a vector providing the coefficient parameters for a single function evaluation. If the third argument is a vector, the function is evaluated point-wise. The result is numeric if all the arguments are numeric and symbolic if any of the arguments is symbolic.

See Abramowitz and Stegun, *Handbook of Mathematical Functions*, Chapter 15.

**Examples** Compute hypergeometric functions:

```
syms a z
q = hypergeom([], [], z)
r = hypergeom(1, [], z)
s = hypergeom(a, [], z)
```

The results are:

$$q = \exp(z)$$

$$r = -1/(z - 1)$$

$$s = 1/(1 - z)^a$$

# ifourier

---

**Purpose** Inverse Fourier integral transform

**Syntax**  
`f = ifourier(F)`  
`f = ifourier(F,u)`  
`f = ifourier(F,v,u)`

**Description** `f = ifourier(F)` is the inverse Fourier transform of the scalar symbolic object `F` with default independent variable `w`. The default return is a function of `x`. The inverse Fourier transform is applied to a function of `w` and returns a function of `x`.

$$F = F(w) \Rightarrow f = f(x).$$

If `F = F(x)`, `ifourier` returns a function of `t`:

$$f = f(t)$$

By definition

$$f(x) = 1/(2\pi) \int_{-\infty}^{\infty} F(w)e^{iwx} dw.$$

`f = ifourier(F,u)` makes `f` a function of `u` instead of the default `x`.

$$f(u) = 1/(2\pi) \int_{-\infty}^{\infty} F(w)e^{iwu} dw.$$

Here `u` is a scalar symbolic object.

`f = ifourier(F,v,u)` takes `F` to be a function of `v` and `f` to be a function of `u` instead of the default `w` and `x`, respectively.

$$f(u) = 1/(2\pi) \int_{-\infty}^{\infty} F(v)e^{ivu} dv.$$

**Examples**

Inverse Fourier Transform	MATLAB Commands
$f(w) = e^{-w^2/(4a^2)}$ $F^{-1}[f](x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(w)e^{ixw} dw$ $= \frac{ a }{\sqrt{\pi}} e^{-(ax)^2}$	<pre>syms a w real; f = exp(-w^2/(4*a^2)); F = ifourier(f); F = simple(F)</pre> <p>returns</p> <pre>F = abs(a)/(pi^(1/2)*exp(a^2*x^2))</pre>
$g(x) = e^{- x }$ $F^{-1}[g](t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} g(x)e^{itx} dx$ $= \frac{1}{\pi(1+t^2)}$	<pre>syms x real; g = exp(-abs(x)); ifourier(g)</pre> <p>returns</p> <pre>ans = 1/(pi*(t^2 + 1))</pre>
$f(w) = 2e^{- w } - 1$ $F^{-1}[f](t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(w)e^{itw} dw$ $= -\text{dirac}(t) + \frac{2}{\pi(1+t^2)}$	<pre>syms w t real; f = 2*exp(-abs(w)) - 1; simplify(ifourier(f,t))</pre> <p>returns</p> <pre>ans = 2/(pi*(t^2 + 1)) - dirac(t)</pre>

**See Also**

fourier | ilaplace | iztrans

# ilaplace

---

**Purpose** inverse Laplace transform  
Inverse Laplace transform

**Syntax**  
 $F = \text{ilaplace}(L)$   
 $F = \text{ilaplace}(L, y)$   
 $F = \text{ilaplace}(L, y, x)$

**Description**  $F = \text{ilaplace}(L)$  computes the inverse Laplace transform of the symbolic expression  $L$ . This syntax assumes that  $L$  is a function of the variable  $s$ , and the returned value  $F$  is a function of  $t$ .

$$L = L(s) \Rightarrow F = F(t)$$

If  $L = L(t)$ , `ilaplace` returns a function of  $x$ .

$$F = F(x)$$

By definition, the inverse Laplace transform is

$$F(t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} L(s)e^{st} ds,$$

where  $c$  is a real number selected so that all singularities of  $L(s)$  are to the left of the line  $s = c$ ,  $i$ .

$F = \text{ilaplace}(L, y)$  computes the inverse Laplace transform  $F$  as a function of  $y$  instead of the default variable  $t$ .

$$F(y) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} L(y)e^{sy} ds$$

$F = \text{ilaplace}(L, y, x)$  computes the inverse Laplace transform and lets you specify that  $F$  is a function of  $x$  and  $L$  is a function of  $y$ .



$$F(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} L(y)e^{xy} dy$$

### Examples

Inverse Laplace Transform	MATLAB Command
$f(s) = \frac{1}{s^2}$ $L^{-1}[f] = \frac{1}{2\pi i} \int_{c-ivo}^{c+ivo} f(s)e^{st} ds$ $= t$	<pre>syms s; f = 1/s^2; ilaplace(f)</pre> <p>returns</p> <pre>ans = t</pre>
$g(t) = \frac{1}{(t-a)^2}$ $L^{-1}[g] = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} g(t)e^{xt} dt$ $= xe^{ax}$	<pre>syms a t; g = 1/(t-a)^2; ilaplace(g)</pre> <p>returns</p> <pre>ans = x*exp(a*x)</pre>
$f(u) = \frac{1}{u^2 - a^2}$ $L^{-1}[f] = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} g(u)e^{xu} du$ $= \frac{\sinh(xa)}{a}$	<pre>syms x u; syms a real; f = 1/(u^2-a^2); simplify(ilaplace(f,x))</pre> <p>returns</p> <pre>ans = sinh(a*x)/a</pre>

# ilaplace

---

## See Also

[ifourier](#) | [iztrans](#) | [laplace](#)

**Purpose**            Imaginary part of complex number

**Syntax**            `imag(Z)`

**Description**        `imag(Z)` is the imaginary part of a symbolic `Z`.

**See Also**            `conj` | `real`

# int

---

**Purpose** Integrate symbolic expression

**Syntax**  
`int(expr)`  
`int(expr, v)`  
`int(expr, a, b)`  
`int(expr, v, a, b)`

**Description** `int(expr)` returns the indefinite integral of `expr` with respect to its symbolic variable as defined by `symvar`.

`int(expr, v)` returns the indefinite integral of `expr` with respect to the symbolic scalar variable `v`.

`int(expr, a, b)` returns the definite integral from `a` to `b` of `expr` with respect to the default symbolic variable. `a` and `b` are symbolic or double scalars.

`int(expr, v, a, b)` returns the definite integral of `expr` with respect to `v` from `a` to `b`.

**Examples** Find indefinite integral of the following single-variable expression:

```
syms x;  
int(-2*x/(1 + x^2)^2)
```

The result is:

```
ans =  
1/(x^2 + 1)
```

---

Find indefinite integral of the following multivariable expression with respect to `z`:

```
syms x z;  
int(x/(1 + z^2), z)
```

The result is:

```
ans =  
x*atan(z)
```

---

Integral the following expression from 0 to 1:

```
syms x;  
int(x*log(1 + x), 0, 1)
```

The result is:

```
ans =  
1/4
```

---

Integral the following expression from  $\sin(t)$  to 1:

```
syms x t;  
int(2*x, sin(t), 1)
```

The result is:

```
ans =  
cos(t)^2
```

---

Find indefinite integrals for the expressions listed as the elements of a matrix:

```
syms x t z;  
alpha = sym('alpha');  
int([exp(t), exp(alpha*t)])
```

The result is:

```
ans =  
[ exp(t), exp(alpha*t)/alpha]
```

# int

---

## **See Also**

`diff` | `symsum` | `symvar`

**Purpose** Convert symbolic matrix to signed integers

**Syntax** int8(S)  
int16(S)  
int32(S)  
int64(S)

**Description** int8(S) converts a symbolic matrix S to a matrix of signed 8-bit integers.  
int16(S) converts S to a matrix of signed 16-bit integers.  
int32(S) converts S to a matrix of signed 32-bit integers.  
int64(S) converts S to a matrix of signed 64-bit integers.

---

**Note** The output of int8, int16, int32, and int64 does not have data type symbolic.

---

The following table summarizes the output of these four functions.

Function	Output Range	Output Type	Bytes per Element	Output Class
int8	-128 to 127	Signed 8-bit integer	1	int8
int16	-32,768 to 32,767	Signed 16-bit integer	2	int16
int32	-2,147,483,648 to 2,147,483,647	Signed 32-bit integer	4	int32
int64	-9,223,372,036,854,775,808 to 9,223,372,036,854,775,807	Signed 64-bit integer	8	int64

**See Also** sym | vpa | single | double | uint8 | uint16 | uint32 | uint64

# inv

---

**Purpose** Compute symbolic matrix inverse

**Syntax** `R = inv(A)`

**Description** `R = inv(A)` returns inverse of the symbolic matrix A.

**Examples** Compute the inverse of the following matrix of symbolic numbers:

```
A = sym([2, -1, 0; -1, 2, -1; 0, -1, 2]);
inv(A)
```

The result is:

```
ans =
[ 3/4, 1/2, 1/4]
[ 1/2, 1, 1/2]
[ 1/4, 1/2, 3/4]
```

---

Compute the inverse of the following symbolic matrix:

```
syms a b c d
A = [a b; c d];
inv(A)
```

The result is:

```
ans =
[ d/(a*d - b*c), -b/(a*d - b*c)]
[ -c/(a*d - b*c), a/(a*d - b*c)]
```

---

Compute the inverse of the symbolic Hilbert matrix:

```
inv(sym(hilb(4)))
```

The result is:



```
ans =  
[ 16, -120, 240, -140]  
[ -120, 1200, -2700, 1680]  
[ 240, -2700, 6480, -4200]  
[ -140, 1680, -4200, 2800]
```

**See Also**

eig | det | rank

inverse z-transform

**Purpose** Inverse z-transform

**Syntax**  
 $f = \text{iztrans}(F)$   
 $f = \text{iztrans}(F, k)$   
 $f = \text{iztrans}(F, w, k)$

**Description**  $f = \text{iztrans}(F)$  computes the inverse z-transform of the symbolic expression  $F$ . This syntax assumes that  $F$  is a function of the variable  $z$ , and the returned value  $f$  is a function of  $n$ .

$$f(n) = \frac{1}{2\pi i} \oint_{|z|=R} F(z)z^{n-1} dz, \quad n = 1, 2, \dots$$

where  $R$  is a positive number, such that the function  $F(z)$  is analytic on and outside the circle  $|z| = R$ .

If  $F = F(n)$ ,  $\text{iztrans}$  computes the inverse z-transform  $f$  as a function of the variable  $k$ .

$$f = f(k)$$

$f = \text{iztrans}(F, k)$  computes the inverse z-transform  $f$  as a function of the variable  $k$  instead of the default variable  $n$ .

$f = \text{iztrans}(F, w, k)$  computes the inverse z-transform and lets you specify that  $F$  is a function of  $w$  and  $f$  is a function of  $k$ .

$$F = F(w) \Rightarrow f = f(k)$$

## Examples

Inverse Z-Transform	MATLAB Operation
$f(z) = \frac{2z}{(z-2)^2}$ $Z^{-1}[f] = \frac{1}{2\pi i} \oint_{ z =R} f(s)z^{n-1} dz$ $= n2^n$	<pre>syms z f = 2*z/(z-2)^2; iztrans(f)  returns  ans = 2^n + 2^n*(n - 1)</pre>
$g(n) = \frac{n(n+1)}{n^2 + 2n + 1}$ $Z^{-1} g  = \frac{1}{2\pi i} \oint_{ n =R} g(n)n^{k-1} dn$ $= -1^k$	<pre>syms n g = n*(n+1)/(n^2+2*n+1); iztrans(g)  returns  ans = (-1)^k</pre>
$f(z) = \frac{z}{z-a}$ $Z^{-1}[f] = \frac{1}{2\pi i} \oint_{ z =R} f(z)z^{k-1} dz$ $= a^k \text{ if } a \neq 0$	<pre>syms z a k f = z/(z-a); simplify(iztrans(f,k))  returns  ans = piecewise([a &lt;&gt; 0, a^k])</pre>

## See Also

ifourier | ilaplace | ztrans

# jacobian

---

**Purpose** Jacobian matrix  
Jacobian matrix

**Syntax** `jacobian(f, v)`

**Description** `jacobian(f, v)` computes the Jacobian matrix of the scalar or vector `f` with respect to `v`. The  $(i, j)$ -th entry of the result is  $\partial f(i) / \partial v(j)$ . If `f` is a scalar, the Jacobian matrix of `f` is the gradient of `f`. If `v` is a scalar, the result equals to `diff(f, v)`.

**Examples** Compute the Jacobian matrix for each of these vectors:

```
syms x y z
f = [x*y*z; y; x + z];
v = [x, y, z];
R = jacobian(f, v)
b = jacobian(x + z, v)
```

The results are:

```
R =
[ y*z, x*z, x*y]
[  0,  1,  0]
[  1,  0,  1]

b =
[ 1, 0, 1]
```

**See Also** `diff`

	Jordan form
<b>Purpose</b>	Jordan form of matrix
<b>Syntax</b>	$J = \text{jordan}(A)$ $[V, J] = \text{jordan}(A)$
<b>Description</b>	<p><math>J = \text{jordan}(A)</math> computes the Jordan canonical form (also called Jordan normal form) of a symbolic or numeric matrix <math>A</math>. The Jordan form of a numeric matrix is extremely sensitive to numerical errors. To compute Jordan form of a matrix, represent the elements of the matrix by integers or ratios of small integers, if possible.</p> <p><math>[V, J] = \text{jordan}(A)</math> computes the Jordan form <math>J</math> and the similarity transform <math>V</math>. The matrix <math>V</math> contains the generalized eigenvectors of <math>A</math> as columns, and <math>V \backslash A * V = J</math>.</p>

**Examples** Compute the Jordan form and the similarity transform for this numeric matrix. Verify that the resulting matrix  $V$  satisfies the condition  $V \backslash A * V = J$ :

```
A = [1 -3 -2; -1 1 -1; 2 4 5]
[V, J] = jordan(A)
V \ A * V
```

The result is:

```
A =
     1     -3     -2
    -1      1     -1
     2      4      5
```

```
V =
    -1      1     -1
    -1      0      0
     2      0      1
```

```
J =
```

# jordan

---

```
      2   1   0
      0   2   0
      0   0   3
```

```
ans =
      2   1   0
      0   2   0
      0   0   3
```

**See Also**    eig | inv | poly

**Purpose** Lambert W function

**Syntax**  
`W = lambertw(X)`  
`W = lambertw(K,X)`

**Description** `W = lambertw(X)` evaluates the Lambert W function at the elements of X, a numeric matrix or a symbolic matrix. The Lambert W function solves the equation

$$we^w = x$$

for  $w$  as a function of  $x$ .

`W = lambertw(K,X)` is the  $K$ -th branch of this multi-valued function.

**Examples** Compute the Lambert W function:

```
lambertw([0 -exp(-1); pi 1])
```

The result is:

```
ans =
      0   -1.0000
 1.0737   0.5671
```

The statements

```
syms x y
lambertw([0 x;1 y])
```

return

```
ans =
[          0, lambertw(0, x)]
[ lambertw(0, 1), lambertw(0, y)]
```

## References

[1] Corless, R.M, G.H. Gonnet, D.E.G. Hare, and D.J. Jeffrey, *Lambert's W Function in Maple™*, Technical Report, Dept. of Applied Math., Univ. of Western Ontario, London, Ontario, Canada.

[2] Corless, R.M, Gonnet, G.H. Gonnet, D.E.G. Hare, and D.J. Jeffrey, *On Lambert's W Function*, Technical Report, Dept. of Applied Math., Univ. of Western Ontario, London, Ontario, Canada.

Both papers are available by anonymous FTP from

`cs-archive.uwaterloo.ca`



**Purpose** Laplace transform  
Laplace transform

**Syntax** `laplace(F)`  
`laplace(F, t)`  
`laplace(F, w, z)`

**Description** `L = laplace(F)` computes the Laplace transform of the symbolic expression `F`. This syntax assumes that `F` is a function of the variable `t`, and the returned value `L` as a function of `s`.

$$F = F(t) \Rightarrow L = L(s)$$

If `F = F(s)`, `laplace` returns a function of `t`.

$$L = L(t)$$

By definition, the Laplace transform is

$$L(s) = \int_0^{\infty} F(t)e^{-st} dt$$

`L = laplace(F, t)` computes the Laplace transform `L` as a function of `t` instead of the default variable `s`.

$$L(t) = \int_0^{\infty} F(x)e^{-tx} dx$$

`L = laplace(F, w, z)` computes the Laplace transform `L` and lets you specify that `L` is a function of `z` and `F` is a function of `w`.

$$L(z) = \int_0^{\infty} F(w)e^{-zw} dw$$

# laplace

## Examples

Laplace Transform	MATLAB Command
$f(t) = t^4$ $L[f] = \int_0^{\infty} f(t)e^{-ts} dt$ $= \frac{24}{s^5}$	<pre>syms t; f = t^4; laplace(f)</pre> <p>returns</p> <pre>ans = 24/s^5</pre>
$g(s) = \frac{1}{\sqrt{s}}$ $L[g](t) = \int_0^{\infty} g(s)e^{-st} ds$ $= \sqrt{\frac{\pi}{t}}$	<pre>syms s; g = 1/sqrt(s); laplace(g)</pre> <p>returns</p> <pre>ans = pi^(1/2)/t^(1/2)</pre>
$f(t) = e^{-at}$ $L[f](x) = \int_0^{\infty} f(t)e^{-tx} dt$ $= \frac{1}{x+a}$	<pre>syms t a x; f = exp(-a*t); laplace(f,x)</pre> <p>returns</p> <pre>ans = 1/(a + x)</pre>

## See Also

[fourier](#) | [ilaplace](#) | [ztrans](#)

<b>Purpose</b>	LaTeX representation of symbolic expression
<b>Syntax</b>	<code>latex(S)</code>
<b>Description</b>	<code>latex(S)</code> returns the LaTeX representation of the symbolic expression <code>S</code> .
<b>Examples</b>	<p>The statements</p> <pre> syms x f = taylor(log(1+x)); latex(f) </pre> <p>return</p> <pre> ans = \frac{x^5}{5} - \frac{x^4}{4} + \frac{x^3}{3} - \frac{x^2}{2} + x </pre> <p>The statements</p> <pre> H = sym(hilb(3)); latex(H) </pre> <p>return</p> <pre> ans = \left(\begin{array}{ccc} 1 &amp; \frac{1}{2} &amp; \frac{1}{3} \\ \frac{1}{2} &amp; \frac{1}{3} &amp; \frac{1}{4} \\ \frac{1}{3} &amp; \frac{1}{4} &amp; \frac{1}{5} \end{array}\right) </pre> <p>The statements</p> <pre> syms t; alpha = sym('alpha'); A = [alpha t alpha*t]; latex(A) </pre>

```
return
```

```
ans =  
\left(\begin{array}{ccc} \mathrm{\alpha} & t & \mathrm{\alpha}\backslash, t...  
\end{array}\right)
```

You can use the `latex` command to annotate graphs:

```
syms x  
f = taylor(log(1+x));  
ezplot(f)  
hold on  
title(['$' latex(f) '$'],'interpreter','latex')  
hold off
```

## See Also

[pretty](#) | [ccode](#) | [fortran](#)

**Purpose** Compute limit of symbolic expression

**Syntax**

```
limit(expr, x, a)
limit(expr, a)
limit(expr)
limit(expr, x, a, 'left')
limit(expr, x, a, 'right')
```

**Description**

`limit(expr, x, a)` computes bidirectional limit of the symbolic expression `expr` when `x` approaches `a`.

`limit(expr, a)` computes bidirectional limit of the symbolic expression `expr` when the default variable approaches `a`.

`limit(expr)` computes bidirectional limit of the symbolic expression `expr` when the default variable approaches 0.

`limit(expr, x, a, 'left')` computes the limit of the symbolic expression `expr` when `x` approaches `a` from the left.

`limit(expr, x, a, 'right')` computes the limit of the symbolic expression `expr` when `x` approaches `a` from the right.

**Examples** Compute bidirectional limits for the following expressions:

```
syms x h;
limit(sin(x)/x)
limit((sin(x + h) - sin(x))/h, h, 0)
```

The results are

```
ans =
1

ans =
cos(x)
```

---

Compute the limits from the left and right for the following expressions:

# limit

---

```
syms x;  
limit(1/x, x, 0, 'right')  
limit(1/x, x, 0, 'left')
```

The results are

```
ans =  
Inf  
  
ans =  
-Inf
```

Compute the limit for the functions presented as elements of a vector:

```
syms x a;  
v = [(1 + a/x)^x, exp(-x)];  
limit(v, x, inf)
```

The result is

```
ans =  
[ exp(a), 0]
```

## See Also

[diff](#) | [taylor](#)

<b>Purpose</b>	Logarithm base 10 of entries of symbolic matrix
<b>Syntax</b>	$Y = \log_{10}(X)$
<b>Description</b>	$Y = \log_{10}(X)$ returns the logarithm to the base 10 of $X$ . If $X$ is a matrix, $Y$ is a matrix of the same size, each entry of which is the logarithm of the corresponding entry of $X$ .
<b>See Also</b>	<code>log2</code>

# log2

---

**Purpose**            Logarithm base 2 of entries of symbolic matrix

**Syntax**             $Y = \log_2(X)$

**Description**         $Y = \log_2(X)$  returns the logarithm to the base 2 of  $X$ . If  $X$  is a matrix,  $Y$  is a matrix of the same size, each entry of which is the logarithm of the corresponding entry of  $X$ .

**See Also**            `log10`



**Purpose**

Convert symbolic expression to function handle or file

**Syntax**

```
g = matlabFunction(f)
g = matlabFunction(f1,f2,...)
g = matlabFunction(f,param1,value1,...)
```

**Description**

`g = matlabFunction(f)` converts the symbolic expression `f` to a MATLAB function with the handle `g`.

`g = matlabFunction(f1,f2,...)` converts a list of the symbolic expressions `f1`, `f2`, ... to a MATLAB function with multiple outputs. The function handle is `g`.

`g = matlabFunction(f,param1,value1,...)` converts the symbolic expression `f` to a MATLAB function with the handle `g`. The command accepts the following options for parameter/value pairs:

- `Parameter = 'file'` allows you to generate a file with *optimized* code. The generated file can accept double or matrix arguments and evaluate the symbolic expression applied to the arguments. Optimized means intermediate variables are automatically generated to simplify or speed the code. MATLAB generates intermediate variables as a lowercase letter `t` followed by an automatically generated number, for example `t32`. The value of this parameter must be a string representing the path to the file. If the string is empty, `matlabFunction` generates an anonymous function. If the string does not end in `.m`, the function appends `.m`.
- `Parameter = 'outputs'` allows you to set the names of the output variables. `value` should be a cell array of strings. The default names of output variables coincide with the names you use calling `matlabFunction`. If you call `matlabFunction` using an expression instead of individual variables, the default names of output variables consist of the word `out` followed by the number, for example, `out3`.
- `Parameter = 'vars'` allows you to set the order of the input variables or symbolic vectors in the resulting function handle or the file. The default order is alphabetical. The value of this parameter must be either a cell array of strings or symbolic arrays, or a vector of

# matlabFunction

---

symbolic variables. The number of value entries should equal or exceed the number of free variables in the symbolic expression `f`.

---

**Tip** To convert a MuPAD expression or function to a MATLAB function, use `f = evalin(symengine, 'MuPAD_Expression')` or `f = feval(symengine, 'MuPAD_Function', x1, ..., xn)`. `matlabFunction` cannot correctly convert some MuPAD expressions to MATLAB functions. These expressions do not trigger an error message. When converting a MuPAD expression or function that is not on the MATLAB vs. MuPAD Expressions list, always check the results of conversion. To verify the results, execute the resulting function.

---

## Examples

Convert the following symbolic expression to a MATLAB function with the handle `ht`:

```
syms x y
r = sqrt(x^2 + y^2);
ht = matlabFunction(sin(r)/r)

ht =
@(x,y)sin(sqrt(x.^2+y.^2)).*1.0./sqrt(x.^2+y.^2)
```

The following example generates a file:

```
syms x y z
r = x^2 + y^2 + z^2;
f = matlabFunction(log(r)+r^(-1/2), 'file', 'myfile');
```

If the file `myfile.m` already exists in the current folder, `matlabFunction` replaces the existing function with the converted symbolic expression. You can open and edit the resulting file:

```
function out1 = myfile(x,y,z)
%MYFILE
%    OUT1 = MYFILE(X,Y,Z)
```

```

t2 = x.^2;
t3 = y.^2;
t4 = z.^2;
t5 = t2 + t3 + t4;
out1 = log(t5) + 1.0./sqrt(t5);

```

If the string is empty, matlabFunction generates an anonymous function:

```

syms x y z
r = x^2 + y^2 + z^2;
f = matlabFunction(log(r)+r^(-1/2),'file','')

f =
 @(x,y,z)log(x.^2+y.^2+z.^2)+1.0./sqrt(x.^2+y.^2+z.^2)

```

You can change the order of the input variables:

```

syms x y z
r = x^2 + y^2 + z^2;
matlabFunction(r, 'file', 'my_function',...
'vars', [y z x]);

```

The created my\_function accepts variables in the required order:

```

function r = my_function(y,z,x)
%MY_FUNCTION
%   R = MY_FUNCTION(Y,Z,X)

r = x.^2 + y.^2 + z.^2;

```

You can specify that the input arguments are vectors:

```

syms x y z t
r = (x^2 + y^2 + z^2)*exp(-t);
matlabFunction(r, 'file', 'my_function',...
'vars', {t, [x y z]});

```

The resulting function operates on vectors:

# matlabFunction

---

```
function r = my_function(t,in2)
%MY_FUNCTION
%   R = MY_FUNCTION(T,IN2)

x = in2(:,1);
y = in2(:,2);
z = in2(:,3);
r = exp(-t).*(x.^2+y.^2+z.^2);
```

You can specify the names of the output variables:

```
syms x y z
r = x^2 + y^2 + z^2;
q = x^2 - y^2 - z^2;
f = matlabFunction(r, q, 'file', 'my_function',...
'outputs', {'name1','name2'});
```

The generated function returns *name1* and *name2*:

```
function [name1,name2] = my_function(x,y,z)
%MY_FUNCTION
%   [NAME1,NAME2] = MY_FUNCTION(X,Y,Z)

t9 = x.^2;
t10 = y.^2;
t11 = z.^2;
name1 = t10 + t11 + t9;
if nargin > 1
    name2 = -t10 - t11 + t9;
end
```

Also, you can convert MuPAD expressions:

```
syms x y;
f = evalin(symengine, 'arcsin(x) + arccos(y)');
matlabFunction(f, 'file', 'my_function');
```

The created file contains the same expressions written in the MATLAB language:

```
function f = my_function(x,y)
%MY_FUNCTION
%   F = MY_FUNCTION(X,Y)

f = asin(x) + acos(y);
```

## See Also

[ccode](#) | [fortran](#) | [subs](#) | [sym2poly](#) | [emlBlock](#)

## How To

- “Generating Code from Symbolic Expressions” on page 3-134

# mfun

---

**Purpose** Numeric evaluation of special mathematical function

**Syntax** `mfun('function',par1,par2,par3,par4)`

**Description** `mfun('function',par1,par2,par3,par4)` numerically evaluates one of the special mathematical functions listed in “Syntax and Definitions of mfun Special Functions” on page 3-109. Each `par` argument is a numeric quantity corresponding to a parameter for `function`. You can use up to four parameters. The last parameter specified can be a matrix, usually corresponding to  $x$ . The dimensions of all other parameters depend on the specifications for `function`. You can access parameter information for mfun functions in “Syntax and Definitions of mfun Special Functions” on page 6-139.

MuPAD software evaluates `function` using 16-digit accuracy. Each element of the result is a MATLAB numeric quantity. Any singularity in `function` is returned as NaN.

**Examples** Evaluate the Fresnel cosine integral:

```
mfun('FresnelC',0:5)
```

The result is:

```
ans =  
0    0.7799    0.4883    0.6057    0.4984    0.5636
```

Evaluate the hyperbolic cosine integral:

```
mfun('Chi',[3*i 0])
```

```
ans =  
0.1196 + 1.5708i    NaN
```

**See Also** `mfunlist`

**Purpose** List special functions for use with mfun

**Syntax** mfunlist

**Description** mfunlist lists the special mathematical functions for use with the mfun function. The following tables describe these special functions.

**Syntax and Definitions of mfun Special Functions** The following conventions are used in the next table, unless otherwise indicated in the **Arguments** column.

x, y	real argument
z, z1, z2	complex argument
m, n	integer argument

**mfun Special Functions**

Function Name	Definition	mfun Name	Arguments
Bernoulli numbers and polynomials	Generating functions: $\frac{e^{xt}}{e^t - 1} = \sum_{n=0}^{\infty} B_n(x) \cdot \frac{t^{n-1}}{n!}$	bernoulli(n) bernoulli(n,t)	$n \geq 0$ $0 <  t  < 2\pi$
Bessel functions	BesselI, BesselJ—Bessel functions of the first kind. BesselK, BesselY—Bessel functions of the second kind.	BesselJ(v,x) BesselY(v,x) BesselI(v,x) BesselK(v,x)	v is real.
Beta function	$B(x,y) = \frac{\Gamma(x) \cdot \Gamma(y)}{\Gamma(x+y)}$	Beta(x,y)	

## mfun Special Functions (Continued)

Function Name	Definition	mfun Name	Arguments
Binomial coefficients	$\binom{m}{n} = \frac{m!}{n!(m-n)!}$ $= \frac{\Gamma(m+1)}{\Gamma(n+1)\Gamma(m-n+1)}$	binomial(m,n)	
Complete elliptic integrals	Legendre's complete elliptic integrals of the first, second, and third kind. This definition uses modulus $k$ . The numerical <code>ellipke</code> function and the MuPAD functions for computing elliptic integrals use the parameter $m = k^2 = \sin^2 \alpha$ .	EllipticK(k) EllipticE(k) EllipticPi(a,k)	$a$ is real, $-\infty < a < \infty$ . $k$ is real, $0 < k < 1$ .
Complete elliptic integrals with complementary modulus	Associated complete elliptic integrals of the first, second, and third kind using complementary modulus. This definition uses modulus $k$ . The numerical <code>ellipke</code> function and the MuPAD functions for computing elliptic integrals use the parameter $m = k^2 = \sin^2 \alpha$ .	EllipticCK(k) EllipticCE(k) EllipticCPi(a,k)	$a$ is real, $-\infty < a < \infty$ . $k$ is real, $0 < k < 1$ .



**mfun Special Functions (Continued)**

<b>Function Name</b>	<b>Definition</b>	<b>mfun Name</b>	<b>Arguments</b>
Complementary error function and its iterated integrals	$erfc(z) = \frac{2}{\sqrt{\pi}} \cdot \int_z^{\infty} e^{-t^2} dt = 1 - erf(z)$ $erfc(-1, z) = \frac{2}{\sqrt{\pi}} \cdot e^{-z^2}$ $erfc(n, z) = \int_z^{\infty} erfc(n-1, t) dt$	erfc(z) erfc(n, z)	$n > 0$
Dawson's integral	$F(x) = e^{-x^2} \cdot \int_0^x e^{t^2} dt$	dawson(x)	
Digamma function	$\Psi(x) = \frac{d}{dx} \ln(\Gamma(x)) = \frac{\Gamma'(x)}{\Gamma(x)}$	Psi(x)	
Dilogarithm integral	$f(x) = \int_1^x \frac{\ln(t)}{1-t} dt$	dilog(x)	$x > 1$
Error function	$erf(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt$	erf(z)	
Euler numbers and polynomials	Generating function for Euler numbers: $\frac{1}{\cosh(t)} = \sum_{n=0}^{\infty} E_n \frac{t^n}{n!}$	euler(n) euler(n, z)	$n \geq 0$ $ t  < \frac{\pi}{2}$

## mfun Special Functions (Continued)

Function Name	Definition	mfun Name	Arguments
Exponential integrals	$Ei(n, z) = \int_1^{\infty} \frac{e^{-zt}}{t^n} dt$ $Ei(x) = PV \left( - \int_{-\infty}^x \frac{e^t}{t} dt \right)$	Ei(n, z) Ei(x)	$n \geq 0$ $\text{Real}(z) > 0$
Fresnel sine and cosine integrals	$C(x) = \int_0^x \cos\left(\frac{\pi}{2} t^2\right) dt$ $S(x) = \int_0^x \sin\left(\frac{\pi}{2} t^2\right) dt$	FresnelC(x) FresnelS(x)	
Gamma function	$\Gamma(z) = \int_0^{\infty} t^{z-1} e^{-t} dt$	GAMMA(z)	
Harmonic function	$h(n) = \sum_{k=1}^n \frac{1}{k} = \Psi(n+1) + \gamma$	harmonic(n)	$n > 0$
Hyperbolic sine and cosine integrals	$Shi(z) = \int_0^z \frac{\sinh(t)}{t} dt$ $Chi(z) = \gamma + \ln(z) + \int_0^z \frac{\cosh(t) - 1}{t} dt$	Shi(z) Chi(z)	

**mfun Special Functions (Continued)**

<b>Function Name</b>	<b>Definition</b>	<b>mfun Name</b>	<b>Arguments</b>
(Generalized) hypergeometric function	$F(n, d, z) = \sum_{k=0}^{\infty} \frac{\prod_{i=1}^j \frac{\Gamma(n_i + k)}{\Gamma(n_i)} \cdot z^k}{\prod_{i=1}^m \frac{\Gamma(d_i + k)}{\Gamma(d_i)} \cdot k!}$ <p>where <math>j</math> and <math>m</math> are the number of terms in <math>n</math> and <math>d</math>, respectively.</p>	hypergeom( $n, d, x$ ) where $n = [n_1, n_2, \dots]$ $d = [d_1, d_2, \dots]$	$n_1, n_2, \dots$ are real. $d_1, d_2, \dots$ are real and nonnegative.
Incomplete elliptic integrals	Legendre's incomplete elliptic integrals of the first, second, and third kind. This definition uses modulus $k$ . The numerical ellipse function and the MuPAD functions for computing elliptic integrals use the parameter $m = k^2 = \sin^2 \alpha$ .	EllipticF( $x, k$ ) EllipticE( $x, k$ ) EllipticPi( $x, a, k$ )	$0 < x \leq \infty$ . $a$ is real, $-\infty < a < \infty$ . $k$ is real, $0 < k < 1$ .
Incomplete gamma function	$\Gamma(a, z) = \int_z^{\infty} e^{-t} \cdot t^{a-1} dt$	GAMMA( $z1, z2$ ) $z1 = a$ $z2 = z$	
Logarithm of the gamma function	$\ln\text{GAMMA}(z) = \ln(\Gamma(z))$	lnGAMMA( $z$ )	
Logarithmic integral	$Li(x) = PV \left\{ \int_0^x \frac{dt}{\ln t} \right\} = Ei(\ln x)$	Li( $x$ )	$x > 1$

## mfun Special Functions (Continued)

Function Name	Definition	mfun Name	Arguments
Polygamma function	$\Psi^{(n)}(z) = \frac{d^n}{dz} \Psi(z)$ <p>where <math>\Psi(z)</math> is the Digamma function.</p>	Psi(n,z)	$n \geq 0$
Shifted sine integral	$Ssi(z) = Si(z) - \frac{\pi}{2}$	Ssi(z)	

The following orthogonal polynomials are available using mfun. In all cases, n is a nonnegative integer and x is real.

### Orthogonal Polynomials

Polynomial	mfun Name	Arguments
Chebyshev of the first and second kind	T(n,x) U(n,x)	
Gegenbauer	G(n,a,x)	a is a nonrational algebraic expression or a rational number greater than -1/2.
Hermite	H(n,x)	
Jacobi	P(n,a,b,x)	a, b are nonrational algebraic expressions or rational numbers greater than -1.
Laguerre	L(n,x)	

**Orthogonal Polynomials (Continued)**

Polynomial	mfun Name	Arguments
Generalized Laguerre	$L(n, a, x)$	a is a nonrational algebraic expression or a rational number greater than -1.
Legendre	$P(n, x)$	

**Examples**

```
mfun('H',5,10)

ans =
    3041200

mfun('dawson',3.2)

ans =
    0.1655
```

**Limitations**

In general, the accuracy of a function will be lower near its roots and when its arguments are relatively large.

Running time depends on the specific function and its parameters. In general, calculations are slower than standard MATLAB calculations.

**References**

[1] Abramowitz, M. and I.A., Stegun, *Handbook of Mathematical Functions*, Dover Publications, 1965.

**See Also**

mfun

# mod

---

**Purpose** Symbolic matrix element-wise modulus

**Syntax**  $C = \text{mod}(A, B)$

**Description**  $C = \text{mod}(A, B)$  for symbolic matrices  $A$  and  $B$  with integer elements is the positive remainder in the elementwise division of  $A$  by  $B$ . For matrices with polynomial entries,  $\text{mod}(A, B)$  is applied to the individual coefficients.

**Examples**

```
ten = sym('10');  
mod(2^ten, ten^3)
```

```
ans =  
24
```

```
syms x  
mod(x^3 - 2*x + 999, 10)
```

```
ans =  
x^3 + 8*x + 9
```

**See Also** `quorem`

<b>Purpose</b>	Start MuPAD notebook
<b>Syntax</b>	<pre>mphandle = mupad mphandle = mupad(file)</pre>
<b>Description</b>	<p><code>mphandle = mupad</code> creates a MuPAD notebook, and keeps a handle (pointer) to the notebook in the variable <code>mphandle</code>. You can use any variable name you like instead of <code>mphandle</code>.</p> <p><code>mphandle = mupad(file)</code> opens the MuPAD notebook or program file named <code>file</code> and keeps a handle (pointer) to the notebook or program file in the variable <code>mphandle</code>. This syntax has the functionality of both <code>openmn</code> and <code>openmu</code>. Also, you can use the argument <code>file#linktargetname</code> to refer to the particular link target inside a notebook. In this case, the <code>mupad</code> function opens the MuPAD notebook or program file (<code>file</code>) and jumps to the beginning of the link target <code>linktargetname</code>. If there are multiple link targets with the name <code>linktargetname</code>, the <code>mupad</code> function uses the last <code>linktargetname</code> occurrence.</p>
<b>Examples</b>	<p>To start a new notebook and define a handle <code>mphandle</code> to the notebook, enter:</p> <pre>mphandle = mupad;</pre> <p>To open an existing notebook named <code>notebook1.mn</code> located in the current folder, and define a handle <code>mphandle</code> to the notebook, enter:</p> <pre>mphandle = mupad('notebook1.mn');</pre> <p>To open a notebook and jump to a particular location, create a link target at that location inside a notebook and refer to it when opening a notebook. For example, if you have the Conclusions section in <code>notebook1.mn</code>, create a link target named <code>conclusions</code> and refer to it when opening the notebook. The <code>mupad</code> function opens <code>notebook1.mn</code> and scroll it to display the Conclusions section:</p> <pre>mphandle = mupad('notebook1.mn#conclusions');</pre>

For information about creating link targets, see the Formatting and Exporting MuPAD Documents and Graphics section in the MuPAD Getting Started documentation.

**See Also**

`getVar` | `mupadwelcome` | `openmn` | `openmu` | `setVar`



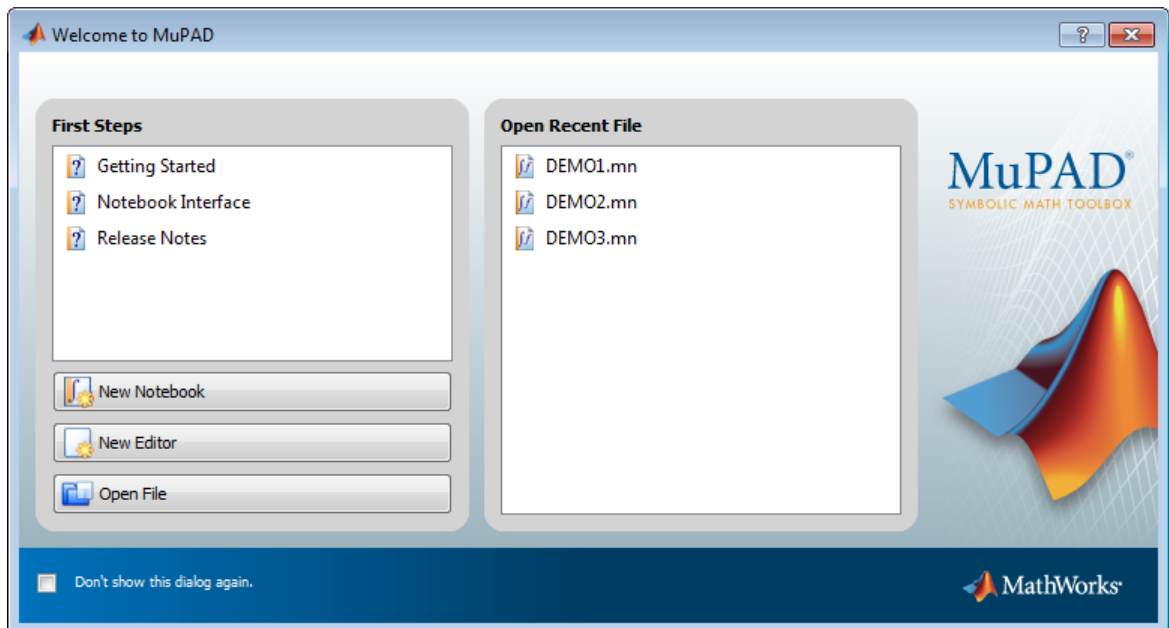
**Purpose** Start MuPAD interfaces

**Syntax** mupadwelcome

**Description** mupadwelcome opens a window that enables you to start various MuPAD interfaces:

- Notebook, for performing calculations
- Editor, for writing programs and libraries
- Help, in the **First Steps** pane

It also enables you to access recent MuPAD files or browse for files.



**See Also** mupad

## **How To**

- “Creating, Opening, and Saving MuPAD Notebooks” on page 4-11

**Purpose** Form basis for null space of matrix

**Syntax** `Z = null(A)`

**Description** `Z = null(A)` returns a list of vectors that form the basis for the null space of a matrix `A`. The product `A*Z` is zero. `size(Z, 2)` is the nullity of `A`. If `A` has full rank, `Z` is empty.

**Examples** Find the basis for the null space and the nullity of the magic square of symbolic numbers. Verify that `A*Z` is zero:

```
A = sym(magic(4));
Z = null(A)
nullityOfA = size(Z, 2)
A*Z
```

The results are:

```
Z =
-1
-3
 3
 1

nullityOfA =
 1

ans =
 0
 0
 0
 0
```

---

Find the basis for the null space of the matrix `B` that has full rank:

```
B = sym(hilb(3))
```

# null

---

```
Z = null(B)
```

The result is:

```
B =  
[ 1, 1/2, 1/3]  
[ 1/2, 1/3, 1/4]  
[ 1/3, 1/4, 1/5]
```

```
Z =  
[ empty sym ]
```

## See Also

[rank](#) | [rref](#) | [size](#) | [svd](#)

**Purpose** Numerator and denominator

**Syntax** `[N,D] = numden(A)`

**Description** `[N,D] = numden(A)` converts each element of `A` to a rational form where the numerator and denominator are relatively prime polynomials with integer coefficients. `A` is a symbolic or a numeric matrix. `N` is the symbolic matrix of numerators, and `D` is the symbolic matrix of denominators.

**Examples** Find the numerator and denominator of the symbolic number:

```
[n, d] = numden(sym(4/5))
```

The result is:

```
n =  
4
```

```
d =  
5
```

Find the numerator and denominator of the symbolic expression:

```
syms x y;  
[n,d] = numden(x/y + y/x)
```

The result is:

```
n =  
x^2 + y^2
```

```
d =  
x*y
```

The statements

```
syms a b
```

# numden

---

```
A = [a, 1/b]
[n,d] = numden(A)
```

```
return
```

```
A =
[a, 1/b]
```

```
n =
[a, 1]
```

```
d =
[1, b]
```

<b>Purpose</b>	Open MuPAD notebook
<b>Syntax</b>	<code>h = openmn(file)</code>
<b>Description</b>	<code>h = openmn(file)</code> opens the MuPAD notebook file named <code>file</code> , and returns a handle to the file in <code>h</code> . The command <code>h = mupad(file)</code> accomplishes the same task.
<b>Examples</b>	To open a notebook named <code>e-e-x.mn</code> in the folder <code>\Documents\Notes</code> of drive <code>H:</code> , enter:  <pre>h = openmn('H:\Documents\Notes\e-e-x.mn');</pre>
<b>See Also</b>	<code>mupad</code>   <code>open</code>   <code>openmu</code>   <code>openmuphlp</code>   <code>openxvc</code>   <code>openxvz</code>

# openmu

---

**Purpose** Open MuPAD program file

**Syntax** `h = openmu(file)`

**Description** `h = openmu(file)` opens the MuPAD program file named `file`, and returns a handle to the file in `h`. The command `h = mupad(file)` accomplishes the same task.

**Examples** To open a program file named `yyx.mu` in the folder `\Documents\Notes` of drive `H:`, enter:

```
h = openmu('H:\Documents\Notes\yyx.mu');
```

**See Also** `mupad` | `open` | `openmn` | `openmuphlp` | `openxvc` | `openxvz`



<b>Purpose</b>	Open MuPAD help file
<b>Syntax</b>	<code>h = openmuphlp(file)</code>
<b>Description</b>	<code>h = openmuphlp(file)</code> opens the MuPAD help file named <code>file</code> , and returns a handle to the file in <code>h</code> . The command <code>h = mupad(file)</code> accomplishes the same task.
<b>Input Arguments</b>	<code>file</code> A MuPAD help file
<b>Output Arguments</b>	<code>h</code> A handle to the file
<b>Examples</b>	To open a help file named <code>helpPage.muphlp</code> in the folder <code>\Documents\Notes</code> of drive <code>H:</code> , enter:  <pre>h = openmuphlp('H:\Documents\Notes\helpPage.muphlp');</pre>
<b>See Also</b>	<code>mupad</code>   <code>open</code>   <code>openmn</code>   <code>openmu</code>   <code>openxvc</code>   <code>openxvz</code>

# openxvc

---

<b>Purpose</b>	Open MuPAD XVC graphics file
<b>Syntax</b>	<code>h = openxvc(file)</code>
<b>Description</b>	<code>h = openxvc(file)</code> opens the MuPAD XVC graphics file named <code>file</code> , and returns a handle to the file in <code>h</code> . The command <code>h = mupad(file)</code> accomplishes the same task.
<b>Input Arguments</b>	<code>file</code> A MuPAD XVC graphics file
<b>Output Arguments</b>	<code>h</code> A handle to the file
<b>Examples</b>	To open a graphics file named <code>image1.xvc</code> in the folder <code>\Documents\Notes</code> of drive <code>H:</code> , enter:  <code>h = openxvc('H:\Documents\Notes\image1.xvc');</code>
<b>See Also</b>	<code>mupad</code>   <code>open</code>   <code>openmn</code>   <code>openmu</code>   <code>openmuphlp</code>   <code>openxvz</code>

<b>Purpose</b>	Open MuPAD XVZ graphics file
<b>Syntax</b>	<code>h = openxvz(file)</code>
<b>Description</b>	<code>h = openxvz(file)</code> opens the MuPAD XVZ graphics file named <code>file</code> , and returns a handle to the file in <code>h</code> . The command <code>h = mupad(file)</code> accomplishes the same task.
<b>Input Arguments</b>	<code>file</code> A MuPAD XVZ graphics file
<b>Output Arguments</b>	<code>h</code> A handle to the file
<b>Examples</b>	To open a graphics file named <code>image1.xvz</code> in the folder <code>\Documents\Notes</code> of drive <code>H:</code> , enter:  <code>h = openxvz('H:\Documents\Notes\image1.xvz');</code>
<b>See Also</b>	<code>mupad</code>   <code>open</code>   <code>openmn</code>   <code>openmu</code>   <code>openmuphlp</code>   <code>openxvc</code>

# poly

---

characteristic polynomial

**Purpose** Characteristic polynomial of matrix

**Syntax** `p = poly(A)`  
`p = poly(A, v)`  
`poly(sym(A))`

**Description** `p = poly(A)` returns the coefficients of the characteristic polynomial of a numeric matrix `A`. For symbolic `A`, `poly(A)` returns the characteristic polynomial of `A` in terms of the default variable `x`. If the elements of `A` already contain the variable `x`, the default variable is `t`. If the elements of `A` contain both `x` and `t`, the default variable is still `t`.

`p = poly(A, v)` returns the characteristic polynomial of a symbolic or numeric matrix `A` in terms of the variable `v`.

`poly(sym(A))` approximately equals `poly2sym(poly(A))` for numeric `A`. The approximation is due to round-off error.

**Examples** Compute characteristic polynomials of one of the MATLAB test matrices:

```
syms z
A = gallery(3)
p = poly(A)
q = poly(sym(A))
s = poly(A, z)
```

The results are:

```
A =
-149   -50  -154
 537   180   546
 -27    -9   -25

p =
 1.0000  -6.0000  11.0000  -6.0000
```

```
q =
x^3 - 6*x^2 + 11*x - 6
```

```
s =
z^3 - 6*z^2 + 11*z - 6
```

---

Compute the characteristic polynomial of the following symbolic matrix in terms of the default variable:

```
syms x y;
B = x*hilb(3)
a = poly(B)
```

The result is:

```
B =
[ x, x/2, x/3]
[ x/2, x/3, x/4]
[ x/3, x/4, x/5]

a =
t^3 - (23*t^2*x)/15 + (127*t*x^2)/720 - x^3/2160
```

Compute the characteristic polynomial of B in terms of the specified variable y:

```
b = poly(B, y)
```

The result is:

```
b =
- x^3/2160 + (127*x^2*y)/720 - (23*x*y^2)/15 + y^3
```

## See Also

[eig](#) | [jordan](#) | [poly2sym](#) | [solve](#)

# poly2sym

---

**Purpose** Polynomial coefficient vector to symbolic polynomial

**Syntax**  
`r = poly2sym(c)`  
`r = poly2sym(c, v)`

**Description** `r = poly2sym(c)` returns a symbolic representation of the polynomial whose coefficients are in the numeric vector `c`. The default symbolic variable is `x`. The variable `v` can be specified as a second input argument. If `c = [c1 c2 ... cn]`, `r = poly2sym(c)` has the form

$$c_1x^{n-1} + c_2x^{n-2} + \dots + c_n$$

`poly2sym` uses `sym`'s default (rational) conversion mode to convert the numeric coefficients to symbolic constants. This mode expresses the symbolic coefficient approximately as a ratio of integers, if `sym` can find a simple ratio that approximates the numeric value, otherwise as an integer multiplied by a power of 2.

`r = poly2sym(c, v)` is a polynomial in the symbolic variable `v` with coefficients from the vector `c`. If `v` has a numeric value and `sym` expresses the elements of `c` exactly, `eval(poly2sym(c))` returns the same value as `polyval(c, v)`.

## Examples

The command

```
poly2sym([1 3 2])
```

returns

```
ans =  
x^2 + 3*x + 2
```

The command

```
poly2sym([.694228, .333, 6.2832])
```

returns

```
ans =  
(6253049924220329*x^2)/9007199254740992 + (333*x)/1000 + 3927/625
```

The command

```
poly2sym([1 0 1 -1 2], y)
```

returns

```
ans =  
y^4 + y^2 - y + 2
```

## See Also

[sym](#) | [sym2poly](#) | [polyval](#)





<b>Purpose</b>	Symbolic matrix element-wise quotient and remainder
<b>Syntax</b>	$[Q, R] = \text{quorem}(A, B)$
<b>Description</b>	$[Q, R] = \text{quorem}(A, B)$ for symbolic matrices $A$ and $B$ with integer or polynomial elements does element-wise division of $A$ by $B$ and returns quotient $Q$ and remainder $R$ so that $A = Q.*B+R$ . For polynomials, $\text{quorem}(A,B,x)$ uses variable $x$ instead of $\text{symvar}(A,1)$ or $\text{symvar}(B,1)$ .

**Examples**

```
syms x
p = x^3 - 2*x + 5;
[q, r] = quorem(x^5, p)

q =
x^2 + 2

r =
- 5*x^2 + 4*x - 10

[q, r] = quorem(10^5, subs(p, '10'))

q = 101
r = 515
```

**See Also**

mod

# rank

---

**Purpose** Compute rank of symbolic matrix

**Syntax** `rank(A)`

**Description** `rank(A)` computes the rank of the symbolic matrix A.

**Examples** Compute the rank of the following numeric matrix:

```
B = magic(4);  
rank(B)
```

The result is:

```
ans =  
    3
```

---

Compute the rank of the following symbolic matrix:

```
syms a b c d  
A = [a b;c d];  
rank(A)
```

The result is:

```
ans =  
    2
```

**See Also** `eig` | `null` | `rref` | `size`

<b>Purpose</b>	Real part of complex symbolic number
<b>Syntax</b>	<code>real(Z)</code>
<b>Description</b>	<code>real(Z)</code> is the real part of a symbolic <code>Z</code> .
<b>See Also</b>	<code>conj</code>   <code>imag</code>

# reset

---

**Purpose** Close MuPAD engine

**Syntax** `reset(symengine)`

**Description** `reset(symengine)` closes the MuPAD engine associated with the MATLAB workspace, and resets all its assumptions. Immediately before or after executing `reset(symengine)` you should clear all symbolic objects in the MATLAB workspace.

**See Also** `symengine`

**Purpose** Symbolic matrix element-wise round

**Syntax** `Y = round(X)`

**Description** `Y = round(X)` rounds the elements of `X` to the nearest integers. Values halfway between two integers are rounded away from zero.

**Examples**

```
x = sym(-5/2);  
[fix(x) floor(x) round(x) ceil(x) frac(x)]  
  
ans =  
[ -2, -3, -3, -2, -1/2]
```

**See Also** `floor` | `ceil` | `fix` | `frac`

# rref

---

**Purpose** Compute reduced row echelon form of matrix

**Syntax** `rref(A)`

**Description** `rref(A)` computes the reduced row echelon form of the symbolic matrix `A`. If the elements of a matrix contain free symbolic variables, `rref` regards the matrix as nonzero.

**Examples** Compute the reduced row echelon form of the magic square matrix:

```
rref(sym(magic(4)))
```

The result is:

```
ans =  
[ 1, 0, 0, 1]  
[ 0, 1, 0, 3]  
[ 0, 0, 1, -3]  
[ 0, 0, 0, 0]
```

---

Compute the reduced row echelon form of the following symbolic matrix:

```
syms a b c;  
A = [a b c; b c a; a + b, b + c, c + a];  
rref(A)
```

The result is:

```
ans =  
[ 1, 0, -(- c^2 + a*b)/(- b^2 + a*c)]  
[ 0, 1, -(- a^2 + b*c)/(- b^2 + a*c)]  
[ 0, 0, 0]
```

**See Also** `eig` | `jordan` | `rank` | `size`

**Purpose** Interactive evaluation of Riemann sums

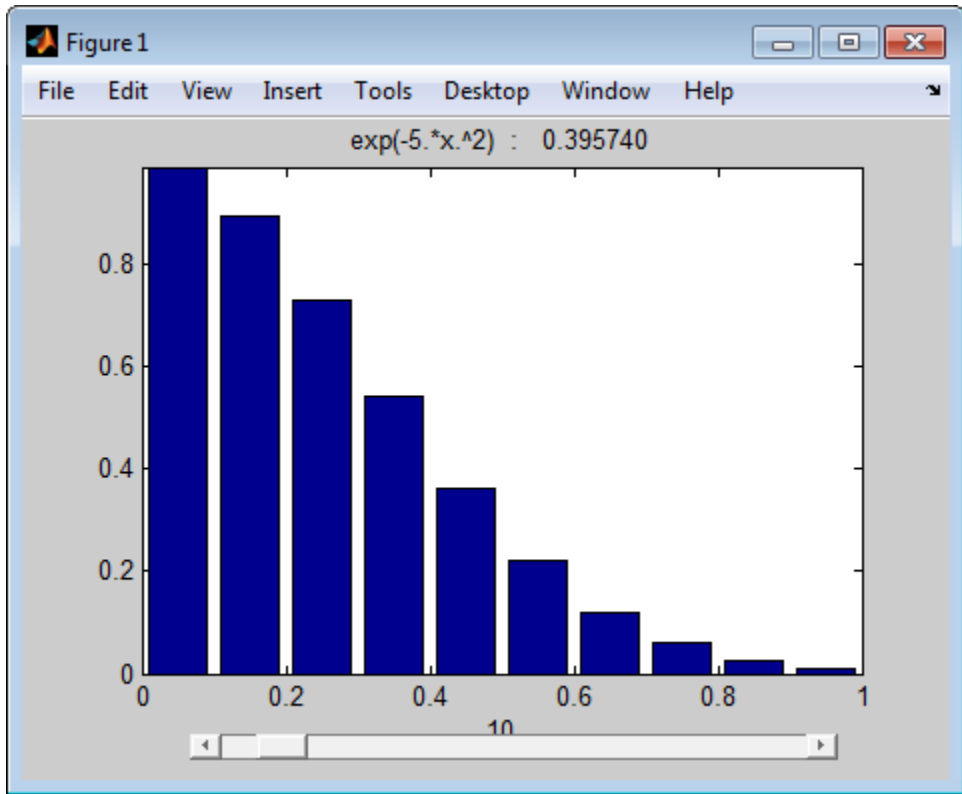
**Syntax**

```
rsums(f)
rsums(f, a, b)
rsums(f, [a, b])
```

**Description** `rsums(f)` interactively approximates the integral of  $f(x)$  by Riemann sums for  $x$  from 0 to 1. `rsums(f)` displays a graph of  $f(x)$  using 10 terms (rectangles). You can adjust the number of terms taken in the Riemann sum by using the slider below the graph. The number of terms available ranges from 2 to 128.  $f$  can be a string or a symbolic expression. The height of each rectangle is determined by the value of the function in the middle of each interval.

`rsums(f, a, b)` and `rsums(f, [a, b])` approximates the integral for  $x$  from  $a$  to  $b$ .

**Examples** Both `rsums('exp(-5*x^2)')` and `rsums exp(-5*x^2)` create the following plot.





**Purpose**

Assign variable in MuPAD notebook

**Syntax**

```
setVar(nb, y)
setVar(nb, 'v', y)
```

**Description**

`setVar(nb, y)` assigns the symbolic expression `y` in the MATLAB workspace to the variable `y` in the MuPAD notebook `nb`.

`setVar(nb, 'v', y)` assigns the symbolic expression `y` in the MATLAB workspace to the variable `v` in the MuPAD notebook `nb`.

**Examples**

```
mpnb = mupad;
syms x;
y = exp(-x);
setVar(mpnb,y)
setVar(mpnb,'z',sin(y))
```

After executing these statements, the MuPAD engine associated with the `mpnb` notebook contains the variables `y`, with value `exp(-x)`, and `z`, with value `sin(exp(-x))`.

**See Also**

`getVar` | `mupad`

# simple

---

**Purpose** Search for simplest form of symbolic expression

**Syntax** `r = simple(S)`  
`[r, how] = simple(S)`

**Description** `r = simple(S)` tries several different algebraic simplifications of the symbolic expression `S`, displays any that shorten the length of `S`'s representation, and returns the shortest. `S` is a `sym`. If `S` is a matrix, the result represents the shortest representation of the entire matrix, which is not necessarily the shortest representation of each individual element. If no return output is given, `simple(S)` displays all possible representations and returns the shortest.

`[r, how] = simple(S)` does not display intermediate simplifications, but returns the shortest found, as well as a string describing the particular simplification. `r` is a `sym`. `how` is a string.

**Examples** Simplify the expressions:

```
syms x;  
f = cos(x)^2 + sin(x)^2;  
f = simple(f)  
g = cos(3*acos(x));  
g = simple(g)
```

The results are:

```
f =  
1  
  
g =  
4*x^3 - 3*x
```

Simplify the expressions displaying all possible simplifications:

```
syms x;  
f = cos(x) + i*sin(x);  
simple(f)
```

The result is:

```
simplify:  
cos(x) + sin(x)*i
```

```
radsimp:  
cos(x) + sin(x)*i
```

```
simplify(100):  
cos(x) + sin(x)*i
```

```
combine(sincos):  
cos(x) + sin(x)*i
```

```
combine(sinhcosh):  
cos(x) + sin(x)*i
```

```
combine(ln):  
cos(x) + sin(x)*i
```

```
factor:  
cos(x) + sin(x)*i
```

```
expand:  
cos(x) + sin(x)*i
```

```
combine:  
cos(x) + sin(x)*i
```

```
rewrite(exp):  
exp(x*i)
```

```
rewrite(sincos):  
cos(x) + sin(x)*i
```

```
rewrite(sinhcosh):  
cosh(x*i) + sinh(x*i)
```

# simple

---

```
rewrite(tan):
(tan(x/2)*2*i)/(tan(x/2)^2 + 1) - (tan(x/2)^2 - 1)/(tan(x/2)^2 + 1)

mwcossin:
sin(x)*i - 2*sin(x/2)^2 + 1

collect(x):
cos(x) + sin(x)*i

ans =
exp(x*i)
```

Simplify the expression and display the simplification method:

```
syms x;
f = (x + 1)*x*(x - 1);
[f, how] = simple(f)

f =
x^3 - x

how =
simplify(100)
```

## See Also

`collect` | `expand` | `factor` | `horner` | `simplify`

---

<b>Purpose</b>	algebraic simplification Algebraic simplification
<b>Syntax</b>	<code>R = simplify(S)</code> <code>R = simplify(S, n)</code>
<b>Description</b>	<code>R = simplify(S)</code> performs algebraic simplification of <code>S</code> using MuPAD simplification rules. If <code>S</code> is a symbolic matrix, this command simplifies each element of <code>S</code> .  <code>R = simplify(S, n)</code> lets you specify the number of steps in the algebraic simplification. Here <code>n</code> is a positive integer. By default, <code>n = 100</code> .
<b>Examples</b>	<p>Simplify the trigonometric expression:</p> <pre>syms x; simplify(sin(x)^2 + cos(x)^2)</pre> <p>The result is:</p> <pre>ans = 1</pre> <p>Simplify the expression:</p> <pre>syms a b c; simplify(exp(c*log(sqrt(a+b))))</pre> <p>The result is:</p> <pre>ans = (a + b)^(c/2)</pre> <p>Simplify the expressions from the list:</p> <pre>S = [(x^2 + 5*x + 6)/(x + 2), sqrt(16)]; R = simplify(S)</pre> <p>The result is:</p>

# simplify

---

```
R =  
[ x + 3, 4]
```

## **See Also**

`collect` | `expand` | `factor` | `horner` | `simple`

**Purpose** Convert symbolic expressions to Simscape language equations

**Syntax** `simscapeEquation(f)`  
`simscapeEquation(LHS, RHS)`

**Description** `simscapeEquation(f)` converts the symbolic expression  $f$  to a Simscape language equation. This function call converts any derivative with respect to the variable  $t$  to the Simscape notation  $X.der$ . Here  $X$  is the time-dependent variable. In the resulting Simscape equation, the variable *time* replaces all instances of the variable  $t$  except for derivatives with respect to  $t$ .

`simscapeEquation(LHS, RHS)` returns a Simscape equation  $LHS == RHS$ .

**Tips** The equation section of a Simscape component file supports a limited number of functions. See the list of Supported Functions for more information. If a symbolic equation contains the functions that are not available in the equation section of a Simscape component file, `simscapeEquation` cannot correctly convert these equations to Simscape equations. Such expressions do not trigger an error message. The following types of expressions are prone to invalid conversion:

- Expressions with infinities
- Expressions returned by `evalin` and `feval`.

If you perform symbolic computations in the MuPAD Notebook Interface and want to convert the results to Simscape equations, use the `generate::Simscape` function in MuPAD.

**Examples** Convert the following expressions to Simscape language equations:

```
syms t
x = sym('x(t)');
y = sym('y(t)');
phi = diff(x)+5*y + sin(t);
```

# simscapeEquation

---

```
simscapeEquation(phi)
simscapeEquation(diff(y),phi)
```

The result is:

```
ans =
phi == sin(time)+y*5.0+x.der;

ans =
y.der == sin(time)+y*5.0+x.der;
```

## See Also

[emlBlock](#) | [matlabFunction](#) | [ccode](#) | [fortran](#)

## How To

- “Generating Simscape Equations” on page 3-144



**Purpose** Convert symbolic matrix to single precision

**Syntax** `single(S)`

**Description** `single(S)` converts the symbolic matrix `S` to a matrix of single-precision floating-point numbers. `S` must not contain any symbolic variables, except `'eps'`.

**See Also** `sym` | `vpa` | `double`

# sinint

---

**Purpose** Sine integral

**Syntax** `Y = sinint(X)`

**Description** `Y = sinint(X)` evaluates the sine integral function at the elements of `X`, a numeric matrix, or a symbolic matrix. The result is a numeric matrix. The sine integral function is defined by

$$Si(x) = \int_0^x \frac{\sin t}{t} dt$$

**Examples** Evaluate sine integral for the elements of the matrix:

```
sinint([pi 0; -2.2 exp(3)])
```

```
ans =  
    1.8519         0  
   -1.6876    1.5522
```

The statement

```
sinint(1.2)
```

returns

```
ans =  
    1.1080
```

The statement

```
syms x;  
diff(sinint(x))
```

returns

```
ans =  
sin(x)/x
```

**See Also**

cosint

# size

---

**Purpose** Symbolic matrix dimensions

**Syntax**  
`d = size(A)`  
`[m, n] = size(A)`  
`d = size(A, n)`

**Description** Suppose  $A$  is an  $m$ -by- $n$  symbolic or numeric matrix. The statement `d = size(A)` returns a numeric vector with two integer components, `d = [m,n]`.

The multiple assignment statement `[m, n] = size(A)` returns the two integers in two separate variables.

The statement `d = size(A, n)` returns the length of the dimension specified by the scalar  $n$ . For example, `size(A, 1)` is the number of rows of  $A$  and `size(A, 2)` is the number of columns of  $A$ .

**Examples** The statements

```
syms a b c d
A = [a b c ; a b d; d c b; c b a];
d = size(A)
r = size(A, 2)
```

return

```
d =
     4     3

r =
     3
```

**See Also** `length` | `ndims`

**Purpose**

Solve equations and systems

**Syntax**

```
S = solve(expr)
S = solve(expr, x)
Y = solve(expr1, expr2, ..., exprn)
[y1, y2, ..., yn] = solve(expr1, expr2, ..., exprn)
Y = solve(expr1, expr2, ..., exprn, x1, x2, ..., xn)
[y1, y2, ..., yn] = solve(expr1, expr2, ..., exprn, x1, x2,
    ..., xn)
```

**Description**

`S = solve(expr)` solves the equation  $\text{expr} = 0$  for its default variable determined by `symvar` and assigns the result to `S`.

`S = solve(expr, x)` solves the equation  $\text{expr} = 0$  for the variable `x` and assigns the result to `S`.

`Y = solve(expr1, expr2, ..., exprn)` solves the system of equations  $\text{expr1} = 0$ ,  $\text{expr2} = 0$ , ...,  $\text{exprn} = 0$  for the variables determined by `symvar` and assigns the result to `Y`.

`[y1, y2, ..., yn] = solve(expr1, expr2, ..., exprn)` solves the system of equations  $\text{expr1} = 0$ ,  $\text{expr2} = 0$ , ...,  $\text{exprn} = 0$  for the variables determined by `symvar` and assigns the solutions `y1`, `y2`, ..., `yn`.

`Y = solve(expr1, expr2, ..., exprn, x1, x2, ..., xn)` solves the system of equations  $\text{expr1} = 0$ ,  $\text{expr2} = 0$ , ...,  $\text{exprn} = 0$  for the variables `x1`, `x2`, ..., `xn` and assigns the result to `Y`.

`[y1, y2, ..., yn] = solve(expr1, expr2, ..., exprn, x1, x2, ..., xn)` solves the system of equations  $\text{expr1} = 0$ ,  $\text{expr2} = 0$ , ...,  $\text{exprn} = 0$  for the variables `x1`, `x2`, ..., `xn` and assigns the solutions to `y1`, `y2`, ..., `yn`.

**Tips**

- If the symbolic solver cannot find a solution of an equation or a system of equations, the toolbox internally calls the numeric solver that tries to find a numeric approximation. For polynomial equations and systems, the numeric solver returns all solutions.

For non-polynomial equations and systems that have solutions, the solver returns only one solution.

- If the solution of an equation or a system of equations contains parameters, the solver can choose one or more values of the parameters and return the results corresponding to these values. For some equations and systems, the solver returns parametrized solutions without choosing particular values. In this case, the solver also issues a warning indicating the values of parameters in the returned solutions.
- To solve differential equations, use the `dsolve` function.
- When solving a system of equations, always assign the result to output arguments. Output arguments allow you to access the values of the solutions of a system.

## Input Arguments

`expr`

A symbolic expression or a string. If a symbolic expression or a string represents a symbolic expression (does not contain the equal sign), the solver solves an equation  $\text{expr} = 0$ . If a string represents an equation (contains the equal sign), the solver solves that equation.

`expr1, expr2, ..., exprn`

Symbolic expressions or strings that represent the system of symbolic equations.

`x`

A variable for which you solve an equation. If you do not specify the variable, the solver uses the default variable determined by `symvar`.

`x1, x2, ..., xn`

Variables for which you solve a system of equations. If you do not specify the variables, the solver uses the default variables determined by `symvar`.

**Output Arguments**

S

A symbolic array that contains solutions of an equation. The size of a symbolic array corresponds to the number of the solutions.

Y

A structure array that contains solutions of a system. The number of fields in the structure array corresponds to the number of independent variables in a system.

y1, y2, ..., yn

Variables to which the solver assigns the solutions of a system of equations. The number of output variables or symbolic arrays must be equal to the number of independent variables in a system. The toolbox sorts independent variables alphabetically, and then assigns the solutions for these variables to the output variables or symbolic arrays.

**Examples**

If the right side of an equation is zero, specify the left side as a symbolic expression or a string:

```
syms x;
solve(x^2 - 1)
solve('x^2 + 4*x + 1')
```

```
ans =
     1
    -1
```

```
ans =
 3^(1/2) - 2
- 3^(1/2) - 2
```

If the right side of an equation is not zero, specify the equation as a string:

```
syms x;
solve('x^4 + 1 = 2*x^2 - 1')
```

The solver returns the symbolic array of solutions:

```
ans =  
  (1 + i)^(1/2)  
  (1 - i)^(1/2)  
 -(1 + i)^(1/2)  
 -(1 - i)^(1/2)
```

---

To avoid ambiguities when solving equations with symbolic parameters, specify the variable for which you want to solve an equation:

```
syms a b c x;  
solve(a*x^2 + b*x + c, a)  
solve('a*x^2 + b*x + c', 'b')
```

The result is:

```
ans =  
 -(c + b*x)/x^2  
  
ans =  
 -(a*x^2 + c)/x
```

If you do not specify the variable for which you want to solve the equation, the toolbox chooses a variable by using the `symvar` function. In this example, the solver chooses the variable `x`:

```
syms a b c x;  
solve(a*x^2 + b*x + c)  
  
ans =  
 -(b + (b^2 - 4*a*c)^(1/2))/(2*a)  
 -(b - (b^2 - 4*a*c)^(1/2))/(2*a)
```

---

When solving a system of equations, use one output argument to return the solutions in the form of a structure array:



```
syms x y;
S = solve('x + y = 1', 'x - 11*y = 5')

S =
    x: [1x1 sym]
    y: [1x1 sym]
```

To display the solutions, access the elements of the structure array S:

```
S = [S.x S.y]

S =
[ 4/3, -1/3]
```

When solving a system of equations, use multiple output arguments to assign the solutions directly to output variables:

```
syms a u v;
[solutions_a, solutions_u, solutions_v] =...
solve('a*u^2 + v^2', 'u - v = 1', 'a^2 - 5*a + 6')
```

The solver returns a symbolic array of solutions for each independent variable:

```
solutions_a =
    3
    2
    2
    3

solutions_u =
(3^(1/2)*i)/4 + 1/4
(2^(1/2)*i)/3 + 1/3
1/3 - (2^(1/2)*i)/3
1/4 - (3^(1/2)*i)/4
```

```
solutions_v =  
    (3^(1/2)*i)/4 - 3/4  
    (2^(1/2)*i)/3 - 2/3  
- (2^(1/2)*i)/3 - 2/3  
- (3^(1/2)*i)/4 - 3/4
```

Entries with the same index form the solutions of a system:

```
solutions = [solutions_a, solutions_u, solutions_v]  
  
solutions =  
[ 3, (3^(1/2)*i)/4 + 1/4, (3^(1/2)*i)/4 - 3/4]  
[ 2, (2^(1/2)*i)/3 + 1/3, (2^(1/2)*i)/3 - 2/3]  
[ 2, 1/3 - (2^(1/2)*i)/3, - (2^(1/2)*i)/3 - 2/3]  
[ 3, 1/4 - (3^(1/2)*i)/4, - (3^(1/2)*i)/4 - 3/4]
```

---

Solve the following equation:

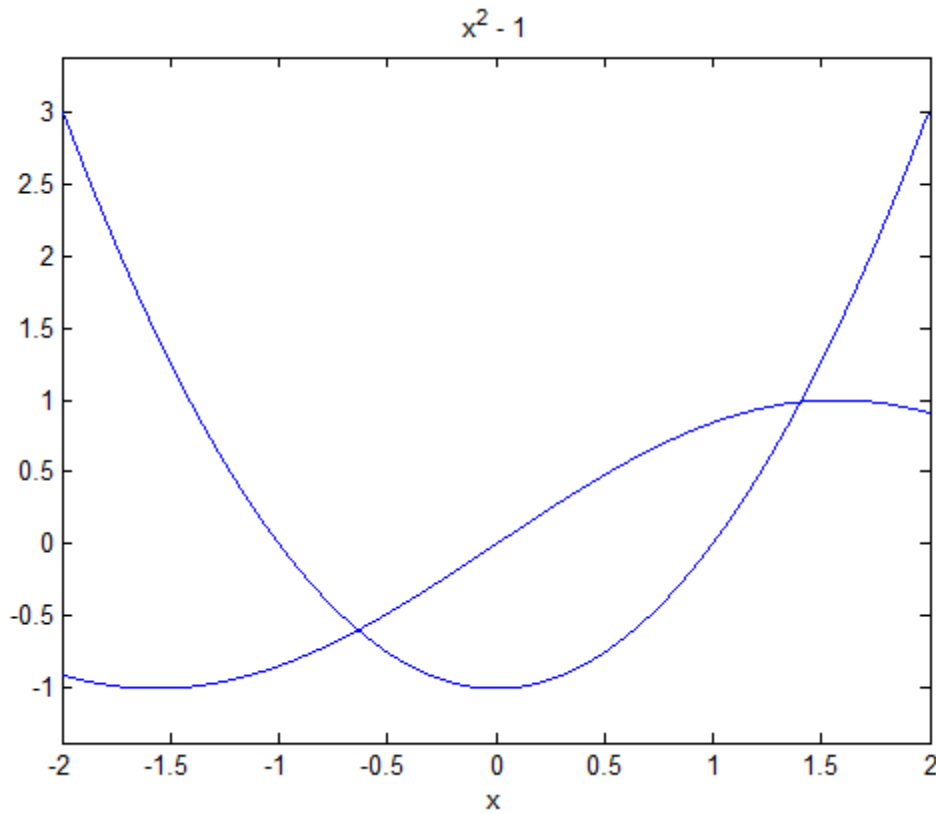
```
syms x;  
solve('sin(x) = x^2 - 1')
```

The symbolic solver cannot find an exact symbolic solution for this equation, and therefore, it calls the numeric solver. Since the equation is not polynomial, an attempt to find all possible solutions can take a lot of time. The numeric solver does not try to find all numeric solutions for this equation. Instead, it returns only the first solution that it finds:

```
ans =  
-0.63673265080528201088799090383828
```

Plotting the left and the right sides of the equation in one graph shows that the equation also has a positive solution:

```
ezplot(sin(x), -2, 2);  
hold on;  
ezplot(x^2 - 1, -2, 2)  
hold off
```



You can find this solution by calling the MuPAD numeric solver directly and specifying the interval where this solution can be found. To call MuPAD commands from the MATLAB Command Window, use the `evalin` or `feval` function:

```
evalin(symengine, 'numeric::solve(sin(x) = x^2  
- 1, x = 0..2)')
```

```
ans =  
1.4096240040025962492355939705895
```

# solve

---

Solve the following trigonometric equations:

```
syms x;  
solve(sin(1/x), x)  
solve(sin(x), x)
```

For the first equation, the solver returns the solution with one parameter and issues a warning indicating the values of the parameter. For the second equation, the solver chooses one value of the parameter and returns the solution corresponding to this value:

```
Warning: The solutions are parametrized by the symbols:  
k = Z_  
  
ans =  
1/(pi*k)  
  
ans =  
0
```

## References

### See Also

[dsolve](#) | [symvar](#)

### How To

- “Solving Equations” on page 3-81

**Purpose**

Sort symbolic vectors, matrices, or polynomials

**Syntax**

```
Y = sort(X)
Y = sort(X, dim)
Y = sort(X, mode)
[Y, I] = sort(X)
```

**Description**

`Y = sort(X)` sorts the elements of a symbolic vector or matrix in ascending order. If `X` is a vector, `sort(X)` sorts the elements of `X` in numerical or lexicographic order. If `X` is a matrix, `sort(X)` sorts each column of `X`.

`Y = sort(X, dim)` sorts the elements of a symbolic vector or matrix along the dimension of `X` specified by the integer `dim`. For two-dimensional matrices, use 1 to sort element of each column and 2 to sort element of each row.

`Y = sort(X, mode)` sorts the elements of a symbolic vector or matrix in the specified direction, depending on the value of `mode`. Use `ascend` to sort in ascending order, and `descend` to sort in descending order.

`[Y, I] = sort(X)` sorts a symbolic vector or a matrix `X`. This call also returns the array `I` that shows the indices that each element of a new vector or matrix `Y` had in the original vector or matrix `X`. If `X` is an `m`-by-`n` matrix, then each column of `I` is a permutation vector of the corresponding column of `X`, such that

```
for j = 1:n
    Y(:,j) = X(I(:,j),j);
end
```

If `X` is a two-dimensional matrix and you sort the elements of each column, the array `I` shows the row indices that the elements of `Y` had in the original matrix `X`. If you sort the elements of each row, `I` shows the original column indices.

**Examples**

Sort the elements of the following symbolic vector in ascending and descending order:

## sort

---

```
syms a b c d e;  
sort([7 e 1 c 5 d a b])  
sort([7 e 1 c 5 d a b], 'descend')
```

The results are:

```
ans =  
[ 1, 5, 7, a, b, c, d, e]
```

```
ans =  
[ e, d, c, b, a, 7, 5, 1]
```

Sort the elements of the following symbolic matrix:

```
X = sym(magic(3))
```

```
X =  
[ 8, 1, 6]  
[ 3, 5, 7]  
[ 4, 9, 2]
```

By default, the `sort` command sorts elements of each column:

```
sort(X)  
  
ans =  
[ 3, 1, 2]  
[ 4, 5, 6]  
[ 8, 9, 7]
```

To sort the elements of each row, use set the value of the `dim` option to 2:

```
sort(X, 2)  
  
ans =  
[ 1, 6, 8]  
[ 3, 5, 7]  
[ 2, 4, 9]
```

Sort the elements of each row of X in descending order:

```
sort(X, 2, 'descend')
```

```
ans =  
[ 8, 6, 1]  
[ 7, 5, 3]  
[ 9, 4, 2]
```

Sort the matrix X returning the array with indices that each element of the resulting matrix had in X:

```
[Y, I] = sort(X)
```

```
Y =  
[ 3, 1, 2]  
[ 4, 5, 6]  
[ 8, 9, 7]
```

```
I =  
     2     1     3  
     3     2     1  
     1     3     2
```

## See Also

[sym2poly](#) | [coeffs](#)

# subexpr

---

**Purpose** Rewrite symbolic expression in terms of common subexpressions

**Syntax** `[Y, SIGMA] = subexpr(X, SIGMA)`  
`[Y, SIGMA] = subexpr(X, 'SIGMA')`

**Description** `[Y, SIGMA] = subexpr(X, SIGMA)` or `[Y, SIGMA] = subexpr(X, 'SIGMA')` rewrites the symbolic expression `X` in terms of its common subexpressions.

**Examples** The statements

```
h = solve('a*x^3+b*x^2+c*x+d = 0');  
[r,s] = subexpr(h,'s')
```

return the rewritten expression for `t` in `r` in terms of a common subexpression, which is returned in `s`:

```
r =  
s^(1/3) - b/(3*a) - (- b^2/(9*a^2) + c/(3*a))/s^(1/3)  
(- b^2/(9*a^2) + c/(3*a))/(2*s^(1/3)) - s^(1/3)/2 + ...  
(3^(1/2)*(s^(1/3) + (- b^2/(9*a^2) + c/(3*a))/s^(1/3))*i)/2 - b/(3*a)  
(- b^2/(9*a^2) + c/(3*a))/(2*s^(1/3)) - s^(1/3)/2 - ...  
(3^(1/2)*(s^(1/3) + (- b^2/(9*a^2) + c/(3*a))/s^(1/3))*i)/2 - b/(3*a)  
  
s =  
((d/(2*a) + b^3/(27*a^3) - (b*c)/(6*a^2))^2 + ...  
(- b^2/(9*a^2) + c/(3*a))^3)^(1/2) - b^3/(27*a^3) - ...  
d/(2*a) + (b*c)/(6*a^2)
```

**See Also** `pretty` | `simple` | `subs`



**Purpose**

Symbolic substitution in symbolic expression or matrix

**Syntax**

```
R = subs(S)
R = subs(S, new)
R = subs(S, old, new)
```

**Description**

`R = subs(S)` replaces all occurrences of variables in the symbolic expression `S` with values obtained from the calling function, or the MATLAB workspace.

`R = subs(S, new)` replaces the default symbolic variable in `S` with `new`.

`R = subs(S, old, new)` replaces `old` with `new` in the symbolic expression `S`. `old` is a symbolic variable or a string representing a variable name. `new` is a symbolic or numeric variable or expression. That is, `R = subs(S,old,new)` evaluates `S` at `old = new`. The substitution is first attempted as a MATLAB expression resulting in the computation being done in double-precision arithmetic if all the values in `new` are double precision. Convert the new values to `sym` to ensure symbolic or variable-precision arithmetic.

If `old` and `new` are vectors or cell arrays of the same size and type, each element of `old` is replaced by the corresponding element of `new`. If `S` and `old` are scalars and `new` is an array or cell array, the scalars are expanded to produce an array result. If `new` is a cell array of numeric matrices, the substitutions are performed element-wise (i.e., `subs(x*y, {x, y}, {A, B})` returns `A.*B` when `A` and `B` are numeric).

If `subs(s,old,new)` does not change `s`, `subs(s,new,old)` is tried. This provides backwards compatibility with previous versions and eliminates the need to remember the order of the arguments. `subs(s,old,new,0)` does not switch the arguments if `s` does not change.

---

**Tip** If  $A$  is a matrix, the command `subs(S, x, A)` replaces all occurrences of the variable  $x$  in the symbolic expression  $S$  with the matrix  $A$ , and replaces the constant term in  $S$  with the constant times a matrix of all ones. To evaluate  $S$  in the matrix sense, use the command `polyvalm(sym2poly(S), A)`, which replaces the constant term with the constant times an identity matrix.

---

## Examples

### Single Input

Suppose  $a = 980$  and  $C2 = 3$  exist in the workspace.

The statement

```
y = dsolve('Dy = -a*y')
```

produces

```
y =  
C2/exp(a*t)
```

Then the statements

```
a = 980; C2 = 3; subs(y)
```

produce

```
ans =  
3/exp(980*t)
```

### Single Substitution

```
syms a b;  
subs(a + b, a, 4)
```

returns

```
ans =  
b + 4
```

### Multiple Substitutions

```
syms a b;  
subs(cos(a) + sin(b), {a, b}, {sym('alpha'), 2})
```

returns

```
ans =  
sin(2) + cos(alpha)
```

### Scalar Expansion Case

```
syms t;  
subs(exp(a*t), 'a', -magic(2))
```

returns

```
ans =  
[ 1/exp(t), 1/exp(3*t)]  
[ 1/exp(4*t), 1/exp(2*t)]
```

### Multiple Scalar Expansion

```
syms x y;  
subs(x*y, {x, y}, {[0 1; -1 0], [1 -1; -2 1]})
```

returns

```
ans =  
 0  -1  
 2   0
```

### See Also

[simplify](#) | [subexpr](#)

**Purpose** Compute singular value decomposition of symbolic matrix

**Syntax**

```
sigma = svd(A)
sigma = svd(vpa(A))
[U, S, V] = svd(A)
[U, S, V] = svd(vpa(A))
```

**Description** `sigma = svd(A)` returns a symbolic vector containing the singular values of a symbolic matrix `A`. With symbolic inputs, `svd` does not accept complex values as inputs.

`sigma = svd(vpa(A))` returns a vector with the numeric singular values using variable-precision arithmetic.

`[U, S, V] = svd(A)` and `[U, S, V] = svd(vpa(A))` return numeric unitary matrices `U` and `V` with the columns containing the singular vectors and a diagonal matrix `S` containing the singular values. The matrices satisfy  $A = U*S*V'$ . The `svd` command does not compute symbolic singular vectors. With multiple outputs, `svd` does not accept complex values as inputs.

**Examples** Compute the symbolic and numeric singular values and the numeric singular vectors of the following magic square:

```
digits(5)
A = sym(magic(4));
svd(A)
svd(vpa(A))
[U, S, V] = svd(A)
```

The results are:

```
ans =
      0
  2*5^(1/2)
  8*5^(1/2)
      34
```

```
ans =  
      34.0  
      17.889  
      4.4721  
      2.8024*10^(-7)  
  
U =  
[ 0.5, 0.67082, 0.5, 0.22361]  
[ 0.5, -0.22361, -0.5, 0.67082]  
[ 0.5, 0.22361, -0.5, -0.67082]  
[ 0.5, -0.67082, 0.5, -0.22361]  
  
S =  
[ 34.0, 0, 0, 0]  
[ 0, 17.889, 0, 0]  
[ 0, 0, 4.4721, 0]  
[ 0, 0, 0, 0]  
  
V =  
[ 0.5, 0.5, 0.67082, 0.22361]  
[ 0.5, -0.5, -0.22361, 0.67082]  
[ 0.5, -0.5, 0.22361, -0.67082]  
[ 0.5, 0.5, -0.67082, -0.22361]
```

**See Also**[digits](#) | [eig](#) | [inv](#) | [vpa](#)

**Purpose** Define symbolic objects

**Syntax**

```
S = sym(A)
x = sym('x')
x = sym('x', 'real')
k = sym('x', 'positive')
x = sym('x', 'clear')
A = sym('A', [m n])
A = sym('A', n)
A = sym(A, 'real')
A = sym(A, 'positive')
A = sym(A, 'clear')
S = sym(A, flag)
```

**Description** `S = sym(A)` constructs an object `S`, of the `sym` class, from `A`. If the input argument is a string, the result is a symbolic number or variable. If the input argument is a numeric scalar or matrix, the result is a symbolic representation of the given numeric values.

`x = sym('x')` creates the symbolic variable with name `x` and stores the result in `x`.

`x = sym('x', 'real')` creates the symbolic variable `x` and assumes that `x` is real, so that `conj(x)` is equal to `x`. `alpha = sym('alpha')` and `r = sym('Rho', 'real')` are other examples.

Similarly, `k = sym('x', 'positive')` creates the symbolic variable `x` and assumes that `x` is real and positive.

`x = sym('x', 'clear')` clears all previously set assumptions on the variable `x`. Ensures that `x` is neither real nor positive. See also the reference pages on `syms`. For compatibility with previous versions of the software, `x = sym('x', 'unreal')` has the same effect as `x = sym('x', 'clear')`.

Statements like `pi = sym('pi')` and `delta = sym('1/10')` create symbolic numbers that avoid the floating-point approximations inherent in the values of `pi` and `1/10`. The `pi` created in this way temporarily replaces the built-in numeric function with the same name.

`A = sym('A', [m n])` creates a  $m$ -by- $n$  matrix of symbolic variables. The dimensions  $m$  and  $n$  of a matrix must be integers. You can use this syntax to create a 1-by- $n$  or an  $n$ -by-1 vector. The `sym` function does not allow you to use symbolic variables without assigned numeric values as dimensions. By default, the generated names of elements of a vector use the form  $A_k$ , and the generated names of elements of a matrix use the form  $A_{i_j}$ . The base,  $A$ , must be a valid variable name. (To verify whether the name is a valid variable name, use `isvarname`.) The values of  $k$ ,  $i$ , and  $j$  range from 1 to  $m$  or 1 to  $n$ . To specify other form for generated names of matrix elements, use `'%d'` in the first input. For example, `A = sym('A%d%d', [3 3])` generates the 3-by-3 symbolic matrix  $A$  with the elements  $A_{11}$ ,  $A_{12}$ , ...,  $A_{33}$ .

`A = sym('A', n)` creates an  $n$ -by- $n$  matrix.

`A = sym(A, 'real')` sets an assumption that all elements of a symbolic matrix (or a symbolic vector)  $A$  are real. To create a symbolic vector or a symbolic matrix  $A$ , use `A = sym('A', [m n])` or `A = sym('A', n)`.

`A = sym(A, 'positive')` sets an assumption that all elements of a symbolic matrix (or a symbolic vector)  $A$  are real and positive. To create a symbolic vector or a symbolic matrix  $A$ , use `A = sym('A', [m n])` or `A = sym('A', n)`.

`A = sym(A, 'clear')` clears assumptions previously set on the symbolic matrix (or the symbolic vector)  $A$ .

`S = sym(A, flag)` where `flag` is one of `'r'`, `'d'`, `'e'`, or `'f'`, converts a numeric scalar or matrix to symbolic form. The technique for converting floating-point numbers is specified by the optional second argument, which can be `'f'`, `'r'`, `'e'` or `'d'`. The default is `'r'`.

`'f'` stands for “floating-point.” All values are represented in the form  $N \cdot 2^e$  or  $-N \cdot 2^e$ , where  $N$  and  $e$  are integers,  $N \geq 0$ . For example, `sym(1/10, 'f')` is `3602879701896397/36028797018963968`.

`'r'` stands for “rational.” Floating-point numbers obtained by evaluating expressions of the form  $p/q$ ,  $p \cdot \pi/q$ ,  $\sqrt{p}$ ,  $2^q$ , and  $10^q$  for modest sized integers  $p$  and  $q$  are converted to the corresponding symbolic form. This effectively compensates for the round-off error

involved in the original evaluation, but may not represent the floating-point value precisely. If no simple rational approximation can be found, an expression of the form  $p \cdot 2^q$  with large integers  $p$  and  $q$  reproduces the floating-point value exactly. For example, `sym(4/3, 'r')` is `'4/3'`, but `sym(1+sqrt(5), 'r')` is `7286977268806824*2^(-51)`.

'e' stands for "estimate error." The 'r' form is supplemented by a term involving the variable 'eps', which estimates the difference between the theoretical rational expression and its actual floating-point value. For example, `sym(3*pi/4, 'e')` is `3*pi/4*(1+3143276*eps/65)`.

'd' stands for "decimal." The number of digits is taken from the current setting of `digits` used by `vpa`. Fewer than 16 digits loses some accuracy, while more than 16 digits may not be warranted. For example, with `digits(10)`, `sym(4/3, 'd')` is `1.333333333`, while with `digits(20)`, `sym(4/3, 'd')` is `1.3333333333333332593`, which does not end in a string of 3s, but is an accurate decimal representation of the floating-point number nearest to  $4/3$ .

## Examples

Create the 3-by-4 symbolic matrix A with the auto-generated elements `A1_1`, ..., `A3_4`:

```
A = sym('A', [3 4])

A =
[ A1_1, A1_2, A1_3, A1_4]
[ A2_1, A2_2, A2_3, A2_4]
[ A3_1, A3_2, A3_3, A3_4]
```

Now, create the 4-by-4 matrix B with the elements `x_1_1`, ..., `x_4_4`:

```
B = sym('x_%d_%d', [4 4])

B =
[ x_1_1, x_1_2, x_1_3, x_1_4]
[ x_2_1, x_2_2, x_2_3, x_2_4]
[ x_3_1, x_3_2, x_3_3, x_3_4]
[ x_4_1, x_4_2, x_4_3, x_4_4]
```



This syntax does not define elements of a symbolic matrix as separate symbolic objects. To access an element of a matrix, use parentheses:

```
A(2, 3)
B (4, 2)
```

```
ans =
A2_3
```

```
ans =
x_4_2
```

You can use symbolic matrices and vectors generated by the `sym` function to define other matrices:

```
A = diag(sym('A',[1 4]))
```

```
A =
[ A1,  0,  0,  0]
[  0, A2,  0,  0]
[  0,  0, A3,  0]
[  0,  0,  0, A4]
```

Perform operations on symbolic matrices by using the operators that you use for numeric matrices. For example, find the determinant and the trace of the matrix `A`:

```
det(A)
```

```
ans =
A1*A2*A3*A4
```

```
trace(A)
```

```
ans =
A1 + A2 + A3 + A4
```

Also, use the `sym` function to set assumptions on all elements of a symbolic matrix. You cannot create a symbolic matrix and set an

assumption on all its elements in one `sym` function call. Use two separate `sym` function calls: the first call creates a matrix, and the second call specifies an assumption:

```
A = sym('A%d%d', [2 2]);  
A = sym(A, 'positive')
```

```
A =  
[ A11, A12]  
[ A21, A22]
```

Now, MATLAB assumes that all elements of `A` are positive:

```
solve(A(1, 1)^2 - 1, A(1, 1))  
  
ans =  
1
```

To clear all previously set assumptions on elements of a symbolic matrix, also use the `sym` function:

```
A = sym(A, 'clear');  
solve(A(1, 1)^2 - 1, A(1, 1))  
  
ans =  
1  
-1
```

The `sym` function lets you choose the conversion technique by specifying the optional second argument, which can be `'r'`, `'f'`, `'d'` or `'e'`. The default is `'r'`. For example, convert the number  $1/3$  to a symbolic object:

```
r = sym(1/3)  
f = sym(1/3, 'f')  
d = sym(1/3, 'd')  
e = sym(1/3, 'e')  
  
r =  
1/3
```

---

f =  
6004799503160661 / 18014398509481984

d =  
0.3333333333333333148296162562473909929395

e =  
1/3 - eps/12

**See Also**

[digits](#) | [double](#) | [findsym](#) | [reset](#) | [syms](#) | [symvar](#) | [eps](#)

# symengine

---

**Purpose** Return symbolic engine

**Syntax** `s = symengine`

**Description** `s = symengine` returns the currently active symbolic engine.

**Examples** To see which symbolic computation engine is currently active, enter:

```
s = symengine
```

The result is:

```
s =  
MuPAD symbolic engine
```

Now you can use the variable `s` in function calls that require symbolic engine:

```
syms a b c x  
p = a*x^2 + b*x + c;  
feval(s,'polylib::discrim', p, x)
```

The result is:

```
ans =  
b^2 - 4*a*c
```

**See Also** `feval`

**Purpose**                    Shortcut for constructing symbolic objects

**Syntax**                    `syms arg1 arg2 ...`  
`syms arg1 arg2 ... real`  
`syms arg1 arg2 ... clear`  
`syms arg1 arg2 ... positive`

**Description**            `syms arg1 arg2 ...` is a shortcut for

```
arg1 = sym('arg1');  
arg2 = sym('arg2'); ...
```

`syms arg1 arg2 ... real` is a shortcut for

```
arg1 = sym('arg1','real');  
arg2 = sym('arg2','real'); ...
```

`syms arg1 arg2 ... clear` is a shortcut for

```
arg1 = sym('arg1','clear');  
arg2 = sym('arg2','clear'); ...
```

`syms arg1 arg2 ... positive` is a shortcut for

```
arg1 = sym('arg1','positive');  
arg2 = sym('arg2','positive'); ...
```

Each input argument must begin with a letter and can contain only alphanumeric characters. For compatibility with previous versions of the software, `syms arg1 arg2 ... unreal` has exactly the same effect as `syms arg1 arg2 ... clear`.

In functions and scripts, do not use the `syms` command to create symbolic variables with the same names as MATLAB functions. For these names MATLAB does not create symbolic variables, but keeps the names assigned to the functions. If you want to create a symbolic variable with the same name as some MATLAB function inside a function or a script, use the `sym` command. For example:

```
alpha = sym('alpha')
```

## Examples

`syms x y real` is equivalent to

```
x = sym('x','real');  
y = sym('y','real');
```

To clear the symbolic objects `x` and `y` of 'real' status, type

```
syms x y clear
```

Note that `clear x` will *not* clear the symbolic object of its 'real' status. You can achieve this using

- `syms x clear` to remove the 'real' status from `x` without affecting any other symbolic variables.
- `reset(symengine)` to reset the MuPAD engine.
- `clear all` to clear all objects in the MATLAB workspace and resets the MuPAD engine.

## See Also

`findsym` | `reset` | `sym` | `symvar`

## How To

- “Clearing Assumptions and Resetting the Symbolic Engine” on page 4-51

**Purpose** Symbolic-to-numeric polynomial conversion

**Syntax** `c = sym2poly(s)`

**Description** `c = sym2poly(s)` returns a row vector containing the numeric coefficients of a symbolic polynomial. The coefficients are ordered in descending powers of the polynomial's independent variable. In other words, the vector's first entry contains the coefficient of the polynomial's highest term; the second entry, the coefficient of the second highest term; and so on.

**Examples** The command

```
syms x u v
sym2poly(x^3 - 2*x - 5)
```

returns

```
ans =
     1     0    -2    -5
```

The command

```
sym2poly(u^4 - 3 + 5*u^2)
```

returns

```
ans =
     1     0     5     0    -3
```

and the command

```
sym2poly(sin(pi/6)*v + exp(1)*v^2)
```

returns

```
ans =
  2.7183    0.5000         0
```

# sym2poly

---

## See Also

`poly2sym` | `subs` | `sym` | `polyval`



**Purpose** Evaluate symbolic sum of series

**Syntax**

```
r = symsum(expr)
r = symsum(expr, v)
r = symsum(expr, a, b)
r = symsum(expr, v, a, b)
```

**Description**

`r = symsum(expr)` evaluates the sum of the symbolic expression `expr` with respect to the default symbolic variable `defaultVar` determined by `symvar`. The value of the default variable changes from 0 to `defaultVar - 1`.

`r = symsum(expr, v)` evaluates the sum of the symbolic expression `expr` with respect to the symbolic variable `v`. The value of the variable `v` changes from 0 to `v - 1`.

`r = symsum(expr, a, b)` evaluates the sum of the symbolic expression `expr` with respect to the default symbolic variable `defaultVar` determined by `symvar`. The value of the default variable changes from `a` to `b`.

`r = symsum(expr, v, a, b)` evaluates the sum of the symbolic expression `expr` with respect to the symbolic variable `v`. The value of the default variable changes from `a` to `b`.

**Examples** Evaluate the sum of the following symbolic expressions `k` and `k^2`:

```
syms k
symsum(k)
symsum(k^2)
```

The results are

```
ans =
k^2/2 - k/2

ans =
k^3/3 - k^2/2 + k/6
```

---

Evaluate the sum of the following expression specifying the limits:

```
symsum(k^2, 0, 10)
```

The result is

```
ans =  
385
```

---

Evaluate the sum of the following multivariable expression with respect to k:

```
syms x;  
symsum(x^k/sym('k!'), k, 0, inf)
```

The result is

```
ans =  
exp(x)
```

## See Also

`int` | `syms` | `symvar`

## How To

- “Symbolic Summation” on page 3-18

**Purpose**

Find symbolic variables in symbolic expression or matrix

**Syntax**

```
symvar(s)
symvar(s,n)
```

**Description**

`symvar(s)` returns a vector containing all the symbolic variables in `s`. The variables are returned in the alphabetical order with uppercase letters preceding lowercase letters. If there are no symbolic variables in `s`, then `symvar` returns the empty vector. `symvar` does not consider the constants `pi`, `i`, and `j` to be variables.

`symvar(s,n)` returns a vector containing the `n` symbolic variables in `s` that are alphabetically closest to 'x':

- 1 The variables are sorted by the first letter in their names. The ordering is `x y w z v u ... a X Y W Z V U ... A`. The name of a symbolic variable cannot begin with a number.
- 2 For all subsequent letters, the ordering is alphabetical, with all uppercase letters having precedence over lowercase: `0 1 ... 9 A B ... Z a b ...z`.

---

**Note** `symvar(s)` can return variables in different order than `symvar(s,n)`.

---

**Examples**

```
syms wa wb wx yx ya yb
f = wa + wb + wx + ya + yb + yx;
symvar(f)
```

The result is:

```
ans =
[ wa, wb, wx, ya, yb, yx]
```

```
syms x y z a b
w = x^2/(sin(3*y - b));
```

```
symvar(w)
```

The result is:

```
ans =  
[ b, x, y]
```

```
symvar(w, 3)
```

The result is:

```
ans =  
[ x, y, b]
```

`symvar(s,1)` returns the variable closest to `x`. When performing differentiation, integration, substitution or solving equations, MATLAB uses this variable as a default variable.

```
syms v z  
g = v + z;  
symvar(g, 1)
```

The result is:

```
ans =  
z
```

```
syms aaa aab  
g = aaa + aab;  
symvar(g, 1)
```

The result is:

```
ans =  
aaa
```

```
syms X1 x2 xa xb  
g = X1 + x2 + xa + xb;  
symvar(g, 1)
```

The result is:

```
ans =  
x2
```

**See Also**

[findsym](#) | [sym](#) | [syms](#)

# taylor

---

<b>Purpose</b>	Taylor series expansion Taylor series expansion
<b>Syntax</b>	<code>taylor(f)</code> <code>taylor(f, n)</code> <code>taylor(f, a)</code> <code>taylor(f, n, v)</code> <code>taylor(f, n, v, a)</code>
<b>Description</b>	<p><code>taylor(f)</code> computes the Taylor series expansion of <code>f</code> up to the fifth order. The expansion point is 0 (Maclaurin series expansion).</p> <p><code>taylor(f, n)</code> computes the Taylor series expansion of <code>f</code> up to the <math>(n-1)</math>-order, where <code>n</code> is a positive integer. The expansion point is 0 (Maclaurin series expansion).</p> <p><code>taylor(f, a)</code> computes the Taylor series expansion of <code>f</code> up to the fifth order around the expansion point <code>a</code>. Here <code>a</code> is a real number. If <code>a</code> is a positive integer or if you want to change the truncation order, use <code>taylor(f, n, a)</code> to specify the base point and the truncation order.</p> <p><code>taylor(f, n, v)</code> computes the Taylor series expansion of <code>f</code> up to the <math>(n-1)</math>-order with respect to <code>v</code>. Here <code>f</code> is a symbolic expression representing a function, and <code>v</code> specifies the independent variable in the expression. <code>v</code> can be a string or symbolic variable. The expansion point is 0 (Maclaurin series expansion).</p> <p><code>taylor(f, n, v, a)</code> computes the Taylor series expansion of <code>f</code> around the expansion point <code>a</code>. The argument <code>a</code> can be a numeric value, a symbol, or a string representing a numeric value or an unknown. If <code>a</code> is a symbol or a string, do not omit <code>v</code>.</p> <p>If <code>a</code> is neither an integer nor a symbol or a string, you can supply the arguments <code>n</code>, <code>v</code>, and <code>a</code> in any order. <code>taylor</code> determines the purpose of the arguments from their position and type.</p> <p>You also can omit any of the arguments <code>n</code>, <code>v</code>, and <code>a</code>. If you do not specify <code>v</code>, <code>taylor</code> uses <code>symvar</code> to determine the function's independent variable. <code>n</code> defaults to 6, and <code>a</code> defaults to 0.</p>

The following expression represent the Taylor series for an analytic function  $f(x)$  about the base point  $x=a$ :

$$f(x) = \sum_{m=0}^{\infty} (x-a)^m \cdot \frac{f^{(m)}(a)}{m!}$$

### Examples

This table describes various uses of the `taylor` command and its relation to Maclaurin and Taylor series expansions. Before using the `taylor` command, define the function you want to expand. For example:

```
syms x
f = exp(x^2);
```

Mathematical Operation	MATLAB Operation
$\sum_{m=0}^5 x^m \cdot \frac{f^{(m)}(0)}{m!}$	<code>taylor(f)</code>
$\sum_{m=0}^{n-1} x^m \cdot \frac{f^{(m)}(0)}{m!}$ $n$ is a positive integer	<code>taylor(f,n)</code> $n$ is a positive integer.
$\sum_{m=0}^5 (x-a)^m \cdot \frac{f^{(m)}(a)}{m!}$ $a$ is a real number	<code>taylor(f,a)</code> $a$ is a real number.
$\sum_{m=0}^{n-1} (x-a)^m \cdot \frac{f^{(m)}(a)}{m!}$ $n$ is a positive integer and $a$ is real. Also, $a$ can be a positive integer.	<code>taylor(f,n,a)</code> $a$ is real and $n$ is a positive integer.

# taylor

In the case where  $f$  is a function of two or more variables ( $f=f(x, y, \dots)$ ), there is an additional parameter that allows you to select the variable for the Taylor expansion. Look at this table for illustrations of this feature.

Mathematical Operation	MATLAB Operation
$\sum_{m=0}^5 \frac{y^m}{m!} \cdot \frac{\partial^m}{\partial y^m} f(x, y) \Big _{y=0}$	<code>taylor(f, y)</code>
$\sum_{m=0}^{n-1} \frac{y^m}{m!} \cdot \frac{\partial^m}{\partial y^m} f(x, y) \Big _{y=0}$ <p><math>n</math> is a positive integer</p>	<code>taylor(f, y, n)</code> or <code>taylor(f, n, y)</code> $n$ is a positive integer.
$\sum_{m=0}^5 \frac{(y-a)^m}{m!} \cdot \frac{\partial^m}{\partial y^m} f(x, y) \Big _{y=a}$ <p><math>a</math> is real</p>	<code>taylor(f, y, a)</code> $a$ is real.
$\sum_{m=0}^{n-1} \frac{(y-a)^m}{m!} \cdot \frac{\partial^m}{\partial y^m} f(x, y) \Big _{y=a}$ <p><math>a</math> is real and <math>n</math> is a positive integer</p>	<code>taylor(f, n, y, a)</code> $a$ is real and $n$ is a positive integer.

## See Also

`symvar` | `taylorTool`



**Purpose** Taylor series calculator

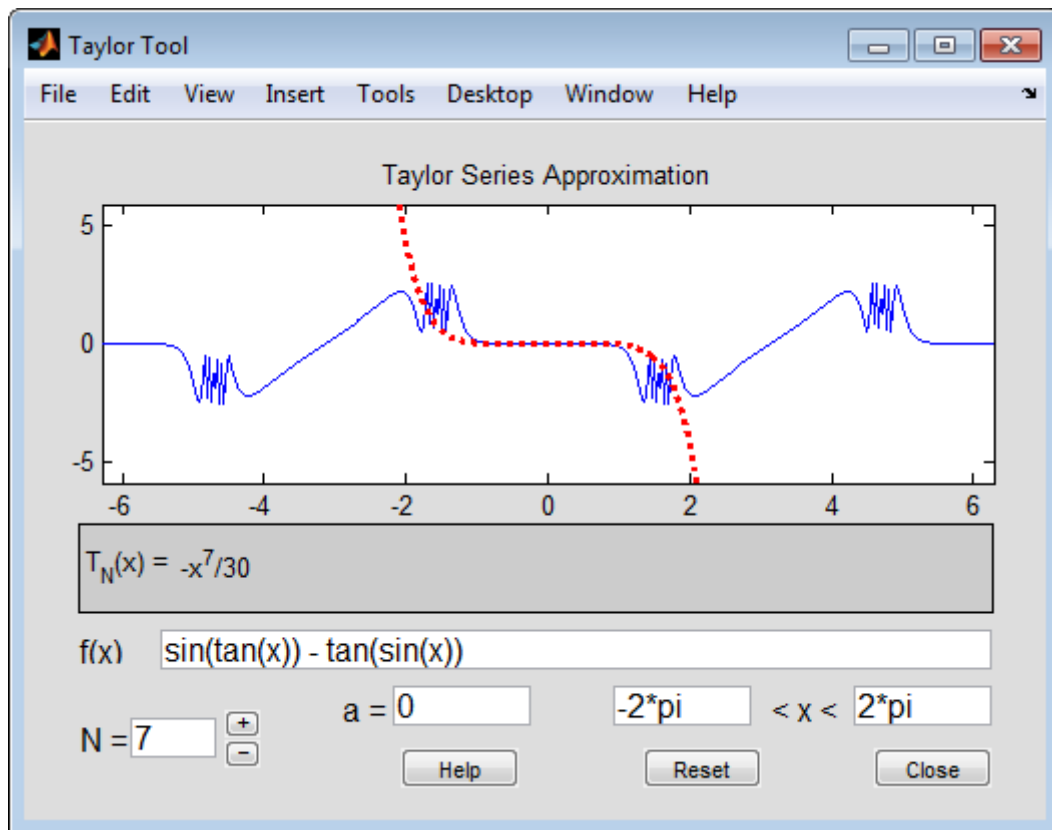
**Syntax** `taylor`tool  
`taylor`tool('f')

**Description** `taylor`tool initiates a GUI that graphs a function against the Nth partial sum of its Taylor series about a base point  $x = a$ . The default function, value of N, base point, and interval of computation for `taylor`tool are  $f = x \cdot \cos(x)$ ,  $N = 7$ ,  $a = 0$ , and  $[-2\pi, 2\pi]$ , respectively.

`taylor`tool('f') initiates the GUI for the given expression f.

**Examples** `taylor`tool('sin(tan(x)) - tan(sin(x))')

# taylortool



**See Also** [funtool](#) | [rsums](#)

**Purpose**

Enable and disable tracing of MuPAD commands

**Syntax**

```
trace(symengine, 'on')  
trace(symengine, 'off')
```

**Description**

`trace(symengine, 'on')` enables tracing of all subsequent MuPAD commands. Tracing means that for each command Symbolic Math Toolbox shows all internal calls to MuPAD functions and the results of these calls.

`trace(symengine, 'off')` disables MuPAD commands tracing.

**See Also**

`evalin` | `feval`

# tril

---

**Purpose** Return lower triangular part of symbolic matrix

**Syntax** `tril(A)`  
`tril(A, k)`

**Description** `tril(A)` returns a triangular matrix that retains the lower part of the matrix A. The upper triangle of the resulting matrix is padded with zeros.

`tril(A, k)` returns a matrix that retains the elements of A on and below the  $k$ -th diagonal. The elements above the  $k$ -th diagonal equal to zero. The values  $k = 0$ ,  $k > 0$ , and  $k < 0$  correspond to the main, superdiagonals, and subdiagonals, respectively.

**Examples** Display the matrix retaining only the lower triangle of the original symbolic matrix:

```
syms a b c
A = [a b c; 1 2 3; a + 1 b + 2 c + 3];
tril(A)
```

The result is:

```
ans =
[    a,    0,    0]
[    1,    2,    0]
[ a + 1, b + 2, c + 3]
```

---

Display the matrix that retains the elements of the original symbolic matrix on and below the first superdiagonal:

```
syms a b c
A = [a b c; 1 2 3; a + 1 b + 2 c + 3];
tril(A, 1)
```

The result is:

```
ans =  
[      a,      b,      0]  
[      1,      2,      3]  
[ a + 1, b + 2, c + 3]
```

---

Display the matrix that retains the elements of the original symbolic matrix on and below the first subdiagonal:

```
syms a b c  
A = [a b c; 1 2 3; a + 1 b + 2 c + 3];  
tril(A, -1)
```

The result is:

```
ans =  
[      0,      0, 0]  
[      1,      0, 0]  
[ a + 1, b + 2, 0]
```

## See Also

[diag](#) | [triu](#)

# triu

---

**Purpose** Return upper triangular part of symbolic matrix

**Syntax** `triu(A)`  
`triu(A, k)`

**Description** `triu(A)` returns a triangular matrix that retains the upper part of the matrix A. The lower triangle of the resulting matrix is padded with zeros.

`triu(A, k)` returns a matrix that retains the elements of A on and above the  $k$ -th diagonal. The elements below the  $k$ -th diagonal equal to zero. The values  $k = 0$ ,  $k > 0$ , and  $k < 0$  correspond to the main, superdiagonals, and subdiagonals, respectively.

**Examples** Display the matrix retaining only the upper triangle of the original symbolic matrix:

```
syms a b c
A = [a b c; 1 2 3; a + 1 b + 2 c + 3];
triu(A)
```

The result is:

```
ans =
[ a, b,    c]
[ 0, 2,    3]
[ 0, 0, c + 3]
```

---

Display the matrix that retains the elements of the original symbolic matrix on and above the first superdiagonal:

```
syms a b c
A = [a b c; 1 2 3; a + 1 b + 2 c + 3];
triu(A, 1)
```

The result is:

```
ans =  
[ 0, b, c]  
[ 0, 0, 3]  
[ 0, 0, 0]
```

---

Display the matrix that retains the elements of the original symbolic matrix on and above the first subdiagonal:

```
syms a b c  
A = [a b c; 1 2 3; a + 1 b + 2 c + 3];  
triu(A, -1)
```

The result is:

```
ans =  
[ a,      b,      c]  
[ 1,      2,      3]  
[ 0, b + 2, c + 3]
```

## See Also

[diag](#) | [tril](#)

# uint8

**Purpose** Convert symbolic matrix to unsigned integers

**Syntax** uint8(S)  
uint16(S)  
uint32(S)  
uint64(S)

**Description** uint8(S) converts a symbolic matrix S to a matrix of unsigned 8-bit integers.  
uint16(S) converts S to a matrix of unsigned 16-bit integers.  
uint32(S) converts S to a matrix of unsigned 32-bit integers.  
uint64(S) converts S to a matrix of unsigned 64-bit integers.

---

**Note** The output of uint8, uint16, uint32, and uint64 does not have type symbolic.

---

The following table summarizes the output of these four functions.

Function	Output Range	Output Type	Bytes per Element	Output Class
uint8	0 to 255	Unsigned 8-bit integer	1	uint8
uint16	0 to 65,535	Unsigned 16-bit integer	2	uint16
uint32	0 to 4,294,967,295	Unsigned 32-bit integer	4	uint32
uint64	0 to 18,446,744,073,709,551,615	Unsigned 64-bit integer	8	uint64

**See Also** sym | vpa | single | double | int8 | int16 | int32 | int64



<b>Purpose</b>	Variable-precision arithmetic
<b>Syntax</b>	R = vpa(A) R = vpa(A, d)
<b>Description</b>	R = vpa(A) uses variable-precision arithmetic (VPA) to compute each element of A to at least d decimal digits of accuracy, where d is the current setting of <code>digits</code> .  R = vpa(A, d) uses at least d significant (nonzero) digits, instead of the current setting of <code>digits</code> .
<b>Tips</b>	<ul style="list-style-type: none"> <li>The toolbox increases the internal precision of calculations by several digits (guard digits).</li> </ul>
<b>Input Arguments</b>	<p>A</p> <p>A symbolic object</p> <p>d</p> <p>An integer greater than 1 and smaller than <math>2^{29} + 1</math></p>
<b>Output Arguments</b>	<p>R</p> <p>A symbolic object representing a floating-point number</p>
<b>Examples</b>	<p>Approximate the following expressions with the 25 digits precision:</p> <pre>old = digits(25); q = vpa('1/2') p = vpa(pi) w = vpa('(1+sqrt(5))/2') digits(old)  q = 0.5  p =</pre>



---

The `vpa` function lets you specify the number of significant (nonzero) digits that is different from the current `digits` setting. For example, compute the ratio  $1/3$  and the ratio  $1/3000$  with 4 significant digits:

```
vpa(1/3, 4)
vpa(1/3000, 4)
```

```
ans =
0.3333
```

```
ans =
0.0003333
```

---

The number of digits that you specify by the `vpa` function or the `digits` function is the minimal number of digits. Internally, the toolbox can use more digits than you specify. These additional digits are called guard digits. For example, set the number of digits to 4, and then display the floating-point approximation of  $1/3$  using 4 digits:

```
old = digits;
digits(4);
a = vpa(1/3, 4)
```

```
a =
0.3333
```

Now, display `a` using 20 digits. The result shows that the toolbox internally used more than 4 digits when computing `a`. The last digits in the following result are incorrect because of the round-off error:

```
vpa(a, 20)
digits(old);
```

```
ans =
0.3333333333333303016843
```



'r', 'f', 'd' or 'e'. The default is 'r'. For example, convert the constant  $\pi=3.141592653589793\dots$  to a symbolic object:

```
r = sym(pi)
f = sym(pi, 'f')
d = sym(pi, 'd')
e = sym(pi, 'e')

r =
pi

f =
884279719003555/281474976710656

d =
3.1415926535897931159979634685442

e =
pi - (198*eps)/359
```

Compute these numbers with the 4 digits VPA precision. Three of the four numeric approximations give the same result:

```
vpa(r, 4)
vpa(f, 4)
vpa(d, 4)
vpa(e, 4)

ans =
3.142

ans =
3.142

ans =
3.142

ans =
```

```
3.142 - 0.5515*eps
```

Now, increase the VPA precision to 40 digits. The numeric approximation of 1/10 depends on the technique that you used to convert 1/10 to the symbolic object:

```
vpa(r, 40)
vpa(f, 40)
vpa(d, 40)
vpa(e, 40)
```

```
ans =
3.141592653589793238462643383279502884197
```

```
ans =
3.141592653589793115997963468544185161591
```

```
ans =
3.1415926535897931159979634685442
```

```
ans =
3.141592653589793238462643383279502884197 - ...
0.5515320334261838440111420612813370473538*eps
```

**See Also**

`digits` | `double`

**How To**

- “Variable-Precision Arithmetic” on page 3-48

**Purpose** Compute Riemann zeta function

**Syntax** `Y = zeta(X)`  
`Y = zeta(n, X)`

**Description** `Y = zeta(X)` evaluates the zeta function at the elements of `X`, a numeric matrix, or a symbolic matrix. The zeta function is defined by

$$\zeta(w) = \sum_{k=1}^{\infty} \frac{1}{k^w}$$

`Y = zeta(n, X)` returns the `n`-th derivative of `zeta(X)`.

**Examples** Compute the Riemann zeta function for the number:

```
zeta(1.5)
```

The result is:

```
ans =
    2.6124
```

Compute the Riemann zeta function for the matrix:

```
zeta(1.2:0.1:2.1)
```

The result is:

```
ans =
Columns 1 through 6

    5.5916    3.9319    3.1055    2.6124    2.2858    2.0543

Columns 7 through 10

    1.8822    1.7497    1.6449    1.5602
```

Compute the Riemann zeta function for the matrix of the symbolic expressions:

```
syms x y;  
zeta([x 2; 4 x + y])
```

The result is:

```
ans =  
[ zeta(x),      pi^2/6]  
[ pi^4/90, zeta(x + y)]
```

Differentiate the Riemann zeta function:

```
diff(zeta(x), x, 3)
```

The result is:

```
ans =  
zeta(3, x)
```



	z-transform
<b>Purpose</b>	z-transform
<b>Syntax</b>	$F = \text{ztrans}(f)$ $F = \text{ztrans}(f, w)$ $F = \text{ztrans}(f, k, w)$
<b>Description</b>	<p><math>F = \text{ztrans}(f)</math> computes the <math>z</math>-transform of the symbolic expression <math>f</math>. This syntax assumes that <math>f</math> is a function of the variable <math>n</math>, and the returned value <math>F</math> as a function of <math>z</math>.</p>

If  $f = f(z)$ , then  $\text{ztrans}(f)$  returns a function of  $w$ .

$$F = F(w)$$

$$f = f(n) \Rightarrow F = F(z)$$

By definition, the  $z$ -transform is

$$F(z) = \sum_{n=0}^{\infty} \frac{f(n)}{z^n}$$

$F = \text{ztrans}(f, w)$  computes the  $z$ -transform  $F$  as a function of  $w$  instead of the default variable  $z$ .

$$F(w) = \sum_{n=0}^{\infty} \frac{f(n)}{w^n}$$

$F = \text{ztrans}(f, k, w)$  computes the  $z$ -transform and lets you specify that  $f$  is a function of  $k$  and  $L$  is a function of  $w$ .

$$F(w) = \sum_{k=0}^{\infty} \frac{f(k)}{w^k}$$

## Examples

Z-Transform	MATLAB Operation
$f(n) = n^4$ $Z[f] = \sum_{n=0}^{\infty} f(n)z^{-n}$ $= \frac{z(z^3 + 11z^2 + 11z + 1)}{(z-1)^5}$	<pre>syms n; f = n^4; ztrans(f)  ans = (z^4 + 11*z^3 + 11*z^2 + z)/(z - 1)^5</pre>
$g(z) = a^z$ $Z[g] = \sum_{z=0}^{\infty} g(z)w^{-z}$ $= \frac{w}{w-a}$	<pre>syms a z; g = a^z; ztrans(g)  ans = -w/(a - w)</pre>
$f(n) = \sin(an)$ $Z[f] = \sum_{n=0}^{\infty} f(n)w^{-n}$ $= \frac{w \sin a}{1 - 2w \cos a + w^2}$	<pre>syms a n w; f = sin(a*n); ztrans(f, w)  ans = (w*sin(a))/(w^2 - 2*cos(a)*w + 1)</pre>

## See Also

fourier | iztrans | laplace

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